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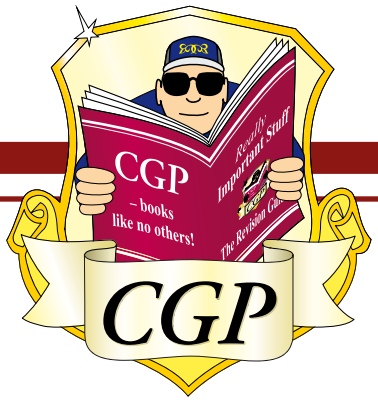

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Mathematics

For the Grade 9-1 Course

The Workbook

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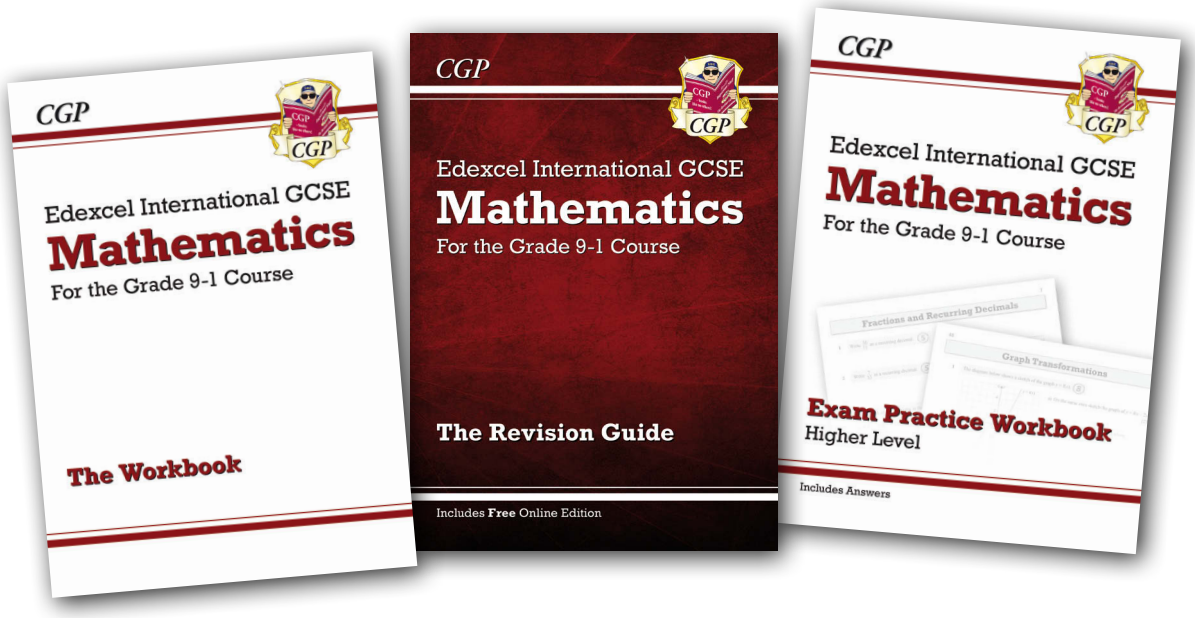

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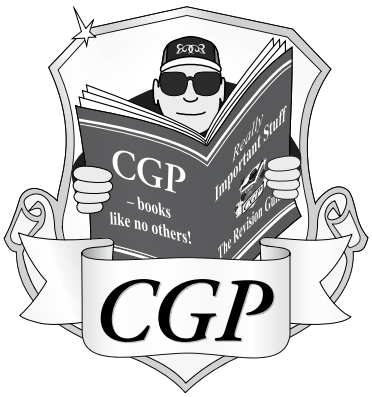

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Contents

Section One — Numbers

Section Two — Algebra

Section Three — Graphs, Functions and Calculus

Section Four — Geometry and Measure

Section Five — Pythagoras and Trigonometry

Section Six — Statistics and Probability

Throughout the book, the more challenging questions are marked like this: **Q1**

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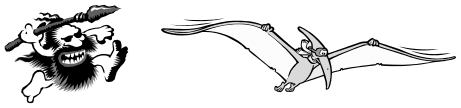
Order of Operations

Nothing weird here. Just a load of sums to do — while you're at it remember BODMAS. Both Ocelots Demand Myriad Antique Similes. Hang on, that's not right... Brackets, Other, Division, Multiplication, Addition, Subtraction. That's better.

Q1 Calculate the following. Give your answers to 2 d.p. where necessary.

a) $6 + 4 \div 2$
b) $(6 + 4) \div 2$
c) $(2 \times 6) - (7 \div 3)^2$
d) $2 \times 6 - 7 \div 3^2$
e) $8 \div (9 \times 3^2) - 1$
f) $4 \times 4^2 + (4 \times 4)^2 - (4 \div 4)^3 \div 4^3$
g) $7 \div (6 \div 3) + 2$
h) $10^3 - (-5)^2 + (64 \div 8)$
i) $(3 \times 3) + 24 \div (2 - 14)^2$
j) $11 \times (77 \div 7) + 121 - 2 \times (10 + 1)^2$



Q2 Calculate the following. Give your answers to 2 d.p. where necessary.

a) $10 + \sqrt{16} - (42 \div 7)$
b) $\sqrt{36} \div 2 + (5 \times 14)$
c) $91 + (\sqrt{289} \times 2) - 12$
d) $\sqrt{1} \times 42 \div 6$
e) $2 + (\sqrt{81} + 3)^2 \div 7$
f) $(\sqrt{400} - 18)^3 + (4 \div 6)$
g) $\sqrt{9} \div 3 \times 2 - 1$
h) $\sqrt{7} + \sqrt{5} - (\sqrt{3} \times 2)$
i) $\sqrt{626} - (4 + 56 \div 3)^2$
j) $(12 - \sqrt{4912}) \div 15^2$

Q3 Calculate the following. Give your answers to 2 d.p. where necessary.

a) $\dfrac{21}{2+5}$
b) $\dfrac{3^2}{9+9^2}$
c) $\dfrac{\sqrt{16}}{3 \div 12}$
d) $\dfrac{5^2}{\sqrt{49}-4}$
e) $\dfrac{1-\sqrt{17}}{\sqrt{45}+22^2}$
f) $\dfrac{18^2}{2+\sqrt[3]{17}}$
g) $\dfrac{15^2-2^3}{\sqrt{12}-\sqrt{22}}$
h) $\dfrac{4 \times \sqrt[3]{3}}{24+\sqrt[3]{30}}$
i) $\dfrac{14+(3 \div 7)}{(3^3-\sqrt{5})}$
j) $\dfrac{\sqrt[3]{177}-\sqrt{2}}{\sqrt{15} \times 4}$
k) $\dfrac{\sqrt{(81 \div 4)}}{7^3-(31 \times 0.3)}$
l) $\sqrt{\dfrac{7}{4^3}} - (53 \div 2^4)^2$

Types of Number

There are a few special number sequences that you really need to know — SQUARE, CUBE and PRIME NUMBERS, as well as POWERS and ODD and EVEN NUMBERS.

Q1 Jon thinks of a number between 1 and 10. The cube of the number is equal to the square of double the number. What is the number Jon is thinking of?

Q2 On a certain day the temperature at midday was 14°C. By midnight the temperature had fallen by 17°C. What was the temperature at midnight?

Q3 Which of these are rational and which are irrational?

a) $\frac{6}{2}$ **b)** $\sqrt{16}$ **c)** $\sqrt{5}$ **d)** $\frac{3}{8}$ **e)** $\sqrt[3]{25}$ **f)** 0.2413

Q4 1 is the first odd number. It is also the first square number and the first cube number. Which is greater:

a) the third odd number, the third square number or the third cube number?
b) the sixth odd number, the fourth square number or the second cube number?

Q5 Using any or all of the figures **1, 2, 5, 9** write down:

a) the smallest prime number
b) a prime number greater than 20
c) a prime number between 10 and 20
d) two prime numbers whose sum is 21
e) a number that is not prime.

Remember — 1 is not a prime.
Look, it just isn't, OK.

Q6 **a)** In the ten by ten square opposite, ring all the prime numbers. (The first three have been done for you.)

b) Among the prime numbers between 10 and 100, find three which are still prime when their digits are reversed.

c) Give a reason for 27 not being a prime number.

1	(2)	(3)	4	(5)	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Q7 What is the largest prime less than 120?

Q8 How many prime numbers are even?

Square Roots and Cube Roots

Square root just means "WHAT NUMBER TIMES ITSELF (e.g. 2×2) GIVES..."
The square roots of 64 are 8 and –8 because 8×8=64 and -8×-8=64.
Cube root means "WHAT NUMBER TIMES ITSELF TWICE (e.g. 2×2×2) GIVES ..."
The cube root of 27 is 3 because 3×3×3=27.
Square roots always have a + and – answer, cube roots only have 1 answer.

Q1 Use the $\sqrt{\ }$ button on your calculator to find the following positive square roots to 1 d.p.

a) $\sqrt{60}$ **e)** $\sqrt{520}$ **i)** $\sqrt{170}$

b) $\sqrt{19}$ **f)** $\sqrt{75}$ **j)** $\sqrt{7220}$

c) $\sqrt{34}$ **g)** $\sqrt{750}$ **k)** $\sqrt{1\,000\,050}$

d) $\sqrt{200}$ **h)** $\sqrt{0.9}$ **l)** $\sqrt{27}$

Q2 Without using a calculator, write down both answers to each of the following:

a) $\sqrt{4}$ **d)** $\sqrt{49}$ **g)** $\sqrt{144}$

b) $\sqrt{16}$ **e)** $\sqrt{25}$ **h)** $\sqrt{64}$

c) $\sqrt{9}$ **f)** $\sqrt{100}$ **i)** $\sqrt{81}$

Q3 Use your calculator to find the following:

a) $\sqrt[3]{4096}$ **c)** $\sqrt[3]{1331}$ **e)** $\sqrt[3]{1}$

b) $\sqrt[3]{1728}$ **d)** $\sqrt[3]{1\,000\,000}$ **f)** $\sqrt[3]{0.125}$

Q4 Without using a calculator, find the value of the following:

a) $\sqrt[3]{64}$ **c)** $\sqrt[3]{125}$ **e)** $\sqrt[3]{216}$

b) $\sqrt[3]{512}$ **d)** $\sqrt[3]{1000}$ **f)** $\sqrt[3]{8000}$

Q5 Nida is buying a small storage box online. She sees a cube box with a volume of 343 cm^3. What is the length of each box edge?

Q6 A farmer is buying fencing to surround a square field of area 3600 m^2. What length of fencing does he need to buy?

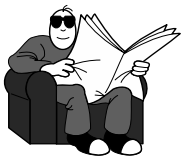
Q7 Sarah thinks of a number. She calculates that the square of the number is 256. What is the square root of the number?

Multiples, Factors and Prime Factors

Q1 1 3 6 9 12

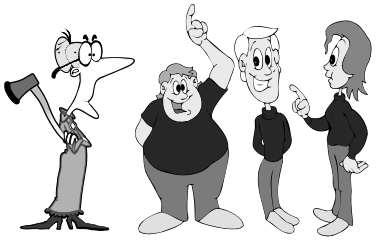
From the numbers above, write down:

a) a multiple of 4
b) the prime number
c) two square numbers
d) three factors of 27
e) two numbers, P and Q, that satisfy both $P = 2Q$ and $P = \sqrt{144}$



This is real basic stuff — you just have to know your times tables. And your primes, of course...

Q2 48 students went on a geography field trip
Their teachers split them into equal groups
Suggest five different ways that the teachers might have split up the students.



Q3 A school ran 3 evening classes: Conversational French, Cake Making and Woodturning. The Conversational French class had 29 students, Cake Making had 27 students, and the Woodturning class had 23. For which classes did the teacher have difficulty dividing the students into equal groups?

Q4 **a)** Write down the first five cube numbers.
b) Which of the numbers given in part **a)** are multiples of 2?
c) Which of the numbers given in part **a)** are multiples of 3?
d) Which of the numbers given in part **a)** are multiples of 4?
e) Which of the numbers given in part **a)** are multiples of 5?

Q5 Write down the prime factorisation of:

a) 18
b) 140
c) 47

The tricky bit is remembering that a prime factorisation includes all the prime factors that multiply to make that number — so you've got to repeat some of them.

Q6 **a)** List the first five prime numbers.
b) If added together, what is their total?
c) Write down the prime factorisation of the answer to part **b)**.

Q7 **a)** List the first five odd numbers.
b) If added together, what is their total?
c) Write down the prime factorisation of the answer to part **b)**.

Multiples, Factors and Prime Factors

Q8 The prime factor decomposition of a certain number is $3^2 \times 5 \times 11$.

a) Write down the number.

b) Write down the prime factor decomposition of 165.

Q9 **a)** Write down the first ten square numbers.

b) From your list, pick out all the multiples of 2.

c) From your list, pick out all the multiples of 3.

d) From your list, pick out any cube numbers.

e) Add the numbers in your list together and write down the prime factor decomposition of the total.

Q10 Gordon is doing some woodwork and needs to calculate the volume of a wooden rectangular block (a cuboid). The length of the block is 50 cm, the height 25 cm and the width 16 cm.

a) What is the volume (in cm^3) of the wooden block?

b) What is the prime factorisation of the number found in part **a)**?

c) Gordon needs to cut the block into smaller blocks with dimensions 4 cm × 5 cm × 5 cm. What is the maximum number of small blocks Gordon can make from the larger block? Make sure you show all your working.

Q11 The prime factor decomposition of a certain number is $2^3 \times 5 \times 17$.

a) What is the number?

b) What is the prime factor decomposition of half of this number?

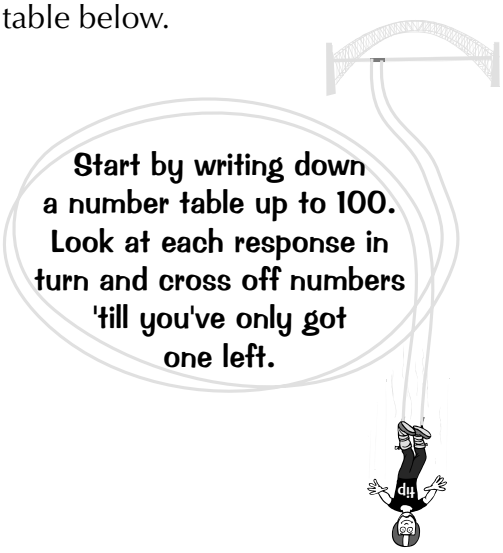
c) What is the prime factor decomposition of a quarter of the number?

d) What is the prime factor decomposition of an eighth of the number?

Q12 Bryan and Sue were playing a guessing game. Sue thought of a number between 1 and 100 which Bryan had to guess. Bryan was allowed to ask five questions, which are listed with Sue's responses in the table below.

Bryan's Questions	Sue's Responses
Is it prime?	No
Is it odd?	No
Is it less than 50?	Yes
Is it a multiple of 3?	Yes
Is it a multiple of 7?	Yes

What is the number that Sue thought of?



LCM and HCF

LCM is the Lowest Common Multiple and HCF is the Highest Common Factor. The basic method is to list the multiples or factors of each number and pick out the answer. But sometimes the question will ask you to use prime factors to calculate the LCM or HCF — so make sure you can use this method too.

Q1 **a)** List the first ten multiples of 6, starting at 6.
b) List the first ten multiples of 5, starting at 5.
c) What is the LCM of 5 and 6?

Q2 **a)** List all the factors of 30.
b) List all the factors of 48.
c) What is the HCF of 30 and 48?

Q3 For each set of numbers find the HCF.

a) 40, 60
b) 10, 40, 60
c) 10, 24, 40, 60
d) 15, 45
e) 15, 30, 45
f) 15, 20, 30, 45
g) 32, 64
h) 32, 48, 64
i) 16, 32, 48, 64

Q4 For each set of numbers find the LCM.

a) 40, 60
b) 10, 40, 60
c) 10, 24, 40, 60
d) 15, 45
e) 15, 30, 45
f) 15, 20, 30, 45
g) 32, 64
h) 32, 48, 64
i) 16, 32, 48, 64

Q5 **a)** Express 15 and 18 as the product of their prime factors.
b) Using your answer to part **a)**, find the LCM of 15 and 18.

Q6 **a)** Express 90 and 120 as the product of their prime factors.
b) Using your answer to part **a)**, find the HCF of 90 and 120.

Q7 **a)** Given that $48 = 2^4 \times 3$ and $72 = 2^3 \times 3^2$, find the LCM of 48 and 72.
b) Given that $60 = 2^2 \times 3 \times 5$ and $126 = 2 \times 3^2 \times 7$, find the LCM of 60 and 126.

Q8 **a)** Given that $120 = 2^3 \times 3 \times 5$ and $150 = 2 \times 3 \times 5^2$, find the HCF of 120 and 150.
b) Given that $140 = 2^2 \times 5 \times 7$ and $600 = 2^3 \times 3 \times 5^2$, find the HCF of 140 and 600.

Q9 Lars, Rita and Alan regularly go swimming. Lars goes every 2 days, Rita goes every 3 days and Alan goes every 5 days. They <u>all</u> went swimming together on Friday 1st June.
a) On what <u>date</u> will Lars and Rita next go swimming together?
b) On what <u>date</u> will Rita and Alan next go swimming together?
c) On what <u>day of the week</u> will all three next go swimming together?
d) Which of the three (if any) will go swimming on 15th June?

This is just a LCM question in disguise.

Q10 Maggie has a lot of toy birds — 36 parrots, 42 puffins and 84 penguins. She wants to give them all away to some friends so that they each receive exactly the same number of toys, but they each get only one type of bird. What is the least number of friends she can give the toys away to?

Fractions

Answer the following questions without using a calculator.

Q1 Carry out the following multiplications, giving your answers in their lowest terms:

a) $\frac{1}{8} \times \frac{1}{8}$ **c)** $\frac{3}{18} \times \frac{1}{3}$ **e)** $1\frac{1}{4} \times 4\frac{1}{8}$

b) $\frac{2}{3} \times \frac{1}{6}$ **d)** $1\frac{1}{4} \times 3\frac{1}{8}$ **f)** $\frac{9}{10} \times \frac{9}{100} \times \frac{1}{100}$

Q2 Carry out the following divisions, giving your answers in their lowest terms:

a) $\frac{1}{8} \div \frac{1}{8}$ **c)** $\frac{3}{18} \div \frac{1}{3}$ **e)** $1\frac{1}{4} \div 4\frac{1}{8}$

b) $\frac{2}{3} \div \frac{1}{6}$ **d)** $1\frac{1}{4} \div 3\frac{1}{8}$ **f)** $\left(\frac{9}{10} \div \frac{9}{100}\right) \div \frac{1}{100}$

Q3 Evaluate the following, giving your answers in their lowest terms:

a) $\frac{1}{8} + \frac{1}{8}$ **c)** $\frac{3}{18} + \frac{1}{3}$ **e)** $1\frac{1}{4} + 4\frac{1}{8}$

b) $\frac{1}{6} + \frac{2}{3}$ **d)** $1\frac{1}{4} + 3\frac{1}{8}$ **f)** $\frac{9}{10} + \frac{9}{100} + \frac{1}{100}$

Q4 Caley is making some punch for her birthday party. She mixes $\frac{1}{2}$ litre of cranberry juice, $1\frac{1}{2}$ litres of apple juice, $\frac{2}{3}$ litre of orange juice and $\frac{4}{5}$ litres of pineapple juice.
She has a bowl that will hold 4 litres. Will this be big enough to contain all of the punch?

Q5 Evaluate the following, giving your answers in their lowest terms:

a) $\frac{1}{8} - \frac{1}{8}$ **c)** $\frac{3}{18} - \frac{1}{3}$ **e)** $1\frac{1}{8} - 4\frac{1}{4}$

b) $\frac{2}{3} - \frac{1}{6}$ **d)** $3\frac{1}{8} - 1\frac{1}{4}$ **f)** $\left(\frac{9}{10} - \frac{9}{100}\right) - \frac{1}{100}$

Q6 Evaluate the following, giving your answers in their lowest terms:

a) $\frac{1}{2} + \frac{1}{4}$ **e)** $6 \times \frac{2}{3}$ **i)** $3 + \frac{8}{5}$

b) $\frac{2}{3} - \frac{1}{4}$ **f)** $\frac{4}{5} \div \frac{2}{3}$ **j)** $\frac{2}{3}\left(\frac{3}{4} + \frac{4}{5}\right)$

c) $\frac{1}{5} + \frac{2}{3} - \frac{2}{5}$ **g)** $\frac{5}{12} \times \frac{3}{2}$ **k)** $\left(\frac{1}{7} + \frac{3}{14}\right) \times \left(3 - \frac{1}{5}\right)$

d) $5 - \frac{1}{4}$ **h)** $\frac{5}{6} - \frac{7}{8}$ **l)** $\left(\frac{3}{4} - \frac{1}{5}\right) \div \left(\frac{7}{8} + \frac{1}{16}\right)$

Fractions

The cunning bit with long wordy questions is picking out the important bits and then translating them into numbers. It's not that easy at first, but you'll get better — I guess you've just gotta learn to ignore the waffly stuff.

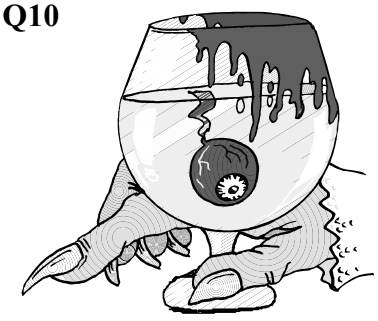
Answer these without using your calculator:

Q7 What fraction of 1 hour is:
- **a)** 5 minutes
- **b)** 15 minutes
- **c)** 40 minutes?

Q8 If a TV programme lasts 40 minutes, what fraction of the programme is left after:
- **a)** 10 minutes
- **b)** 15 minutes
- **c)** 35 minutes?

Q9 A café employs eighteen girls and twelve boys to wait at tables. Another six boys and nine girls work in the kitchen.
What fraction of the kitchen staff are girls?
What fraction of the employees are boys?

Q10



In a survey, people were asked if they liked a new cola drink. One in five thought it was great, four out of fifteen felt there was no difference in taste, three in ten disliked it and the rest offered no opinion.
What fraction of people offered no opinion?

Forget all about cola drinks and red trousers — just write it all as a sum, then do the calculation. Nowt to it.

Q11 Neil wore red trousers on a total of 12 days in November.
- **a)** On what fraction of the total number of days in November did Neil wear red trousers?
- **b)** For 1/5 of the days in November Neil wore a blue shirt. How many days is this?

Q12

The Sandwich Club of Great Britain are going on their annual picnic.
- **a)** The boxes they use to transport their sandwiches are 10 inches high and are the width of a single sandwich. Each sandwich is 5/8 inch thick. How many boxes will they need for 80 sandwiches?
- **b)** How tall would the box need to be if 40 sandwiches were to be stacked inside?

Fractions

You can use your calculator for these.

Q13 The population of Australia was 18 million in 1995, of which 3.5 million people lived in Sydney and 1 million people lived in Perth.

a) What fraction of the population lived in Perth?

b) What fraction of the population lived in Perth or Sydney?

This stuff is pretty easy — but you don't want to go making silly mistakes in the exam. Get the practice in now.

Q14 Green Island is split into six regions A, B, C, D, E and F. The areas of the six regions are 12, 2, 3, 18, 4, and 9 km^2 respectively.

a) What is the total area of the island?

b) What fraction of the island's area is taken up by the two largest regions?

Q15 In a consumer survey, 100 people stated their favourite vegetable. 25 people chose peas, 35 carrots and 32 runner beans.

a) How many of the 100 people chose a vegetable other than peas, carrots or runner beans?

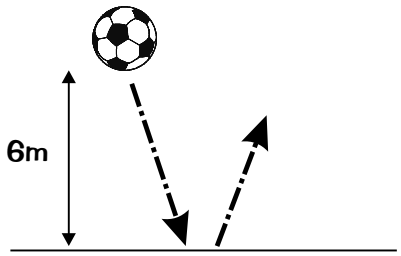
b) What fraction of the 100 people chose carrots as their favourite vegetable?

c) What fraction of the 100 people chose peas as their favourite vegetable?

d) At least how many people chose a green vegetable as their favourite?

e) Not more than how many people chose a green vegetable as their favourite?

Q16 A ball is dropped from a height of 6 m.



After each bounce the ball rises to 2/3 of its previous height. What height will it reach after the third bounce?

Q17 George wants to make a cake. The recipe requires 150 g each of flour, sugar and butter, and 3 eggs. George only has 2 eggs so he decides to make a smaller cake with the same proportions.

a) How much flour will George need to use?

b) If each egg weighs 25 g, how much will the cake weigh before it goes in the oven?

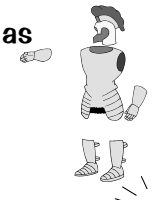
c) What fraction of the uncooked weight is flour?

d) If the cake loses 1/7 of its weight during baking (due to moisture loss) what will it weigh after baking?

Q18 Jenny goes shopping. She gets 1/3 off a bag priced £28, 1/5 off a dress priced £62 and 2/3 off a hat priced £14. How much money does she save?

Fractions, Decimals and Percentages

I reckon that converting decimals to percentages is about as easy as it gets — so make the most of it.



Q1 Express each of the following as a percentage:

a) 0.25 **b)** 0.5 **c)** 0.75 **d)** 0.1 **e)** 0.4152 **f)** 0.8406 **g)** 0.3962 **h)** 0.2828

All you're doing is multiplying by 100 — it really couldn't be easier.

Q2 Express each percentage as a decimal:

a) 50% **b)** 12% **c)** 40% **d)** 34% **e)** 60.2% **f)** 54.9% **g)** 43.1% **h)** 78.8%

Now you're dividing by 100 — so just move the decimal point to the left.

Q3 Express each of the following as a percentage:

a) $\frac{1}{2}$ **b)** $\frac{1}{4}$ **c)** $\frac{1}{8}$ **d)** $\frac{3}{4}$ **e)** $\frac{1}{25}$ **f)** $\frac{2}{3}$ **g)** $\frac{4}{15}$ **h)** $\frac{2}{7}$

Q4 Express each percentage as a fraction in its lowest terms:

a) 25% **b)** 60% **c)** 45% **d)** 30% **e)** 8.2% **f)** 49.6% **g)** 88.6% **h)** 32.4%

Best thing to do with e)-h) is to put them over 100, then get rid of the decimal point by multiplying top and bottom by 10. Then just cancel down as normal.

Q5 119 out of 140 houses on an estate have DVD players. What percentage is this?

Q6 In an exam Tina scored 52/80. The grade she receives depends on the percentage scored. What grade will Tina get?

Grades	
51-60%	5
61-70%	6
71-80%	7
81-90%	8
91-100%	9

Fractions, Decimals and Percentages

Decimals are just another way of writing fractions — so it's easy to convert between the two...

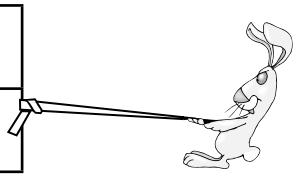
Q7 Without using a calculator, write the following fractions as decimals:

a) $\frac{3}{10}$ **b)** $\frac{37}{100}$ **c)** $\frac{2}{5}$ **d)** $\frac{3}{8}$

e) $\frac{14}{8}$ **f)** $\frac{8}{64}$ **g)** $\frac{24}{40}$ **h)** $\frac{4}{80}$

Q8 Fill in the gaps in the following conversion table.

Decimal	0.5		0.125	1.6				0.45
Fraction	$\frac{1}{2}$	$\frac{1}{5}$			$\frac{1}{4}$	$\frac{7}{2}$	$\frac{3}{20}$	



Q9 Write the following fractions as recurring decimals:

a) $\frac{5}{6}$ **b)** $\frac{7}{9}$ **c)** $\frac{7}{11}$ **d)** $\frac{47}{99}$

e) $\frac{10}{11}$ **f)** $\frac{29}{63}$ **g)** $\frac{478}{999}$ **h)** $\frac{5891}{9999}$

Q10 Write the following decimals as fractions in their simplest form:

a) 0.6 **b)** 0.75 **c)** 0.95 **d)** 0.128

e) $0.\dot{3}$ **f)** $0.\dot{6}$ **g)** $0.\dot{1}$ **h)** $0.\dot{1}\dot{6}$

Q11 Write the following recurring decimals as fractions in their simplest form:

a) 0.222... **b)** 0.444... **c)** 0.888... **d)** 0.808080...

e) 0.121212... **f)** 0.545545545... **g)** 0.753753753... **h)** 0.156156156...

Percentages

Finding "something %" of "something-else" is really quite simple — so you'd better be sure you know how. You also need to be able to give "something" as a percentage of "something else". Try this mix for size...

Q1 Find:

a) 8% of £16 **b)** 85% of 740 kg **c)** 40% of 40 minutes

Q2 Express each percentage as a decimal:

a) 20% **b)** 35% **c)** 2% **d)** 62.5%

Q3 Express each percentage as a fraction in its lowest terms:

a) 20% **b)** 3% **c)** 70% **d)** 84.2%

Q4 Express each of the following as a percentage:

a) $\frac{1}{8}$ **b)** 0.23 **c)** $\frac{12}{40}$ **d)** 0.34

Q5 In a French test, Lauren scored 17/20. What percentage is this?

Q6 87 out of 120 pupils at Backwater School have access to a computer. What percentage is this?

Q7 Donald earns an annual wage of £23 500. He doesn't pay tax on the first £6400 that he earns. How much income tax does he pay a year if the rate of tax is:

a) 25%

b) 40%?

Q8 There are approximately 6000 fish and chip shops in the UK. On average, a fish and chip shop gets about 160 visitors each day. Given that the population of the UK is roughly 60 million, approximately what percentage of the population visit a fish and chip shop each day?

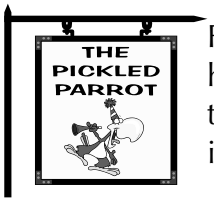
Q9 At birth, Veronica was 0.3 m tall. By adulthood she had grown to 1.5 m tall. Calculate her height now as a percentage of her height at birth.

Percentages

These questions are simple too — you just need to "find the new amount after a % increase or decrease". Once you've worked out the percentage, remember to add it back on to (or subtract it from) the original amount.

Q10 John bought a new TV. The tag in the shop said it cost £299 + VAT.
If VAT is charged at 20%, how much did he pay (to the nearest penny)?

Q11



Four friends stay at the Pickled Parrot Hotel for a night and each have an evening meal. Bed and Breakfast costs £37 per person and the evening meal costs £15 per person. How much is the total cost, if a service charge is added at 17½%?

Q12 The owners of a museum are expecting a 14% increase in visitors next year.
This year they had 20 200 visitors.
How many visitors should they expect next year?

Q13 Tim is choosing between two cars to buy.
The first car is priced at £8495 and has 15% off.
The second car is priced at £8195 and has 12% off.
Which car is the cheapest? Show your working.

Q14 Tanya paid £6500 for her new car. Each year its value decreased by 8%.
How much was it worth when it was one year old?

Q15 Jeremy wanted a new sofa for his lounge. A local furniture shop had just what he was looking for — and for only £130.00 + delivery. Jeremy had £150 pounds in his bank account. If delivery was charged at 17½%, could Jeremy afford the sofa?

Now it's time for some trickier types of percentage question. First up is finding the percentage change. Don't forget to find the difference in values first.

Q16 During a rainstorm, a water butt increased in weight from 10.4 kg to 13.6 kg.
What was the percentage increase (to the nearest percent)?

Q17 An electrical store reduces the price of a particular camera from £90.00 to £78.30.
What is the percentage reduction?

Percentages

Q18 Desmond's GCSE maths exam is next week. As part of his revision he attempted 31 questions on his least favourite topic of percentages. He got 21 questions fully right on the first attempt. Two days later he tried all 31 questions again and this time got 29 correct.

a) What percentage of questions did he get correct on his first attempt?

b) What percentage of questions did he get correct on his second attempt?

c) What is the percentage improvement in Desmond's results?

Finding the original value always looks a bit confusing at first. The bit most people get wrong is deciding whether the value given represents more or less than 100% of the original — so always check your answer makes sense.

Q19 In the new year sales Robin bought a tennis racket for £68.00. The original price had been reduced by 15%. What was the original price?

Q20 There are 360 people living in a certain village. The population of the village has grown by 20% over the past year.

a) How many people lived in the village one year ago?

b) If the village continues to grow at the same rate, how many whole years from today will it be before the population is more than twice its current size?

Ooh... here's an interest question. This one's just about simple interest (don't fret, there's heaps on compound interest on the next page).

Q21 I invest £1000 for a period of three years in an account paying 7.875% per annum. At the end of each year I withdraw the interest. Calculate the total interest I will receive over the three years.

Some percentage questions can be a bit weird and wonderful, so you'll need to get your thinking cap on. But don't panic — as long as you remember all your percentage skills, you'll be fine.

Q22 If $L = MN$, what is the percentage increase in L if M increases by 15% and N increases by 20%?

Q23 An electrical shop buys a stereo from a wholesaler for £x and increases its price by 35% to make a profit. When the stereo doesn't sell, the shop reduces its price by 20%. If the stereo now sells, calculate the shop's overall percentage profit.

Q24 A couple bought their house 2 years ago for £y. In the first year, house prices in their area rose by 10%. In the second year, house prices in their area fell by 5%. Calculate the percentage profit they would make if they sold their house now.

Interest and Depreciation

You may not have a lot of interest in this page, but it will be useful to invest some time in it. Ah-ha!

Q1 A financial advisor is asked to calculate the future value of his clients' investments. Calculate the amount in each of these accounts if:

a) £200 is invested for 10 years at 9% compound interest per annum
b) £500 is invested for 3 years at 7% compound interest per annum
c) £750 is invested for 6 months at 8% compound interest per annum
d) £1000 is invested for 12 months at 6.5% compound interest per annum.

Q2 Mrs Smith decides to invest £7000 in a savings account. She has the choice of putting all her money into an account paying 5% compound interest per annum or she can put half of her investment into an account paying 6% compound interest per annum and the remaining half into an account paying 4% compound interest per annum. If she left the investment alone for 3 years, which is her best option and by how much?

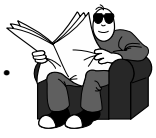
Q3 A used car salesman is buying stock at an auction. Before the auction, he estimates the value of each car on offer using their original price, their age, and a depreciation of 14% each year. This value is the maximum amount he will bid for each car. Calculate the maximum amount he should bid on these used cars:

a) a car which cost £8495 six months ago
b) a car which cost £34 000 eighteen months ago
c) a car which cost £13 495 two years ago
d) a car which cost £14 395 two years ago
e) a car which cost £11 295 three years ago
f) a car which cost £6795 twelve months ago.

Q4 Julia is opening a bank account. There are three options available — an account that pays 4% compound interest per year, an account that pays 5% simple interest per year and an account that pays £5 per month. Which should she choose if she invests:

a) £1000 for 1 year?
b) £3000 for 10 years?
c) £2200 for 25 years?
Show your workings for each answer.

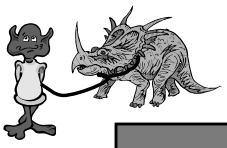
I'd put my money in Victorian rolling pins, myself...



Q5 An accountant is looking at some financial records. She needs to work out the principal amount that was invested a year ago. What was the principal amount invested in:

a) an account containing £278.10 that paid 3% interest?
b) an account containing £837.40 that paid 6% interest?
c) an account containing £1175.20 that paid 4% interest?
d) shares that are worth £8172.24 and have increased in value by 2%?
e) shares that are worth £5049 and have decreased in value by 1%?

Ratios



I don't want to spoil the surprise, but you're going to need your calculator for this bit — get your finger on that fraction button...

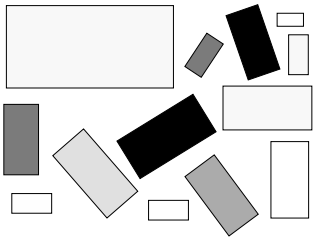
RATIOS are like FRACTIONS which are like DECIMALS

We can treat the RATIO 3:4 like the FRACTION $\frac{3}{4}$, which is 0.75 as a DECIMAL.

Watch out though — this <u>isn't</u> $\frac{3}{4}$ of the <u>total</u>:
If there are girls and boys in the ratio 3:4, it means there's $\frac{3}{4}$ as many girls as boys.
So if there's 8 boys, there's $\frac{3}{4} \times 8 = 6$ girls.

Q1 Write these ratios in their simplest forms:

a) 6:8 **c)** 1.5:3 **e)** 2 weeks:4 days
b) 5:20 **d)** 2¼:4 **f)** £1.26:14p



Q2 A rectangle has sides in the ratio 1:2. Calculate the length of the longer side if the shorter side is:

a) 3 cm **b)** 5.5 cm **c)** 15.2 m

Calculate the length of the shorter side if the longer side is:

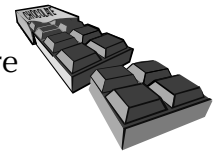
d) 3 cm **e)** 5.5 cm **f)** 15.2 m

Q3 Divide the following amounts in the ratios given:

a) £20 in the ratio 2:3 **c)** 500 g in the ratio 1:2:2
b) 150 m in the ratio 8:7 **d)** 8 hrs in the ratio 1:2:3

For these you add up the ratio numbers to find the total number of parts and <u>divide</u> by this. Then <u>multiply</u> by each number in the ratio separately to find the different amounts.

Q4 John and Peter share a bar of chocolate marked into 16 squares. They share it in the ratio 1:3 respectively. How many squares does each boy get?



Q5

A 2 litre bottle of cola is to be shared between three girls in the ratio 2:3:5. How many <u>millilitres</u> will each girl get?

<u>Watch out for your units</u> — you'll have to change them over for this one — and your answer should be in <u>millilitres</u>.

Q6 Oak and ash saplings are planted along a roadside in the ratio 2:3 respectively. If there are 20 oak saplings, how many ash saplings are there?

Q7 Tony gives £100 to be shared by Jane, Holly and Rosemary in a ratio according to their <u>age</u>. Jane is 10, Holly is 12 and Rosemary is 3 years old. How much will each child get?

Q8 Sunil, Donalda and Paul work in a restaurant. As they work different hours, they split their tips in the ratio 3:4:2. One night, Donalda got £8 more than Paul. How much did Sunil get?

Ratios

Q9 The ratio of girls to boys in a school is 7:6.
If there are 455 pupils in total, how many are
a) girls?
b) boys?

Q10 Sarah works as a waitress. Each week, she splits her wage into spending money and savings in the ratio 7:3.
a) One week, Sarah earns £130.
How much should she put in her savings that week?
b) The next week, Sarah put £42 into her savings.
How much did she earn in total that week?

Q11 An architect is drawing the plan of a house to a scale of 1 cm to 3 m.
a) Write this ratio in its simplest form.
b) How wide is a room that appears as 2 cm on the drawing?
c) The hall is 10 m long. How long will the architect need to make it on the drawing?

Make sure you convert to the same units when you're working out the ratio.

Q12 Concrete is mixed using cement, sand and gravel in the ratio 1:3:6. If Dave uses a 5 kg bag of cement, how much:
a) sand does he need?
b) gravel does he need?
c) If Dave needs 80 kg of concrete,
how much of each substance does he need?

Q13

I picked some strawberries after a few wet days. Some were nibbled by snails, some were mouldy and some fine. The ratio was 2:3:10 respectively. If 9 strawberries were mouldy how many:
a) were fine?
b) were not fine?
c) What fraction of the total amount were fine?

Q14 Salt & Vinegar, Cheese & Onion and Prawn Cocktail flavour crisps were sold in the school tuck shop in the ratio 5:3:2. If 18 bags of Prawn Cocktail were sold, how many bags:
a) of Salt & Vinegar were sold?
b) were sold altogether?

Proportion

Q1 If 3 minibuses can carry 51 students, how many students can 5 minibuses carry?

Q2 If 17 textbooks cost £150.45, how much will 28 cost?

Q3 If it takes 4 people 28 hours to complete a task, how long would it take just one person?

Q4 A person earns £6.20 an hour. How much do they earn for 15½ hours work?

Q5 Wool from 8 sheep is needed to make a scarf.
How many are needed to make 12 scarves?

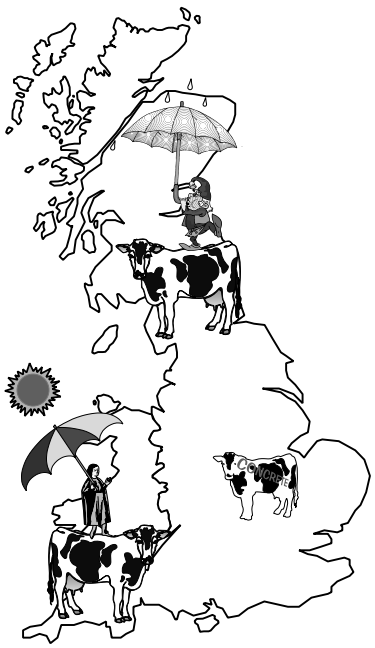
Q6 On a map, 2 cm represents 3 km.
a) If two towns are 14 km apart, what is the distance between them on the map?
b) If two road junctions are 20.3 cm apart on the map, what is their real distance apart?

Q7 Isla is making a chocolate cake using the recipe shown on the right. She wants to make the cake for 10 people.
a) How much sugar will she need?
b) How much flour will she need?
c) Isla only has 320 g of butter.
Will this be enough for her cake?
d) Matt makes a smaller version of the cake using 160 g of sugar.
How many people would his cake serve?

Chocolate cake
(serves 6)

180 g	flour
240 g	sugar
210 g	butter
3	eggs
60 g	cocoa powder

Q8 A herd of 7 cows produces 161 litres of milk a day.
Find the smallest number of cows it would take to produce at least 1000 litres per day.



Q9 A scientist proposes that rainfall is proportional to latitude. It rains an average of 53.1 cm a year in Milton Keynes, which is at latitude 52.0° N. If the scientist is correct, what is the average rainfall in:
a) Ulverston — latitude 54.2° N?
b) Boscastle — latitude 50.4° N?
If average daytime temperature in July is inversely proportional to latitude and it's 21.0 °C in Boscastle, what will the average daytime temperature in July be in:
c) Milton Keynes?
d) Ulverston?

Rounding Numbers

With all these rounding methods, you need to identify the last digit — e.g. if you're rounding 23.41 to 1 decimal place the last digit is 4. Then look at the next digit to the right. If it's 5 or more you round up, if it's 4 or less you round down.

Q1 Round these numbers to the required number of decimal places:

a) 62.1935 (1 d.p.)
b) 62.1935 (2 d.p.)
c) 62.1935 (3 d.p.)
d) 19.624328 (5 d.p.)
e) 6.2999 (3 d.p.)
f) π (3 d.p.)

Q2 Round these numbers to the required number of significant figures.

a) 1329.62 (3 s.f.)
b) 1329.62 (4 s.f.)
c) 1329.62 (5 s.f.)
d) 120 (1 s.f.)
e) 0.024687 (1 s.f.)
f) 0.024687 (4 s.f.)

Remember — the first significant figure is the first digit which isn't zero.

Q3 $K = 456.9873$
Write K correct to:

a) one decimal place
b) two decimal places
c) three decimal places
d) three significant figures
e) two significant figures
f) one significant figure.

Q4 Calculate the square root of 8. Write your answer to two decimal places.

Q5 Calculate, giving your answers to a sensible degree of accuracy:

a) $\dfrac{42.65 \times 0.9863}{24.6 \times 2.43}$

b) $\dfrac{13.63 + 7.22}{13.63 - 7.22}$

Rounding Numbers

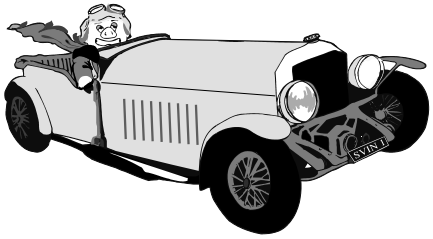
Q6 Round these prices to the nearest pound:

a) £1100.45
b) £87.61
c) £299.50
d) £2.55
e) £376.49
f) £44.19

Q7 A bumper bag of icing sugar weighs 23.4 kg.
What is this correct to the nearest kilogram?

Q8 David divides £15.20 by 3. What is the answer to the nearest penny?

Q9 The great racing driver Speedy Wheelman covered 234.65 km during the course of one of his races.
Give this distance correct to the nearest km.



Q10 Jack's company pays his travel expenses.
They round the distance he drives to the nearest km, and then pay 20p for every km.
In one week, Jack drives 95.45 km. How much money can Jack claim back?

Q11 A pack of three model cars costs £14.30. John wants to work out what one model car would cost. What is the answer correct to the nearest penny?

Q12 Pru measured the length of her bedroom as 2.345 metres.
Give this measurement correct to the nearest centimetre.

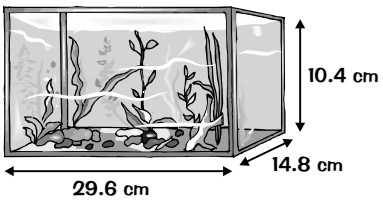
Q13 Jessy jumps 4.65 m in the long jump.
What is this to the nearest ten centimetres?

Q14 A baby sea turtle weighs 1814.46 g.
How much does it weigh to 3 significant figures?

Q15 Milo takes 26.99 seconds to run 200 m. Assuming he ran at an even pace, how long did it take him to run each 100 m? Give your answer to the nearest second.

Estimating

Q1 Mark wants to buy some tropical fish.
The pet shop owner tells him that he will need a tank with a volume of at least 7000 cm³.
Estimate whether Mark's tank will be big enough.



Q2 Without using your calculator find approximate answers to the following:

a) 6560×1.97
b) 8091×1.456
c) $38.45 \times 1.4237 \times 5.0002$
d) $45.34 \div 9.345$
e) $34504 \div 7133$
f) $\dfrac{55.33 \times 19.345}{9.23}$
g) 7139×2.13
h) $98 \times 2.54 \times 2.033$
i) $21 \times 21 \times 21$
j) $8143 \div 81$
k) $62000 \div 950$
l) $\pi \div 3$

Turn these into nice easy numbers that you can deal with without a calculator.

Q3 At the start of the week, a shop had approximately 15 000 cartons of broccoli juice in stock. The shop sold 1483 cartons on Monday, 2649 on Tuesday, 1539 on Wednesday, 1478 on Thursday and 2958 on Friday. Estimate the number of cartons remaining.

Q4 Showing all your working, estimate the value of the following:

a) $\dfrac{144.5 + 49.1}{153.2 - 41.2}$
b) $\dfrac{18.2 \times 10.7}{\sqrt{398.6}}$
c) $\dfrac{2021.23 \times 4.0436}{20.33 \times 4.902}$
d) $\dfrac{(9.2)^2 \div 10.3}{4.306 \times 5.011}$

Q5 Joan needs to estimate the size of her bedroom so that she can buy enough paint to cover the walls. Two of the walls measure 2.86 m by 3.16 m, and the other two walls measure 2.86 m by 3.42 m.

a) Estimate the area that Joan needs to paint in m².
b) If one tin of paint will cover 15 m², how many tins of paint will Joan need to paint her bedroom?

Q6 Estimate the following square roots, to 1 dp:

a) $\sqrt{48}$ **b)** $\sqrt{118}$ **c)** $\sqrt{84}$
d) $\sqrt{17}$ **e)** $\sqrt{98}$ **f)** $\sqrt{34}$

Start with square roots that you know — and use them to make an educated guess.

Bounds

Whenever a measurement is rounded off to a given unit, the actual measurement can be anything up to half a unit bigger or smaller.

1) 90 m to the nearest metre could be anything between 89.5 m and 90.5 m. (But not exactly equal to 90.5 m, or it would be rounded up to 91 m).
2) 700 people to the nearest 10 people could be anything between 695 people and 704 people. (Because this only involves whole numbers.)

Q1 Jodie weighs herself on some scales that are accurate to the nearest 10 grams. The digital display shows her weight as 64.78 kg.

a) What is the maximum that she could weigh?

b) What is the minimum that she could weigh?

Q2 A rectangular rug is 1.8 metres long and 0.7 metres wide. Both measurements are given correct to one decimal place.

a) State the minimum possible length of the rug.

b) Calculate the maximum possible area of the rug.

Q3 It's Pancake Day and Nigel is making a large batch of pancakes to share with friends. His recipe tells him to add 2.5 litres of milk, but his measuring jug only measures up to 500 ml and is accurate to the nearest 10 ml.

a) What is the maximum volume of milk Nigel could measure out, assuming he is as accurate as he can be?

b) What is the minimum volume of milk Nigel could measure out?

Q4 Sandra has a parcel to post. To find out how much it will cost she weighs it.

a) A set of kitchen scales, that weigh to the nearest 10 g, show that the parcel weighs 90 g. Write down the largest weight that the parcel could be.

b) Next she weighs the parcel on a different set of kitchen scales, which are accurate to the nearest 5 g. The packet weighs 95 g. Write down the upper and lower bounds of the weight of the package according to these scales.

c) The post office weighs the parcel on some electronic scales to the nearest gram. It weighs 98 g. Can all the scales be right?

Q5 $A = 13$, correct to 2 significant figures.
$B = 12.5$, correct to 3 significant figures.

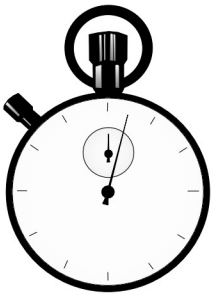
a) For the value of A, write down the upper bound and the lower bound.

b) For the value of B, write down the upper bound and the lower bound.

c) Calculate the upper bound and lower bound for C when $C = AB$.

Bounds

Q6 Jimmy, Sarah and Douglas are comparing their best times for running the 1500 m.
Jimmy's best time is 5 minutes 30 seconds measured to the nearest 10 seconds.
Sarah's best time is also 5 minutes 30 seconds, but measured to the nearest 5 seconds.
Douglas' best time is 5 minutes 26 seconds measured to the nearest second.



a) What are the upper and lower bounds for Sarah's best time?
b) Of the three Douglas thinks that he is the quickest at running the 1500 m. Explain why this may not be the case.

To find the upper or lower bound of a calculation, you've just got to decide which version of the values involved (max or min) to use to get the biggest or smallest overall answer.

Q7 $R = \frac{S}{T}$ is a formula used by stockbrokers.
S = 940, correct to 2 significant figures and T = 5.56, correct to 3 significant figures.
a) For the value of S, write down the upper bound and the lower bound.
b) For the value of T, write down the upper bound and the lower bound.
c) Calculate the upper bound and lower bound for R.
d) Write down the value of R correct to an appropriate number of significant figures.

Remember — you don't always get the maximum value by using the biggest input values.

Q8 Ash wants to put a new carpet in his living room. He has measured the floor as being 3.4 m × 5.2 m to the nearest 10 cm. What area of carpet should Ash buy to make sure he has enough to cover the whole floor?

Q9 A lorry travelled 125 kilometres in 1 hour and 50 minutes. If the time was measured to the nearest 10 minutes and the distance to the nearest five kilometres, what was the maximum value of the average speed of the lorry, in kilometres per hour?

You need to add the errors for all the sides together.

Q10 a) The length of a rectangle is measured as 12 ± 0.1 cm. The width of the same rectangle is measured as 4 ± 0.1 cm. Calculate the perimeter of the rectangle, giving also the maximum possible error.
b) A rectangle measures $A \pm x$ cm in length and $B \pm y$ cm in width. The formula $P = 2(A + B)$ is used to calculate the perimeter, P, of the rectangle. What is the maximum possible error in P?

Standard Form

Writing very big (or very small) numbers gets a bit messy with all those zeros if you don't use this standard index form. But of course, the main reason for knowing about standard form is... you guessed it — it's in the Exam.

Q1 Delilah is doing some calculations for her science homework. She needs to give her answers as ordinary numbers. How should she write the following answers?

a) 3.56×10
b) 3.56×10^{3}
c) 3.56×10^{-1}
d) 3.56×10^{4}
e) 0.082×10^{2}
f) 0.082×10^{-2}
g) 0.082×10
h) 0.082×10^{-1}
i) 157×10
j) 157×10^{-3}
k) 157×10^{3}
l) 157×10^{-1}

Q2 Write in standard form:

a) 2.56
b) 25.6
c) 0.256
d) 25 600
e) 95.2
f) 0.0952
g) 95 200
h) 0.000952
i) 4200
j) 0.0042
k) 42
l) 420.

Q3 Write in standard form:

a) 34.7×10
b) 73.004
c) 0.005×10^{3}
d) 9183×10^{2}
e) 15 million
f) 937.1×10^{4}
g) 0.000075
h) 0.05×10^{-2}
i) 534×10^{-2}
j) 621.03
k) 149×10^{2}
l) 0.003×10^{-4} .

When scientists write about massive things such as the universe, or tiny things such as cells and particles, it's often more convenient to write numbers in standard form. Write the numbers in Questions 4 to 7 in standard form.

Q4 The average diameter of a cell nucleus in a mammal is around 0.006 mm.

Q5 A billion = a thousand million A trillion = a thousand billion.

Q6 A light year is approximately 9 460 000 000 000 km.

Q7 Nautilus covered 69 138 miles before having to refuel.

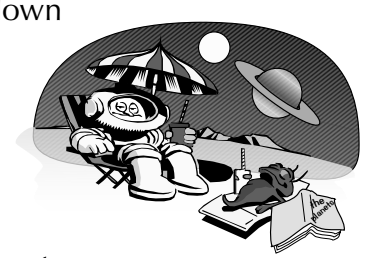
Q8 A tissue sample is three cells thick. Each cell has a thickness of 0.000004 m. What is the thickness of the tissue sample, in mm? Give your answer in standard form.

Q9 This table gives the diameter and distance from the Sun of some planets.

Planet	Distance from Sun (km)	Diameter (km)
Earth	1.5×10^{8}	1.3×10^{4}
Venus	1.085×10^{8}	1.2×10^{4}
Mars	2.28×10^{8}	6.8×10^{3}
Mercury	5.81×10^{7}	4.9×10^{3}
Jupiter	7.8×10^{8}	1.4×10^{5}
Neptune	4.52×10^{9}	4.9×10^{4}
Saturn	1.43×10^{9}	1.2×10^{5}

From the table write down which planet is:

a) smallest in diameter
b) largest in diameter
c) nearest to the Sun
d) furthest from the Sun.



Write down which planets are:

e) nearer to the Sun than the Earth
f) bigger in diameter than the Earth.

Standard Form

Q10 Work out the answers to these calculations. Write your answers in standard form.

a) $(2 \times 10^3) \times (3 \times 10^6)$
b) $(3 \times 10^4) \times (6.3 \times 10^2)$
c) $(8.8 \times 10^{12}) \div (2.2 \times 10^8)$
d) $(2.8 \times 10^5) \div (1.4 \times 10^3)$
e) $(7 \times 10^9) \times (8 \times 10^6)$
f) $(8.9 \times 10^3) + (3.1 \times 10^4)$
g) $(4.35 \times 10^6) - (2.7 \times 10^3)$
h) $(1.8 \times 10^4) - (5.2 \times 10^2)$

Q11 If $x = 4 \times 10^5$ and $y = 6 \times 10^4$ work out the value of
a) xy **b)** $4x$ **c)** $3y$.

Q12 Which is greater, 4.62×10^{12} or 1.04×10^{13}, and by how much?

Q13 Which is smaller 3.2×10^{-8} or 1.3×10^{-9} and by how much?

Q14 The following numbers are not written in standard index form. Rewrite them correctly using standard index form.

a) 42×10^6
b) 38×10^{-5}
c) 10×10^6
d) 11.2×10^{-5}
e) 843×10^3
f) 42.32×10^{-4}
g) 17×10^{17}
h) 28.3×10^{-5}
i) 10×10^{-3}

Don't forget — when you're using a calculator, you've got to write the answer as 3.46×10^{27}, not as 3.46^{27}. If you do it the wrong way, it means something completely different.

Q15 What is 7 million in standard index form?

Q16 The radius of the Earth is 6.38×10^3 km. What is the radius of the Earth measured in cm? Leave your answer in standard form.

Q17 One atomic mass unit is equivalent to 1.661×10^{-27} kg. What are two atomic mass units equivalent to (in standard index form)?

Q18 The length of a light year, the distance light can travel in one year, is 9.461×10^{15} m. How far can light travel in
a) 2 years?
b) 6 months?
Write your answers in standard form.

Q19 a) The surface area of the Earth is approximately 5.1×10^8 km^2. Write this without using standard form.
b) The area of the Earth covered by sea is 362 000 000 km^2. Write this in standard form.
c) What is the approximate area of the Earth covered by land? Write your answer without using standard form.

Sets and Venn Diagrams

Sets aren't scary — they're just collections of things. The tricky bit is showing how these things relate to each other. There are diagrams to draw and symbols to learn. Here are some questions to practise with...

Q1 The elements of set E are all the prime numbers less than 12.

a) Write set E as a rule using set notation.

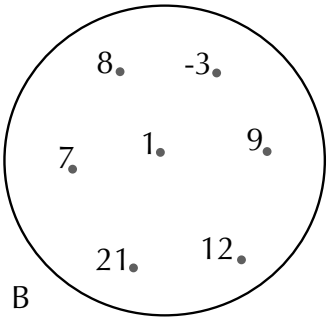
b) Write set E as a complete list of its elements.

Q2 H = {1, 4, 9, 16, 20, 36, 49, 64, 81, 100}
K = {Integers}
L = {Natural numbers ≤ 0}

a) Which one of these sets = ∅

b) Write down a number that is an element of set K and one that is not an element of set K using set notation.

Q3 Set B is shown in the Venn diagram below:



a) Write set B as a complete list of its elements.

b) Draw a Venn diagram for set C = {odd numbers between 18 and 34}.

Q4 Draw a Venn diagram for:

a) A = {odd numbers less than 20} and B = {prime numbers less than 20}

b) C = {integers greater than or equal to -4 and less than or equal to 4} and D = {natural numbers less than 5}

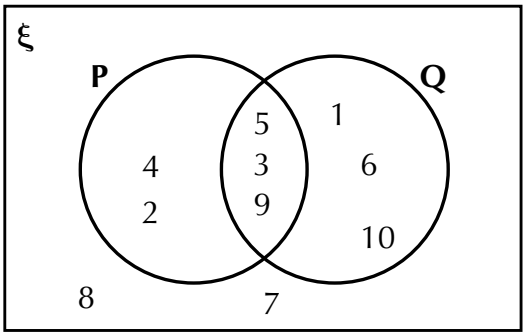
Q5 The Venn diagram on the right shows which numbers between 1 and 10 belong to each of the sets P and Q. Use the Venn diagram to find:

a) n(P)

b) the elements of set Q′

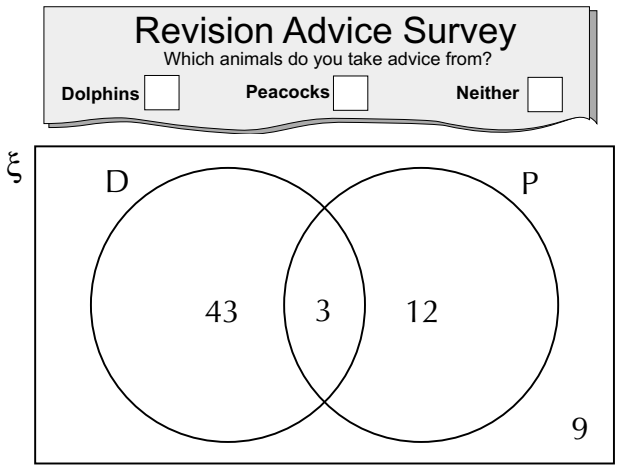
c) n(P ∪ Q)

d) the elements of set P ∩ Q



Sets and Venn Diagrams

Q6 This Venn diagram shows the results of a survey about revision. Set D shows those people who listen to dolphins and set P those who take advice from peacocks.

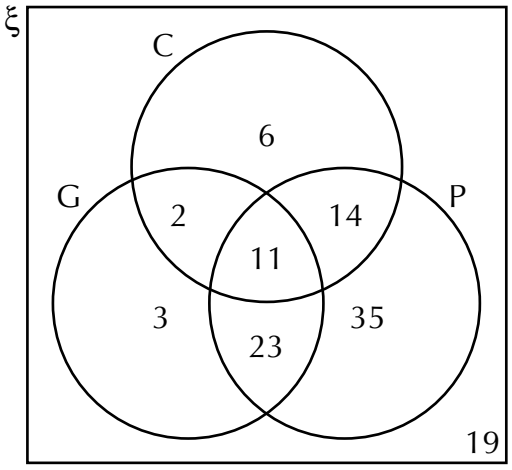


a) What does the universal set, ξ, represent in this diagram?

b) How many people don't take revision advice from dolphins or peacocks?

c) Find n(D ∪ P) and n(D ∩ P).

Q7 John travels around Milton Keynes and counts different types of cows. In addition to real cows he finds artificial cows made from three different materials and some made from a combination of those materials. He records his findings in sets — Concrete (C), Glass (G) and Plastic (P).



a) What is John's universal set?

b) How many real cows did he find?

c) Find n(P).

d) Find n(C ∪ G).

e) Find n(C′).

f) Find n(C ∩ P).

g) Find n(C ∩ G ∩ P).

h) How many cows are made from glass and plastic but not concrete? Write this using set notation.

Q8 $A = \{x: 0 \leq x \leq 18\}$
$B = \{y: y \text{ is a multiple of } 3\}$
$C = \{z: z \text{ is a factor of } 15\}$

Say whether the following statements about sets A, B and C are true or false.

a) $0 \notin A$

b) $0 \in C$

c) $\frac{2}{3} \in A$

d) $B \subset A$

e) $A \subset C$

f) $C \subset B$

g) $C \subset A$

h) $\{6\} \in B$

i) $\{6\} \subset B$

j) $6 \subset B$

k) $(B \cap C) \subset A$

l) $(B \cup C) \subset A$

Powers and Roots

Hang on there. Before you try this page, make sure you know all the rules for dealing with powers...

The small number is called the power or index number. Remember the plural of index is indices.

$5^4 = 5 \times 5 \times 5 \times 5 =$ _____
we say "five to the power four"

$8^3 = 8 \times 8 \times 8 =$ _____
we say "eight to the power three" or "eight cubed"

To save time try using the power button on your calculator [x^y] [y^x] [^]

eg. [5] [x^y] [4] [=]
[8] [x^y] [3] [=]

Q1 Complete the following:

a) $2^4 = 2 \times 2 \times 2 \times 2$ =
b) $10^3 = 10 \times 10 \times 10$ =
c) $3^5 = 3 \times \ldots$ =
d) $4^6 = 4 \times \ldots$ =
e) $1^9 = 1 \times \ldots$ =
f) $5^6 = 5 \times \ldots$ =

Q2 Simplify the following:

a) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
b) $12 \times 12 \times 12 \times 12 \times 12$
c) $x \times x \times x \times x \times x$
d) $m \times m \times m$
e) $y \times y \times y \times y$
f) $z \times z \times z \times z \times z \times z$

Q3 Complete the following (the first one has been done for you):

a) $10^2 \times 10^3 = (10 \times 10) \times (10 \times 10 \times 10) = 10^5$
b) $10^3 \times 10^4 =$ =
c) $10^4 \times 10^2 =$ =
d) $10^5 \times 10^3 =$ =
e) What is the quick method for writing down the final result in **b)**, **c)** and **d)**?

Easy — you'll have learnt this from your power rules.

Q4 Complete the following (the first one has been done for you):

a) $2^4 \div 2^2 = \dfrac{(2 \times 2 \times 2 \times 2)}{(2 \times 2)} = 2^2$
b) $2^5 \div 2^2 = \dfrac{(2 \times 2 \times 2 \times 2 \times 2)}{(2 \times 2)} =$
c) $4^5 \div 4^3 = \dfrac{(4 \times 4 \times 4 \times 4 \times 4)}{} =$
d) $8^5 \div 8^2 =$ =
e) What is the quick method for writing down the final result in **b)**, **c)** and **d)**?

Q5 Which of the following are true?

a) $2^4 \times 2^6 = 2^{10}$
b) $2^2 \times 2^3 \times 2^4 = 2^9$
c) $2^3 \times 2^2 = 2^6$
d) $4^{10} \times 4^4 \times 4^2 = 4^{18}$
e) $2^1 \times 2^3 \times 2^4 = 2^8$
f) $10^4 \times 10^2 = 10^8$
g) $2^{20} \div 2^5 = 2^4$
h) $3^{12} \div 3^4 = 3^8$
i) $4^6 \div 6^4 = 4^2$
j) $10^{20} \div 10^3 = 10^{17}$
k) $4^6 \div (4^2 \times 4^3) = 4^1$
l) $9^2 \times (9^{30} \div 9^{25}) = 9^{10}$

Q6 Remove the brackets from the following and express as a single power:

a) $(3^4 \times 3^2) \div (3^6 \times 3^3)$
b) $(4^{10} \times 4^{12}) \times 4^3$
c) $10^2 \div (10^3 \times 10^{12})$
d) $(3^6)^{-2}$
e) $4^2 \times 4^{-1} \times 4^6 \times (4^2 \div 4^3)$
f) $(5^2 \times 5^3) \div (5^6 \div 5^4)$

Powers and Roots

Simplify the expressions in questions 7 to 11. Give your answers to 3 s.f. where necessary.

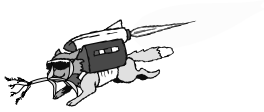
Q7 **a)** $(6.5)^3$
b) $(0.35)^2$
c) $(15.2)^4$
d) $(0.04)^3$
e) $\sqrt{5.6}$
f) $\sqrt[3]{12.4}$
g) $\sqrt{109}$
h) $\sqrt[3]{0.6}$
i) $(1\frac{1}{2})^2$
j) $\sqrt{4\frac{3}{4}}$
k) $\left(\frac{5}{8}\right)^3$
l) $\sqrt[3]{\frac{9}{10}}$

Q8 **a)** $(2.4)^2 + 3$
b) $5.9 - (1.2)^3$
c) $\sqrt[3]{5.6} + (4.2)^2$
d) $(6.05)^3 - \sqrt[3]{8.4}$
e) $6.1[35.4 - (4.2)^2]$
f) $95 - 3(\sqrt[3]{48} - 2.6)$
g) $1\frac{1}{2}\,[4 + (2\frac{1}{4})^2]$
h) $19 - 4[(\frac{1}{4})^2 + (\left(\frac{5}{8}\right)^3)]$
i) $15\frac{3}{5} - 2\frac{1}{2}\left[(1\frac{3}{4})^3 - \sqrt[3]{1\frac{1}{2}}\right]$

Q9 **a)** 5^{-3}
b) 2^{-2}
c) 16^{-4}
d) $(1.5)^{-1}$
e) $5^{\frac{1}{2}}$
f) $6^{\frac{1}{3}}$
g) $9^{\frac{1}{2}}$
h) $(4.2)^{\frac{2}{3}}$
i) $(1\frac{1}{4})^{-3}$
j) $(2\frac{3}{5})^{\frac{1}{5}}$
k) $(5\frac{1}{3})^{-2}$
l) $(10\frac{5}{6})^{\frac{5}{6}}$

Remember — fractional powers mean roots.

Q10 **a)** $\sqrt{(1.4)^2 + (0.5)^2}$
b) $5.9\ [(2.3)^{1/4} + (4.7)^{1/2}]$
c) $2.5 - 0.6[(7.1)^{-3} - (9.5)^{-4}]$
d) $(8.2)^{-2} + (1.6)^4 - (3.7)^{-3}$
e) $\dfrac{3\sqrt{8} - 2}{6}$
f) $\dfrac{15 + 3\sqrt{4.1}}{2.4}$
g) $3\sqrt{4.7} - 4\sqrt{2.1}$
h) $\dfrac{(2\frac{1}{4})^{-2} - (3\frac{1}{2})^{\frac{1}{2}}}{4.4}$



Q11 **a)** $(2\frac{1}{4})^3 - (1.5)^2$
b) $(3.7)^{-2} + (4\frac{1}{5})^{\frac{1}{4}}$
c) $\sqrt[3]{5\frac{1}{3}} \times (4.3)^{-1}$
d) $(7.4)^{\frac{1}{3}} \times (6\frac{1}{4})^3$
e) $\dfrac{\sqrt{22\frac{1}{2}} + (3.4)^2}{(6.9)^3 \times 3.4}$
f) $\dfrac{(15\frac{3}{5})^2 \times (2.5)^{-3}}{3 \times 4\frac{1}{4}}$
g) $5[(4.3)^2 - (2.5)^{1/2}]$
h) $\dfrac{3.5(2\frac{1}{6} - \sqrt{4.1})}{(3.5)^2 \times (3\frac{1}{2})^{-2}}$
i) $\dfrac{1\frac{1}{2} + \frac{1}{4}[(2\frac{2}{3})^2 - (1.4)^2]}{(3.9)^{-3}}$
j) $\sqrt[3]{2.73} + 5\sqrt{2}$

Powers and Roots

Q12 Write the following out in full. E.g. $c^3 = c \times c \times c$

a) k^2
b) p^3q^2
c) gt^2
d) $(gt)^2$
e) $(-t)^2$
f) $-(t)^2$

Q13 Write the following in index notation:

a) $a \times a \times a$
b) $c \times c \times c \times c \times c$
c) $a \times a \times b \times b \times b$
d) $d \times c \times c \times d \times c$
e) $x \times 2x \times 2x$
f) $-y \times 3x \times 2y \times -y$

Q14 Use index laws to simplify the following where possible:

a) $x^3 \times x^2$
b) $x^3 \div x^2$
c) $-(y)^5 \times y^8$
d) $q^{11} \div q^8$
e) $(b^5)(b^4)$
f) $x^3 + x^2$
g) $-(f)^8 \div f$
h) $x^3y^6 \times x^4y$
i) $p^9q^2 \times q^4$
j) $(-x)^2 \div -x$
k) $y^7 \div y^7$
l) k^0
m) $u^3 \times u^0$
n) 1^x
o) $(gt)^1$
p) $r^3 \div r^3$
q) $(k^2)^5$
r) $(p^3)^2$
s) $\frac{v^4}{v^2}$
t) $\frac{i^6}{i^5}$
u) $(gt^2)^7$
v) $(x^3)^3 \times x^2$
w) $\frac{r^4 \times r^4}{r^3 \times r^2}$

Q15 Rewrite the following without using negative powers:

a) k^{-2}
b) $p^{-3}q^2$
c) gt^{-2}
d) $\frac{1}{y^{-2}}$
e) $\frac{a^4}{v^{-2}}$
f) $\frac{v^{-2}a^4}{b}$

Q16 Simplify the following and write your answer without using negative powers:

a) $h^2 \times h^{-8}$
b) $g \div g^4$
c) $(t^2)^4 \times (t^3)^{-4}$
d) $\frac{v^{-4}}{v^{-2}}$
e) $\frac{w^4}{w^5}$
f) $\frac{a^4 \times a^2}{(a^5)^2}$

Q17 These expressions all involve fractional powers. Simplify them as far as possible.

a) $(9a^6)^{\frac{1}{2}}$
b) $(p^8q^{10})^{\frac{1}{2}}$
c) $(x^9y^{12})^{\frac{1}{3}}$
d) $(x^3)^{\frac{2}{3}}$
e) $(64x^3)^{\frac{1}{6}}$
f) $(36j^4)^{-\frac{1}{2}}$
g) $\frac{4b^{\frac{1}{2}} \times b^{\frac{2}{5}}}{32b^{\frac{1}{5}}}$
h) $(81u)^{\frac{1}{4}} \times u^{-\frac{3}{4}}$
i) $\frac{(100d)^{\frac{1}{2}} \div d^{\frac{1}{4}}}{d^{\frac{1}{8}}}$

Algebra Basics

Q1 Work out the following temperature changes:

a) 20 °C to -7 °C
b) -10 °C to -32 °C
c) -17 °C to -5 °C
d) -3 °C to 15 °C
e) -31 °C to -16 °C
f) -5 °C to -17 °C

Q2 Which is larger and by how much?

a) $-12 + 7 - 4 + 6 - 2 + 7$ or **b)** $-30 + 26 - 3 - 7 + 17$

Q3 Simplify: **a)** $4x - 5x + 3x - x + 2x - 7x$ **b)** $30y - 10y + 2y - 3y + 4y - 5y$

Q4 Find the value of xy and $\frac{x}{y}$ for each of the following:

a) $x = -100$ $y = 10$
b) $x = 24$ $y = -4$
c) $x = -48$ $y = -3$
d) $x = 0$ $y = -4$

Q5 Find the value of $(a - b) \div (c + d)$ when $a = 10$, $b = -26$, $c = -5$ and $d = -4$.

Q6 Simplify the following:

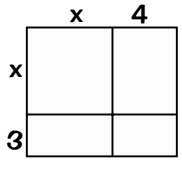
a) $2x \times -3y$
b) $-8a \times 2b$
c) $-4x \times -2x$
d) $4p \times -4p$
e) $-30x \div -3y$
f) $50x \div -5y$
g) $10x \div -2y$
h) $-30x \div -10x$
i) $40ab \div -10ab$
j) $70x^2 \div -7x^2$
k) $-36x^2 \div -9x$
l) $40y^2 \div -5y$

Q7 Simplify the following by collecting like terms together:

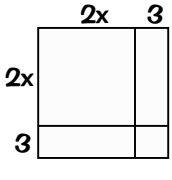
a) $3x^2 + 4x + 12x^2 - 5x$
b) $14x^2 - 10x - x^2 + 5x$
c) $12 - 4x^2 + 10x - 3x^2 + 2x$
d) $20abc + 12ab + 10bac + 4b$
e) $8pq + 7p + q + 10qp - q + p$
f) $15ab - 10a + b - 7a + 2ba$
g) $4pq - 14p - 8q + p - q + 8p$
h) $13x^2 + 4x^2 - 5y^2 + y^2 - x^2$
i) $11ab + 2cd - ba - 13dc + abc$
j) $3x^2 + 4xy + 2y^2 - z^2 + 2xy - y^2 - 5x^2$

Q8 For each of the large rectangles below, write down the area of each of the small rectangles and hence find an expression for the area of each large rectangle.

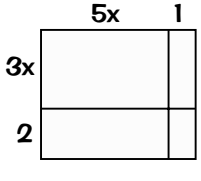
a)



b)



c)



Eeeek — loads of questions...

Formulas from Words

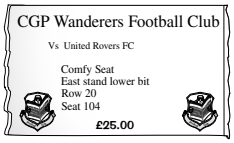
It's no big mystery — algebra is just like normal sums, but with the odd letter or two stuck in for good measure.

Q1 Write an algebraic expression for each of these:

a) To find y add 5 to x
b) To find y multiply x by 7 and add 4
c) To find y subtract 7 from x and divide by 3
d) To find y square 6 and add it to x
e) Square x then divide by 8 to find y
f) y is equal to the square of x divided by 12

Q2 Tickets for a football match cost £25 each.

a) Write a formula to calculate the cost, c, of n tickets.
b) A booking fee of £1.25 is added to each ticket bought online. Write an equation to calculate the cost of n tickets bought online.



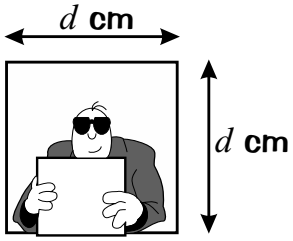
Q3 There are n books in a pile. Write an expression for the number (N) of books in:

a) A pile with 23 more books
b) A pile with 14 fewer books
c) A pile with twice as many books
d) x piles each with n books in them
e) A warehouse filled with a square of book piles x piles long each side.

Q4 **a)** This square has sides of length d cm.

i) What is its perimeter?

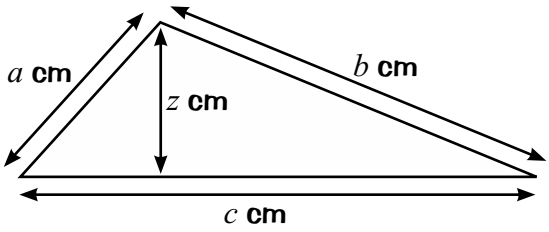
ii) What is its area?



b) For the triangle below write equations for:

i) Its perimeter

ii) Its area.



Q5 The cost (C) of hiring a mountain bike is £10, plus £5 for each hour you use the bike (h). Write down a formula that can be used for working out the cost of hiring a bike.

Q6 The cost per person of a flight from Manchester to Aberdeen is £73 plus £27 tax and an extra £15 for each piece of luggage. Write down a formula to calculate the total cost (T) of a flight for p people with a total of l pieces of luggage.

Q7 The number of sheep (S) that a dragon can eat per day is 3 plus a third of its wingspan in metres (w). Write an expression to calculate how many sheep a dragon can eat in d days.

Multiplying Out Brackets

Remember FOIL for multiplying brackets... don't want to miss any terms now, do you...

Q1 Multiply out the brackets and simplify where possible:

a) $4(x + y - z)$
b) $x(x + 5)$
c) $-3(x - 2)$
d) $7(a + b) + 2(a + b)$
e) $3(a + 2b) - 2(2a + b)$
f) $4(x - 2) - 2(x - 1)$
g) $4e(e + 2f) + 2f(e - f)$
h) $14(2m - n) + 2(3n - 6m)$
i) $4x(x + 2) - 2x(3 - x)$
j) $3(2 + ab) + 5(1 - ab)$
k) $(x - 2y)z - 2x(x + z)$
l) $4(x - 2y) - (5 + x - 2y)$
m) $a - 4(a + b)$
n) $4pq(2 + r) + 5qr(2p + 7)$
o) $x^2(x + 1)$
p) $4x^2\left(x + 2 + \frac{1}{x}\right)$
q) $8ab(a + 3 + b)$
r) $7pq\left(p + q - \frac{1}{p}\right)$
s) $4[(x + y) - 3(y - x)]$

Q2 Multiply out the brackets and simplify your answers where possible:

a) $(x - 3)(x + 1)$
b) $(x - 3)(x + 5)$
c) $(x + 10)(x + 3)$
d) $(x - 5)(x - 2)$
e) $(x + 2)(x - 7)$
f) $(4 - x)(7 - x)$
g) $(2 + 3x)(3x - 1)$
h) $(3x + 2)(2x - 4)$
i) $(x - 3)(4x + 1)$
j) $2(2x + y)(x - 2y)$
k) $4(x + 2y)(3x - 2y)$
l) $(3x + 2y)^2$

Q3 Find the product of $5x - 2$ and $3x + 2$.

Q4 Find the square of $2x - 1$.

For triple brackets, use FOIL on two of the brackets, then multiply the result by the third bracket.

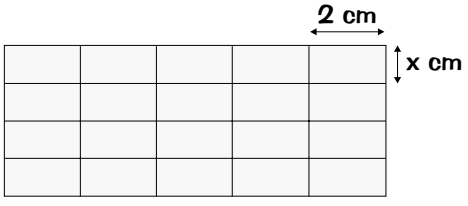
Q5 Multiply out the brackets and simplify your answers where possible.

a) $(x + 1)(x - 2)(x + 2)$
b) $(x + 3)(x + 2)(x + 5)$
c) $(x - 2)(2x - 1)(x - 5)$
d) $(3 - x)(2 + x)(5 + x)$
e) $(1 - 2x)(1 + x)(3 + x)$
f) $(x + 10)(4 + 2x)(1 + 3x)$
g) $(3 + x)^2(3 - x)$
h) $(x - 1)(2x + 1)^2$
i) $(x + 3)^3$

Q6 A rectangular pond has length $(3x - 2)$ m and width $(5 - x)$ m.
Write down a simplified expression for:

a) the pond's perimeter
b) the pond's area.

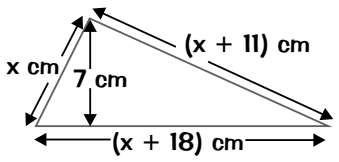
Q7 A rectangular bar of chocolate consists of 20 small rectangular pieces. The size of a small rectangular piece of chocolate is 2 cm by x cm.



a) Write down an expression for the perimeter of the whole bar.
b) Write down an expression for the area of the whole bar.
c) If I ate 6 small rectangular pieces of chocolate, what is the area of the remaining bar?

Q8 Find a simplified expression for the perimeter *and* the area of the following shapes.

a)



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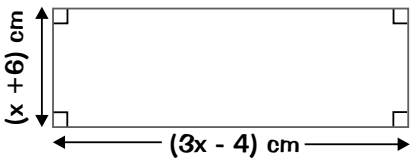

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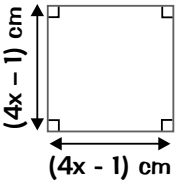
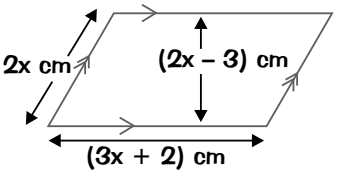


d)



Factorising

Factorising means putting the brackets back in...

Q1 All the expressions below have a^2 as a common factor. Factorise each of them.

a) $a^2b + a^2c$
b) $5a^2 + 13a^2b$
c) $2a^2b + 3a^2c$
d) $a^3 + a^2y$
e) $2a^2x + 3a^2y + 4a^2z$
f) $a^2b^2 + a^3c^2$

Q2 Factorise the following expressions:

a) $x^2 - 5x$
b) $2x + 6$
c) $3x^2 + 12x$
d) $4x^2 - 6x$
e) $3xy + 12x^2y$
f) $9x + 15$
g) $15x^2y - 25x$
h) $4pq^2 - 20pq + 8p^2q$
i) $10x^4 + 6x$
j) $15x^3 - 20x^2$
k) $21x^2 + 14x$
l) $5xyz + 20uxy$

Q3 Factorise and simplify the following:

a) $4xyz + 8xyz$ **b)** $8xyz + 12xyz$ **c)** $8xyz + 16\,x^2yz$ **d)** $20\,x^2y^2z^2 + 16\,xyz^2$

Q4 Using the fact that $a^2 - b^2 = (a + b)(a - b)$, factorise the following expressions:

a) $x^2 - 9$
b) $y^2 - 16$
c) $25 - z^2$
d) $36 - a^2$
e) $4x^2 - 9$
f) $9y^2 - 4$
g) $25 - 16z^2$
h) $1 - 36a^2$
i) $x^4 - 36$
j) $x^4 - y^4$
k) $1 - (ab)^2$
l) $100x^2 - 144y^2$

Q5 Factorise:

a) $x^2 - 4$ **b)** $144 - y^4$ **c)** $1 - 9x^2y^2$ **d)** $49x^4y^4 - 1$

These questions are a mixed bunch so look out for which ones are D.O.T.S and which ones aren't.

Q6 Factorise the following expressions:

a) $64a^2b^3 - 16b^2a^3$
b) $pq + qr - pq^2r$
c) $3m^2 - 24$
d) $b^4 - ab^3 + b^2c$
e) $a^4 - 169$
f) $9ab^2 - 3abc$
g) $81 - z^2$
h) $36m^2 - 25n^2$
i) $m^2n + 3mn - 2mn^3$
j) $121p^2 - 9q^2$
k) $144x^2 - 108y^2 - 60z^2$
l) $64a^2b^2 - 49c^2d^2$

Manipulating Surds

Do these questions <u>without</u> using a calculator.
They're actually not as bad as they might look because you can leave $\sqrt{}$ or π in your answer.

Q1 Simplify:

a) $\sqrt{5} \times \sqrt{3}$ **b)** $\frac{\sqrt{20}}{\sqrt{5}}$ **c)** $(\sqrt{x})^2$

d) $\sqrt{x^2}$ **e)** $\sqrt{8} \times \sqrt{8}$ **f)** $\frac{\sqrt{30}}{\sqrt{6}}$

Q2 A circle has a radius of $\sqrt{3}$ cm. What is its exact area? (Area of circle = πr^2.)

Q3 Simplify these expressions:

Remember — $\sqrt{a} \times \sqrt{b} = \sqrt{(ab)}$.

a) $\sqrt{4} - \sqrt{1}$ **b)** $2\sqrt{3} + 3\sqrt{3}$ **c)** $\sqrt{8}$ **d)** $(2+\sqrt{3})^2$

e) $4\sqrt{5} - \sqrt{5}$ **f)** $\sqrt{50}$ **g)** $\sqrt{8} - \sqrt{2}$ **h)** $\sqrt{18} - \sqrt{9}$

Q4 Are the following expressions rational or irrational?

a) $(1+\sqrt{5})(1-\sqrt{5})$ **b)** $\frac{1+\sqrt{5}}{1-\sqrt{5}}$

Q5 If $x = 1$ and $y = \sqrt{2}$, are the following expressions rational or irrational?

a) $(x + y)(x - y)$ **b)** $\frac{x+y}{x-y}$

Q6 Rationalise the denominators of the following expressions, and then simplify if necessary.

a) $\frac{1}{\sqrt{2}}$ **b)** $\frac{2}{\sqrt{8}}$ **c)** $\frac{a}{\frac{\sqrt{40}}{2}}$ **d)** $\frac{x}{\sqrt{xy}}$

e) $\frac{1}{1+\sqrt{2}}$ **f)** $\frac{6}{3+\sqrt{3}}$ **g)** $\frac{2}{1+\sqrt{6}}$ **h)** $\frac{5+\sqrt{5}}{5-\sqrt{5}}$

Q7 Express $\frac{9}{\sqrt{3}}$ in the form $a\sqrt{3}$.

Remember: rationalising the denominator means getting rid of the square root signs on the bottom of fractions.

Q8 Express $\sqrt{32} + 3\sqrt{2}$ in the form $a\sqrt{2}$.

Q9 Express $(1 + 3\sqrt{2})^2$ in the form $a + b\sqrt{2}$.

Solving Equations

Q1 When 1 is added to a number and the answer then trebled, it gives the same result as doubling the number and then adding 4. Find the number.

Q2 Solve the following:

a) $2x^2 = 18$ **b)** $2x^2 = 72$ **c)** $3x^2 = 27$ **d)** $4x^2 = 36$ **e)** $5x^2 = 5$

Q3 Solve the following:

a) $3x + 1 = 2x + 6$

b) $4x + 3 = 3x + 7$

c) $5x - 1 = 3x + 19$

d) $x + 2 = \frac{1}{2}x - 1$

e) $x + 15 = 4x$

f) $3x + 3 = 2x + 12$

Q4 Solve the following:

a) $3x - 8 = 7$

b) $2(x - 3) = -2$

c) $4(2x - 1) = 60$

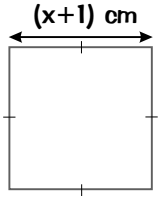
d) $2x - 9 = 25$

e) $\frac{24}{x} + 2 = 6$

f) $5x - 2 = 6x - 7$

g) $30 - \frac{x^2}{2} = 28$

Q5

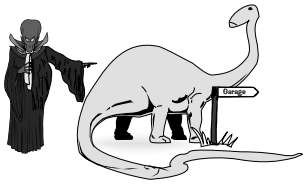


A square has sides of length $(x + 1)$ cm. Find the value of x if:

a) the perimeter of the square is 66 cm

b) the perimeter of the square is 152.8 cm.

With these wordy ones, you just have to write your own equation from the information you're given.



Q6 Mr Smith sent his car to the local garage. He spent £x on new parts, four times this amount on labour and finally £29 for an MOT test. If the total bill was for £106.50, find the value of x.

Q7 Solve:

a) $2(x - 3) - (x - 2) = 5$

b) $5(x + 2) - 3(x - 5) = 29$

c) $2(x + 2) + 3(x + 4) = 31$

d) $10(x + 3) - 4(x - 2) = 7(x + 5)$

e) $5(4x + 3) = 4(7x - 5) + 3(9 - 2x)$

f) $3(7 + 2x) + 2(1 - x) = 19$

g) $\frac{x}{3} + 7 = 12$

h) $\frac{x}{10} + 18 = 29$

i) $17 - \frac{x^2}{3} = 5$

j) $41 - \frac{x}{11} = 35$

k) $\frac{x}{100} - 3 = 4$

l) $\frac{120}{x} = 16$

Q8 Joan, Kate and Linda win £2400 on the National Lottery between them. Joan gets a share of £x, whilst Kate gets twice as much as Joan. Linda's share is £232 less than Joan's amount.

a) Write down an expression for the amounts Joan, Kate and Linda win.

b) Write down an equation in terms of x, and solve it.

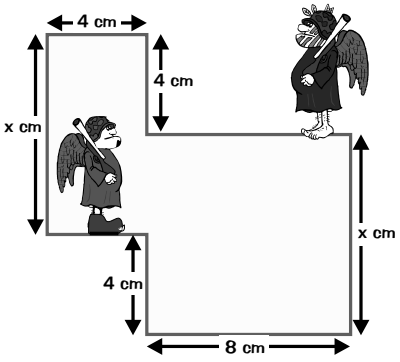
c) Write down the amounts Kate and Linda receive.

Q9 All the angles in the diagram are right angles.

a) Write down an expression for the perimeter of the shape.

b) Write down an expression for the area of the shape.

c) For what value of x will the perimeter and area be numerically equal?



Solving Equations

Q10 Solve the following:

a) $5(x-1)+3(x-4)=-11$

b) $3(x+2)+2(x-4)=x-3(x+3)$

c) $\frac{3x}{2}+3=x$

d) $3(4x+2)=2(2x-1)$

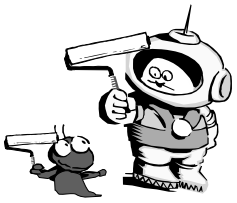
e) $\frac{5x+7}{9}=3$

f) $\frac{2x+7}{11}=3$

It's easy — you just put the 2 bits together and there's your equation. Then all you've got to do is solve it...

Q11 For what value of x is the expression $14-\frac{x}{2}$ equal to the value $\frac{3x-4}{2}$?

Q12 Two men are decorating a room. One has painted 20 m^2 and the other only 6 m^2. They continue painting and manage to paint another x m^2 each. If the first man has painted exactly three times the area painted by the second man, find the value of x.



Q13 Carol's father was 24 years old when Carol was born. Now he is four times as old as Carol. How old is Carol?

Q14 Mr Jones is 4 years older than his wife and 31 years older than his son. All three ages add up to 82 years. If Mr Jones is x years old, find the value of x and find the ages of his wife and son.

Q15 Solve the following:

a) $\frac{y}{2}+2=13$

b) $\frac{3x}{4}-2=4$

c) $\frac{2z}{5}-3=-5$

d) $\frac{1}{5}(x-4)=3$

e) $\frac{2}{3}(x+1)=16$

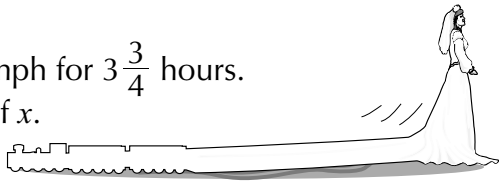
f) $\frac{3}{5}(4x-3)=15$

g) $\frac{8}{x^2}=\frac{32}{36}$

h) $\frac{12}{5x^2}=\frac{3}{20}$

i) $\frac{14}{3x^2}=\frac{2}{21}$

Q16 A train travels at 70 mph for x hours and then at 80 mph for $3\frac{3}{4}$ hours. If the train covers 405 miles of track, find the value of x.



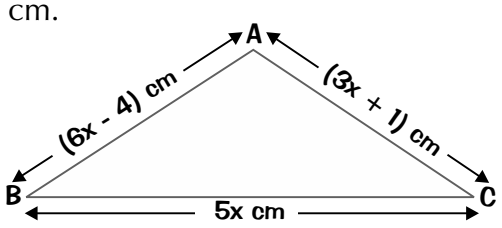
Q17 Solve the following:

a) $\frac{4x+3}{2}+x=\frac{5x+41}{4}$

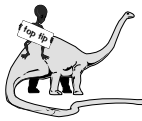
Remember to do the same to the top and the bottom.

b) $\frac{5}{7}(x-2)-\frac{3}{4}(x+3)=-4$

Q18 A triangle has lengths as shown below. Find the length of each side, if the length of AC exceeds that of AB by ½ cm.



Rearranging Formulas



Rearranging is getting the letter you want out of the formula and making it the subject. And it's exactly the same method as for solving equations, which can't be bad.

Q1 Rearrange the following formulas to make the letter in brackets the new subject.

a) $g = 10 - 4h$ (h)

b) $d = \frac{1}{2}(c + 4)$ (c)

c) $j = -2(3 - k)$ (k)

d) $a = \frac{2b}{3}$ (b)

e) $f = \frac{3g}{8}$ (g)

f) $y = \frac{x}{2} - 3$ (x)

g) $s = \frac{t}{6} + 10$ (t)

h) $p = 4q^2$ (q)

Q2 Jason is saving up to go travelling next year and has got a temporary job selling cars. He is paid a basic wage of £500 a month, plus a bonus of £50 for each car he sells. He has a spreadsheet to keep track of his money, which calculates his wages (£w) after working for m months and selling c cars, using the following formula:

$$w = 500m + 50c$$

a) Rearrange the formula to make c the subject.

b) Find the number of cars Jason needs to sell in 11 months to earn £12 100.

Q3 The cost of hiring a car is £28 per day plus 25p per mile.

a) Find the cost of hiring the car for a day and travelling:

i) 40 miles

ii) 80 miles

b) Write down a formula to give the cost of hiring a car (£c) for one day, and travelling n miles.

c) Rearrange the formula to make n the subject.

d) How many miles can you travel, during one day, if you have a budget of:

i) £34, **ii)** £50, **iii)** £56.50.

Q4 Rearrange the following formulas to make the letter in brackets the new subject.

a) $y = x^2 - 2$ (x)

b) $y = \sqrt{(x + 3)}$ (x)

c) $r = \left(\frac{s}{2}\right)^2$ (s)

d) $f = \frac{10 + g}{3}$ (g)

e) $w = \frac{5 - z}{2}$ (z)

f) $v = \frac{1}{3}x^2h$ (x)

g) $v^2 = u^2 + 2as$ (a)

h) $v^2 = u^2 + 2as$ (u)

i) $t = 2\pi\sqrt{\frac{l}{g}}$ (g)

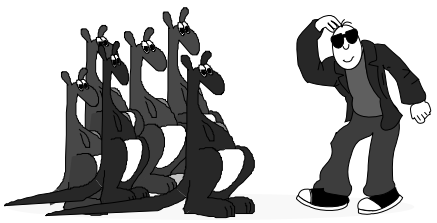
Q5 Mrs Smith buys x jumpers for £J each and sells them in her shop for a total price of £T.

a) Write down an expression for the amount of money she paid for all the jumpers.

b) Using your answer to **a)**, write down a formula for the profit £P Mrs Smith makes selling all the jumpers.

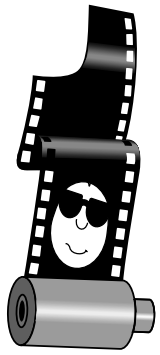
c) Rearrange the formula to make J the subject.

d) Given that Mrs Smith makes a profit of £156 by selling 13 jumpers for a total of £364 find the price she paid for each jumper originally.



Rearranging Formulas

Q6 A website offering digital photo printing charges 12p per print plus 60p postage.



a) Find the cost of ordering:
i) 12 prints.
ii) 24 prints.

b) Write down a formula for the cost C, in pence, of ordering x prints.

c) Rearrange the formula to make x the subject.

d) A regular customer is looking through old receipts to check she has been charged the right amount. How many prints should she have received in each of her last three transactions if she was charged:
i) £4.92
ii) £6.36
iii) £12.12

Q7 Rearrange the following formulas, by collecting terms in x and looking for common factors, to make x the new subject.

a) $xy = z - 2x$
b) $ax = 3x + b$
c) $4x - y = xz$
d) $xy = 3z - 5x + y$
e) $xy = xz - 2$
f) $2(x - y) = z(x + 3)$
g) $xyz = x - y - wz$
h) $3y(x + z) = y(2z - x)$

Q8 Rearrange the following to make the letter in brackets the new subject.

a) $pq = 3p + 4r - 2q$ (p)
b) $fg + 2e = 5 - 2g$ (g)
c) $a(b - 2) = c(b + 3)$ (b)
d) $pq^2 = rq^2 + 4$ (q)
e) $4(a - b) + c(a - 2) = ad$ (a)
f) $\frac{x^2}{3} - y = x^2$ (x)
g) $\sqrt{hk^2 - 14} = k$ (k)
h) $2\sqrt{x} + y = z\sqrt{x} + 4$ (x)
i) $\frac{a}{b} = \frac{1}{3}(b - a)$ (a)
j) $\frac{m + n}{m - n} = \frac{3}{4}$ (m)
k) $\sqrt{\frac{(d - e)}{e}} = 7$ (e)
l) $\frac{x - 2y}{xy} = 3$ (y)

These are getting quite tricky — you've got to <u>collect like terms</u>, before you can make anything else the subject.

Q9 Rearrange the following formulas to make y the new subject.

a) $x(y - 1) = y$
b) $x(y + 2) = y - 3$
c) $x = \frac{y^2 + 1}{2y^2 - 1}$
d) $x = \frac{2y^2 + 1}{3y^2 - 2}$

Factorising Quadratics

Q1 Factorise the quadratics first, and then solve the equations:

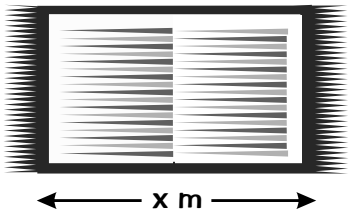
a) $x^2 + 3x - 10 = 0$
b) $x^2 - 5x + 6 = 0$
c) $x^2 - 2x + 1 = 0$
d) $x^2 - 4x + 3 = 0$
e) $x^2 - x - 20 = 0$
f) $2x^2 - 3x - 5 = 0$
g) $3x^2 + 4x - 7 = 0$
h) $x^2 + 14x + 49 = 0$
i) $2x^2 - 7x - 15 = 0$

Q2 Rearrange into the form "$x^2 + bx + c = 0$", then solve by factorising:

a) $x^2 + 5x = 36$
b) $x^2 = 5x$
c) $x^2 - 21 = 4x$
d) $x^2 + 48 = 26x$
e) $x + 5 - \frac{14}{x} = 0$
f) $x^2 - 3(x + 6) = 0$

Q3 Solve $x^2 - \frac{1}{4} = 0$.

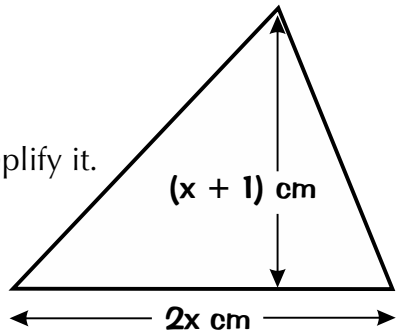
Q4 A rug has length x m. The width is exactly 1 m less than the length.



a) Write down an expression for the area of the rug.
b) If the area of the rug is 6 m^2, find the value of x.

Q5 A triangle has height $(x + 1)$ cm and a base of $2x$ cm.

a) Write down an expression for the area of the triangle and simplify it.
b) If the area of the triangle is 12 cm^2, find the value of x.



Q6 A square room has a floor of sides x metres.
The height of the walls is 3 m. Write down an expression for:

a) the floor area
b) the area of all four walls.
c) If the total area of the floor and the four walls is 64 m^2, form a quadratic equation and solve it to find x.

Q7 A photo has a length of l cm. Its width is $\frac{3}{4}$ cm shorter than the length.

a) Write down an expression for the area of the photo.
b) The photo is enlarged using a scale factor of 4.
i) Write an expression for the area of the enlarged photo.
ii) The area of the enlarged photo is 340 cm^2.
Work out the value of l.

Q8 Simplify the following fractions by factorising first.

a) $\frac{4x + 12}{x^2 + 8x + 15}$
b) $\frac{(x + 2)^2}{x^2 - x - 6}$
c) $\frac{6(x^2 - 2) + x}{9(x^2 - x) - 4}$

The Quadratic Formula

Q1 Find the two values, to 2 d.p, given by each of the following expressions:

a) $\frac{2 \pm \sqrt{3}}{2}$

b) $\frac{4 \pm \sqrt{10}}{3}$

c) $\frac{-2 \pm \sqrt{27}}{2}$

d) $\frac{-3 \pm \sqrt{42}}{3}$

e) $\frac{-10 \pm \sqrt{160}}{5}$

f) $\frac{-27 \pm \sqrt{10}}{2}$

g) $\frac{-8 \pm \sqrt{9.5}}{2.4}$

h) $\frac{10 \pm \sqrt{88.4}}{23.2}$

Q2 The following quadratics can be solved by factorisation, but practise using the formula to solve them.

a) $x^2 + 8x + 12 = 0$
b) $6x^2 - x - 2 = 0$
c) $x^2 - x - 6 = 0$
d) $x^2 - 3x + 2 = 0$
e) $4x^2 - 15x + 9 = 0$
f) $x^2 - 3x = 0$
g) $36x^2 - 48x + 16 = 0$
h) $3x^2 + 8x = 0$
i) $2x^2 - 7x - 4 = 0$
j) $x^2 + x - 20 = 0$
k) $4x^2 + 8x - 12 = 0$
l) $3x^2 - 11x - 20 = 0$
m) $x + 3 = 2x^2$
n) $5 - 3x - 2x^2 = 0$
o) $1 - 5x + 6x^2 = 0$
p) $3(x^2 + 2x) = 9$
q) $x^2 + 4(x - 3) = 0$
r) $x^2 = 2(4 - x)$

Step number 1...
Write out the formula.

Step number 2...
Write down values for a, b and c.

Step number 3... sub a, b and c into the formula. Make sure you divide the whole of the top line by 2a — not just ½ of it.

Q3 Solve the following quadratics using the formula.
Give your answers to no more than two decimal places.

a) $x^2 + 3x - 1 = 0$
b) $x^2 - 2x - 6 = 0$
c) $x^2 + x - 1 = 0$
d) $x^2 + 6x + 3 = 0$
e) $x^2 + 5x + 2 = 0$
f) $x^2 - x - 1 = 0$
g) $3x^2 + 10x - 8 = 0$
h) $x^2 + 4x + 2 = 0$
i) $x^2 - 6x - 8 = 0$
j) $x^2 - 14x + 11 = 0$
k) $x^2 + 3x - 5 = 0$
l) $7x^2 - 15x + 6 = 0$
m) $2x^2 + 6x - 3 = 0$
n) $2x^2 - 7x + 4 = 0$

Oops, forgot to mention step number 4...
check your answers by putting them back in the equation.

The Quadratic Formula

Q4 Rearrange the following in the form "$ax^2 + bx + c = 0$" and then solve by the quadratic formula. Give your answers to two decimal places.

a) $x^2 = 8 - 3x$

b) $(x + 2)^2 - 3 = 0$

c) $3x\,(x - 1) = 5$

d) $2x\,(x + 4) = 1$

e) $x^2 = 4(x + 1)$

f) $(2x - 1)^2 = 5$

g) $3x^2 + 2x = 6$

h) $(x + 2)(x + 3) = 5$

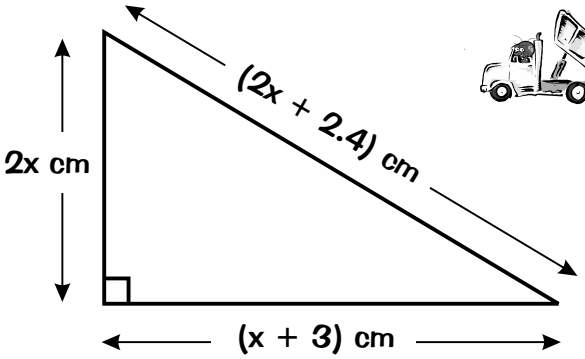
i) $(x - 2)(2x - 1) = 3$

j) $2x + \frac{4}{x} = 7$

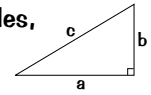
k) $\left(x - \frac{1}{2}\right)^2 = \frac{1}{4}$

l) $4x(x - 2) = -3$

Q5 The sides of a right angled triangle are as shown. Use Pythagoras' theorem to form a quadratic equation in x and then solve it to find x.



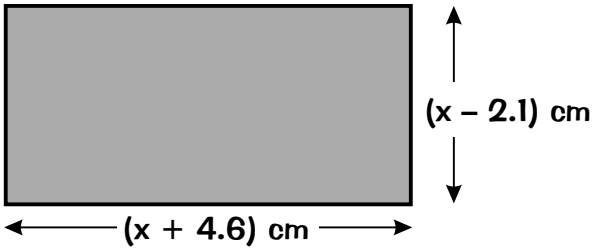
Remember, for right-angled triangles, Pythagoras' theorem is:
$a^2 + b^2 = c^2$



Q6 The area of a rectangle with length $(x + 4.6)$ cm and width $(x - 2.1)$ cm is 134.63 cm^2.

a) Form a quadratic equation and solve it to find x to two decimal places.

b) What is the rectangle's perimeter to one decimal place?



Completing the Square

All you're doing is writing quadratics in the form "$(x + 4)^2 + 2$" instead of "$x^2 + 8x + 18$" — don't let the name put you off.

Q1 Complete the square for the following expressions.

a) $x^2 - 4x - 5$
b) $x^2 - 2x + 1$
c) $x^2 + x + 1$
d) $x^2 - 6x + 9$
e) $x^2 - 6x + 7$
f) $x^2 - 4x$
g) $x^2 + 3x - 4$
h) $x^2 - x - 3$
i) $x^2 - 10x + 25$
j) $x^2 - 10x$
k) $x^2 + 8x + 17$
l) $x^2 - 12x + 35$

Q2 Solve the following quadratic equations by completing the square. Write down your answers correct to 2 d.p.

a) $x^2 + 3x - 1 = 0$
b) $x^2 - x - 3 = 0$
c) $x^2 + 4x - 3 = 0$
d) $x^2 + x - 1 = 0$
e) $x^2 - 3x - 5 = 0$
f) $2x^2 - 6x + 1 = 0$
g) $3x^2 - 3x - 2 = 0$
h) $3x^2 - 6x - 1 = 0$

If you've got an ax^2, take out a factor of 'a' before completing the square, like this: $3x^2 + 12x + 1 = 3(x^2 + 4x) + 1$

Q3 Solve the following quadratic equations by completing the square. Give your answers as surds, simplifying where possible.

a) $x^2 - 8x - 13 = 0$
b) $x^2 - 2x - 5 = 0$
c) $x^2 + 6x - 11 = 0$
d) $x^2 + 8x + 4 = 0$
e) $x^2 + 3x - 2 = 0$
f) $x^2 + 7x + 3 = 0$
g) $x^2 - 5x - 25 = 0$
h) $x^2 - 9x + 9 = 0$
i) $2x^2 + 4x - 7 = 0$
j) $3x^2 - 6x - 1 = 0$
k) $-2x^2 + 7x + 3 = 0$
l) $-3x^2 + 5x - 1 = 0$

Q4 By completing the square, find the coordinates of the minimum point on the graph of each of the following equations.

a) $y = x^2 - 10x - 3$
b) $y = x^2 - 7x - 15$
c) $y = 3x^2 + 12x + 2$
d) $y = 4x^2 - 5x - 1$

The minimum point occurs when the bracket in the completed square equals 0.

Q5 By completing the square, find the coordinates of the turning point on the graph of each of the following equations. In each case, state whether the turning point is a maximum or a minimum.

a) $y = x^2 + 4x - 1$
b) $y = -x^2 + 11x - 6$
c) $y = -x^2 - 3x + 8$
d) $y = 2x^2 + 8x - 11$
e) $y = 2x^2 + 5x - 1$
f) $y = -3x^2 - 6x + 2$

Q6 For each of the following, sketch the graph of the equation and label the turning point and y-intercept with their coordinates.

a) $y = x^2 + 2x - 1$
b) $y = -x^2 + 3x + 2$
c) $y = 2x^2 - x - 5$

Q7 The turning point of the graph of $y = x^2 + mx + n$ is at $(1, -5)$. What are the values of m and n?

Algebraic Fractions

Multiplying and dividing algebraic fractions is just the same as multiplying and dividing normal fractions — to multiply you just multiply the tops and bottoms separately and to divide you just turn the second fraction upside down and then multiply. Simple.

Q1 Simplify the following by cancelling down where possible:

a) $\frac{27x^4y^2z}{9x^3yz^2}$ **b)** $\frac{48a^2b^2}{(2a)^2c}$ **c)** $\frac{3xyz}{9x^2y^3z^4}$ **d)** $\frac{4p^3q^3}{(2pr)^3}$

Q2 Multiply out the following, leaving your answers as simplified as possible:

a) $\frac{x^2}{y} \times \frac{2}{x^3}$

b) $\frac{3a^4}{2} \times \frac{b}{a^2}$

c) $\frac{2x}{y^2} \times \frac{y^3}{4x^3}$

d) $\frac{3pq}{2} \times \frac{4r^2}{9p}$

e) $\frac{10z^3}{xy} \times \frac{4x^3}{5z}$

f) $\frac{30a^2b^2c^2}{7} \times \frac{21c^2}{ab^3}$

g) $\frac{4}{x} \times \frac{x^3}{2} \times \frac{x}{10}$

h) $\frac{2a^2}{3} \times \frac{9b}{a} \times \frac{2a^2b}{5}$

i) $\frac{5a^2b}{b} \times \frac{3a^2c^3}{10bd}$

j) $\frac{p^2}{pq^2} \times \frac{q^2}{p}$

k) $\frac{90r^2}{14t} \times \frac{7t^3}{30r}$

l) $\frac{400d^4}{51e^5} \times \frac{102d^2e^4}{800e^2f}$

It helps if you can cancel some factors before multiplying.

Q3 Divide the following, leaving your answer as simplified as possible:

a) $\frac{4x^3}{y} \div \frac{2x}{y^2}$

b) $\frac{ab}{c} \div \frac{b}{c}$

c) $\frac{30x^3}{y^2} \div \frac{10x}{y}$

d) $\frac{pq}{r} \div \frac{2}{r}$

e) $\frac{e^2f^2}{5} \div \frac{ef}{10}$

f) $\frac{5x^3}{y} \div \frac{1}{y}$

g) $\frac{16xyz}{3} \div \frac{4x^2}{9}$

h) $\frac{20a^3}{b^3} \div \frac{5}{b^2}$

i) $\frac{25a^3}{b^3} \div \frac{5}{b^2}$

j) $\frac{4x}{y^4z^4} \div \frac{2}{y^2z^3}$

k) $\frac{3m}{2n^2} \div \frac{m}{4n}$

l) $\frac{70f^3}{g} \div \frac{10f^4}{g^2}$

Q4 Simplify the following expressions:

a) $\frac{9a^2 - 16}{6a + 8}$ **b)** $\frac{4x^2 - y^2}{8x + 4y}$ **c)** $\frac{25x^2 - 36}{15x - 18}$

Q5 Solve the following equations for x:

a) $\frac{20x^4y^2z^3}{7xy^5} \times \frac{14y^3}{40x^2z^3} = 5$ **b)** $\frac{48x^5y^2}{12z^3} \div \frac{16x^2y^2}{z^3} = 2$

Algebraic Fractions

OK, I guess it gets a bit tricky here — you've got to cross-multiply to get a common denominator before you can get anywhere with adding or subtracting.

Q6 Add the following, simplifying your answers:

a) $\frac{3}{2x} + \frac{y}{2x}$

b) $\frac{1}{x} + \frac{y}{x}$

c) $\frac{4xy}{3z} + \frac{2xy}{3z}$

d) $\frac{(4x+2)}{3} + \frac{(2x-1)}{3}$

e) $\frac{5x+2}{x} + \frac{2x+4}{x}$

f) $\frac{6x}{3} + \frac{2x+y}{6}$

g) $\frac{x}{8} + \frac{2+y}{24}$

h) $\frac{x}{10} + \frac{y-1}{5}$

i) $\frac{2x}{3} + \frac{2x}{4}$

j) $\frac{x}{6} + \frac{5x}{7}$

k) $\frac{x}{3} + \frac{x}{y}$

l) $\frac{zx}{4} + \frac{x+z}{y}$

Q7 Subtract the following, leaving your answers as simplified as possible:

a) $\frac{4x}{3} - \frac{5y}{3}$

b) $\frac{4x+3}{y} - \frac{4}{y}$

c) $\frac{(8x+3y)}{2x} - \frac{(4x+2)}{2x}$

d) $\frac{(9-5x)}{3x} - \frac{(3+x)}{3x}$

e) $\frac{10+x^2}{4x} - \frac{x^2+11}{4x}$

f) $\frac{2x}{3} - \frac{y}{6}$

g) $\frac{z}{5} - \frac{2z}{15}$

h) $\frac{4m}{n} - \frac{m}{3}$

i) $\frac{2b}{a} - \frac{b}{7}$

j) $\frac{(p+q)}{2} - \frac{3p}{5}$

k) $\frac{p-2q}{4} - \frac{2p+q}{2}$

l) $\frac{3x}{y} - \frac{4-x}{3}$

Q8 Simplify the following:

a) $\left(\frac{a}{b} \div \frac{c}{d}\right) \times \frac{ac}{bd}$

b) $\frac{x^2+xy}{x} \times \frac{z}{xz+yz}$

c) $\frac{(p+q)}{r} \times \frac{3}{2(p+q)}$

d) $\frac{m^2n}{p} + \frac{mn}{p^2}$

e) $\frac{1}{x+y} + \frac{1}{x-y}$

f) $\frac{2}{x} - \frac{3}{2x} + \frac{4}{3x}$

g) $\frac{a+b}{a-b} + \frac{a-b}{a+b}$

h) $\frac{1}{4pq} \div \frac{1}{3pq}$

i) $\frac{x}{8} - \frac{x+y}{4} + \frac{x-y}{2}$

Q9 Simplify the following:

You'll need to factorise some of these first.

a) $\frac{a+b}{a-b} + \frac{a-b}{a+b}$

b) $\frac{3x+1}{2x-3} + \frac{x-2}{2x+5}$

c) $\frac{2x+2}{y^2} \times \frac{xy}{x+1}$

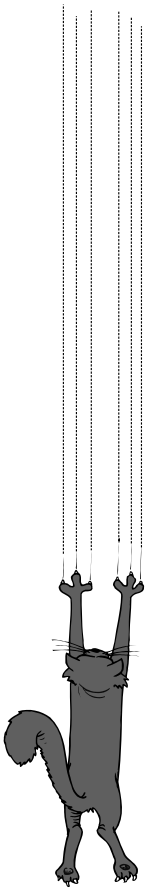
d) $\frac{2x^2+x-3}{3x-3} \div \frac{4x^2y+8xy+3y}{5y}$

e) $\frac{x+3}{x^2+3x-4} \times \frac{x-1}{x^2-9}$

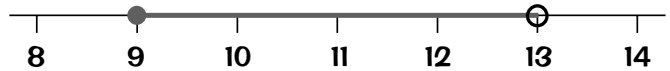
f) $\left(\frac{4x+24}{3x^2+36x+60} \div \frac{x-9}{2x+4}\right) \times (x^2+x-90)$

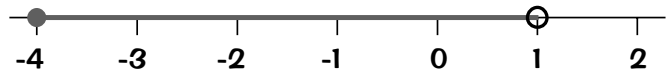
Inequalities

Yet another one of those bits of Maths that looks worse than it is — these are just like equations, really, except for the symbols.

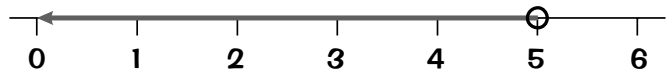


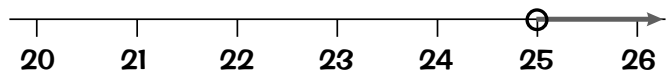
Q1 Write down the inequality represented by each diagram below.

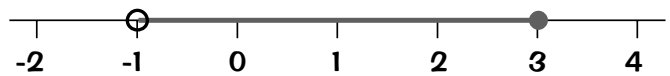
a)


b)


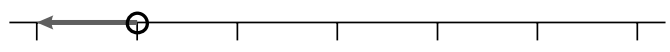
c)


d)


e)


f)


g)


h)


Q2 By drawing an appropriate part of the number line for each question, represent each of the following inequalities.

a) $x > 5$
b) $x \leq 2$
c) $-5 < x < 2$
d) $-2 \leq x < 3$
e) $-2 < x \leq 3$
f) $6 < x \leq 7$
g) $-3 \leq x \leq -2$
h) $-3 < x \leq 0$

Q3 Solve the following:

a) $3x + 2 > 11$
b) $5x + 4 < 24$
c) $5x + 7 \leq 32$
d) $3x + 12 \leq 30$
e) $2x - 7 \geq 8$
f) $17 + 4x < 33$
g) $2(x + 3) < 20$
h) $2(5x - 4) < 32$
i) $5(x + 2) \geq 25$
j) $4(x - 1) > 40$
k) $10 - 2x > 4x - 8$
l) $7 - 2x \leq 4x + 10$
m) $8 - 3x \geq 14$
n) $16 - x < 11$
o) $16 - x > 1$
p) $12 - 3x \leq 18$

Q4 Find the largest integer x, such that $2x + 5 \geq 5x - 2$.

Q5 When a number is subtracted from 11, and this new number is then divided by two, the result is always less than five. Write this information as an inequality and solve it to show the possible values of the number.

Q6 Solve the following:

a) $-2 < x - 3 < 5$
b) $1 \leq x + 1 \leq 9$
c) $7 < 2x + 3 < 11$
d) $7 < 4 - 3x \leq 16$
e) $2 \leq \frac{x}{5} + 1 \leq 3$
f) $-2 < \frac{x - 3}{2} < 10$
g) $1 \leq \frac{2x + 7}{3} < 5$
h) $-10 < \frac{x + 1}{5} \leq -2$
i) $-1 \leq \frac{2 - x}{3} < 6$

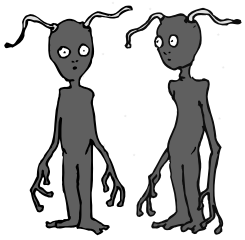
Inequalities

Q7 Two schools are merging and a new school is being built to accommodate all the pupils. There will be 1,130 pupils in total in the new school. No class must have more than 32 pupils.
How many classrooms are needed?
Show this information as an inequality.

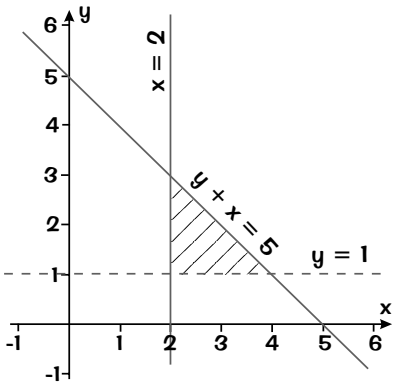
Call the number of classrooms x.

Q8 A couple are planning their wedding. For the reception in a local hotel, they have a budget of £900. If the hotel charges £18 per head, how many guests could be invited? Show this information as an inequality.

Q9 The shaded region satisfies three inequalities.
Write down these inequalities.



Remember the difference between solid and dotted lines.



Q10 Draw a set of axes with the x-axis from –2 to 6 and the y-axis from –1 to 7.
Show on a graph the region enclosed by the following three inequalities.

$$y < 6, \qquad x + y \geq 5 \qquad \text{and} \qquad x \leq 5$$

Q11 Draw and label a number line from –5 to 5 for each of the following inequalities.
Represent the inequalities on your number lines.

a) $x^2 \leq 4$ **c)** $x^2 \leq 9$ **e)** $16 \geq x^2$ **g)** $9 > x^2$
b) $x^2 < 1$ **d)** $25 \geq x^2$ **f)** $x^2 \leq 1$ **h)** $x^2 \leq 0$

Q12 A company are recruiting new members of staff. All applicants must take two online tests. To get an interview, applicants must score higher than 5 on the first test, at least 7 on the second, and have a total combined score of at least 14.

a) Write out three inequalities to represent the three criteria for getting an interview. Use x for the score on the first test and y for the score on the second test.

b) The company want to analyse the quality of applicants by plotting their test scores on a graph, and picking out the ones who satisfy the criteria. Using suitable axes, show on a graph the region enclosed by the three inequalities where suitable candidates would be placed.

Sketching the graphs will help with these.

Q13 Solve the following:

a) $x^2 - x - 6 < 0$
b) $x^2 + 6x + 5 \leq 0$
c) $x^2 + 3x + 2 > 0$
d) $x^2 - 10x + 16 \geq 0$
e) $x^2 - 18 > 7x$
f) $12x + 35 < -x^2$
g) $-x^2 + 9x - 20 \leq 0$
h) $-x^2 - 2x + 3 > 0$
i) $2x^2 + 7x + 6 \leq 0$
j) $9x - 2x^2 < 4$
k) $3x^2 < x + 2$
l) $4x^2 + 11x \geq 20$

Simultaneous Equations and Graphs

Q1 Solve the following simultaneous equations by drawing graphs. Use values $0 \leqslant x \leqslant 6$

a) $y = x$
$y = 9 - 2x$

b) $y = 2x + 1$
$2y = 8 + x$

c) $y = 4 - 2x$
$x + y = 3$

d) $y = 3 - x$
$3x + y = 5$

e) $2x + y = 6$
$y = 3x + 1$

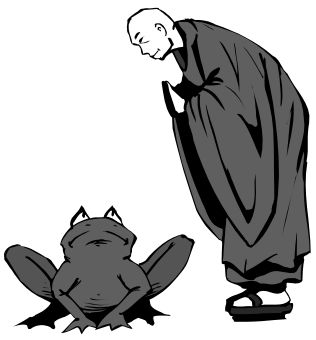
f) $y = 2x$
$y = x + 1$

g) $x + y = 5$
$2x - 1 = y$

h) $2y = 3x$
$y = x + 1$

i) $y = x - 3$
$y + x = 7$

j) $y = x + 1$
$2x + y = 10$



Q2 The diagram shows the graphs:
$y = x^2 - x$
$y = x + 2$
$y = 8$
$y = -2x + 4$

Use the graphs to find the solutions to:

a) $x^2 - x = 0$

b) $x^2 - x = x + 2$

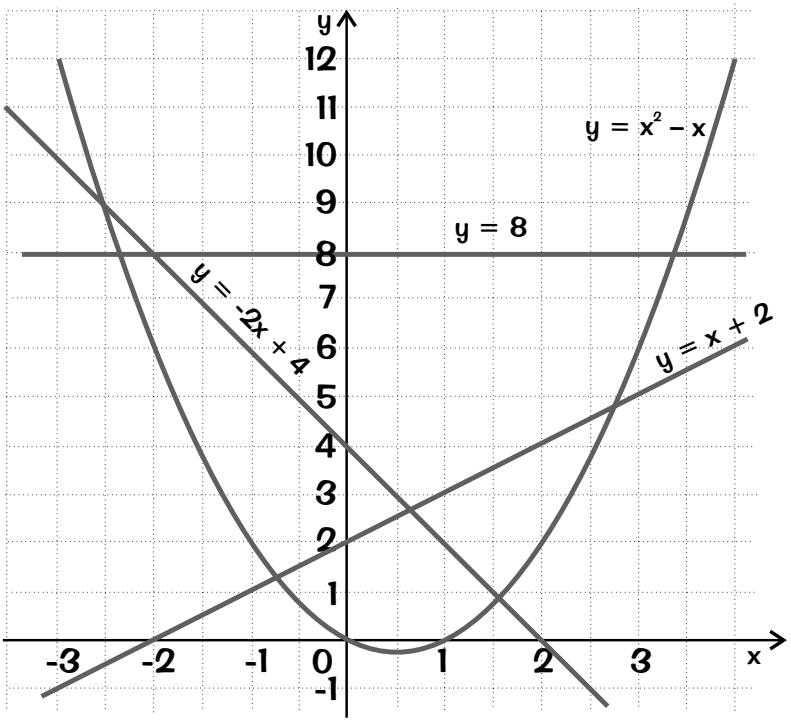
c) $x^2 - x = 8$

d) $x^2 - x = -2x + 4$

e) $-2x + 4 = x + 2$

f) $x^2 - x - 8 = 0$

g) $x^2 + x = 4$



These equations look a bit nasty, but they're just made up of the equations you've got graphs for. And you know how to do the rest of it, don't you...

Q3 Complete this table for $y = -\frac{1}{2}x^2 + 5$:

x	-4	-3	-2	-1	0	1	2	3	4
$-\frac{1}{2}x^2$									
+5									
y									

Draw the graph $y = -\frac{1}{2}x^2 + 5$.
Use your graph to solve the following equations (to 1 d.p.):

a) $-\frac{1}{2}x^2 + 5 = 0$ **b)** $-\frac{1}{2}x^2 + 5 = -3$ **c)** $-\frac{1}{2}x^2 + 5 = x$

Simultaneous Equations

To solve simultaneous equations from scratch, you've got to get rid of either x or y first — to leave you with an equation with just one unknown in it.

Q1 Use the linear equation (the one with no x^2s in it) to find an expression for y. Then substitute it into the quadratic equation (the one with x^2s in it), to solve these equations:

a) $y = x^2 + 2$
$y = x + 14$

b) $y = x^2 - 8$
$y = 3x + 10$

c) $y = 2x^2$
$y = x + 3$

d) $x + 5y = 30$
$x^2 + \frac{4}{5}x = y$

e) $y = 1 - 13x$
$y = 4x^2 + 4$

f) $y = 3(x^2 + 3)$
$14x + y = 1$

Q2 Solve the following simultaneous equations:

a) $4x + 6y = 16$
$x + 2y = 5$

b) $3x + 8y = 24$
$x + y = 3$

c) $3y - 8x = 24$
$3y + 2x = 9$

d) $y = x^2 - 2$
$y = 3x + 8$

e) $y = 3x^2 - 10$
$13x - y = 14$

f) $y + 2 = 2x^2$
$y + 3x = 0$

g) $3y - 10x - 17 = 0$
$\frac{1}{3}y + 2x - 5 = 0$

h) $\frac{x}{2} - 2y = 5$
$12y + x - 2 = 0$

i) $x + y = \frac{1}{2}(y - x)$
$x + y = 2$

Q3 Two farmers are buying livestock at a market. Farmer Ed buys 6 sheep and 5 pigs for £430 and Farmer Jacob buys 4 sheep and 10 pigs for £500.

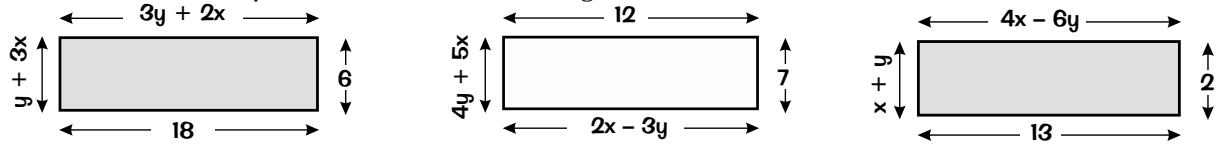
a) If sheep cost £x and pigs cost £y, write down the two purchases as a pair of simultaneous equations.

b) Solve for x and y.

Q4 On Farmer Palmer's farm, the cats have got into the chicken coop and are causing chaos. Farmer Palmer counts, in total, 11 heads and 30 legs. How many cats and how many chickens are in the chicken coop?

Q5 Isobel is buying pick & mix sweets. She weighs out 20 jellies and 30 toffees which come to 230 g. She takes one of each off the scales before they get bagged up, and the weight drops to 221 g. How much does an individual toffee weigh?

Q6 Find the value of x and y for each of the following rectangles, by first writing down a pair of simultaneous equations and then solving them.



Q7 Two customers enter a shop to buy milk and cornflakes. Mrs Smith buys 5 pints of milk and 2 boxes of cornflakes and spends £3.44. Mr Brown buys 4 pints of milk and 3 boxes of cornflakes and receives £6.03 change after paying with a £10 note. Write down a pair of simultaneous equations and solve them to find the price in pence of a pint of milk (m) and a box of cornflakes (c).

Q8 Solve $\frac{3(x-y)}{5} = x - 3y = x - 6$.

Sequences

Ahhh, sequences — you really need to know all about them and their n^{th} terms. They're a doddle once you figure out what's happening in each gap...

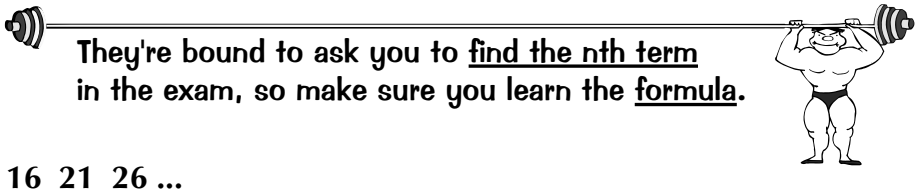
Q1 Write down the next 3 terms in each of these sequences and describe the sequence.

a) 2, 4, 6, 8, ...

b) 1, 3, 5, 7, ...

c) 1, 4, 9, 16, ...

d) 1, 8, 27, 64, ...



They're bound to ask you to find the nth term in the exam, so make sure you learn the formula.

Q2 **6 11 16 21 26 ...**

a) What are the next 3 terms in this sequence?

b) What is the difference between each term?

c) Write down a formula for the nth term of this sequence.

d) Use the formula to find the 20th term of the sequence.

OK then, I'll tell you the formula just this once: $a + (n - 1)d$ (d = difference, a = 1st term) LEARN IT!

Q3 In the following sequences, write down the next 3 terms and the nth term:

a) 2, 4, 6, 8, …

b) 1, 3, 5, 7, …

c) 5, 10, 15, 20, …

d) 5, 8, 11, 14, …

e) 7, 10, 13, 16,...

f) 12, 17, 22, 27,...

g) 6, 16, 26, 36,...

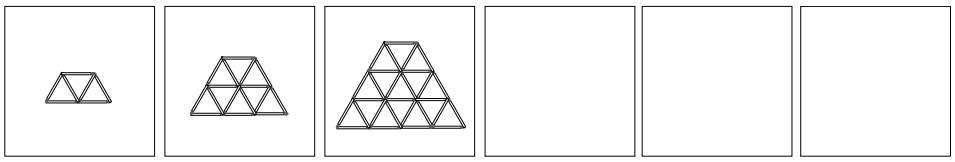
h) 54, 61, 68, 75,...

Q4 **1 5 9 13 ...**

a) Find an expression for the nth term of this sequence.

b) Is 75 a term in this sequence? Explain your answer.

Q5



The pattern above is based on individual triangles.
Work out the number of individual triangles that would be in each of the next three groups.

Q6 10, 20, 15, 17½, 16¼...

a) Write down the next 4 terms.

b) Explain how you would work out the 10th term.

Q7 An arithmetic series starts 7 + 11 + 15 + 19...

Find the sum of the first 16 terms.

The formula for the sum of the first n terms is a nasty one: $S_n = \frac{n}{2}[2a + (n - 1)d]$ Don't worry though — it'll be on your formula sheet in the exams.

Q8 Given that the 10th term in an arithmetic series is 17, and the 20th term is 37:

a) Work out the common difference d and the first term a.

b) Find the sum of the first 100 terms of the series.

Proof

Proofs look pretty scary but they're usually alright once you get started. Remember, you can use 2n to represent an even number and 2n + 1 to represent an odd number.

Q1 Prove that $(2n + 1)^2 - (2n - 1)^2 - 10$ is not a multiple of 8 for positive integer values of n.

Q2 Prove that the sum of any three consecutive numbers is divisible by 3.

Q3 Prove that an even number multiplied by another even number will always result in an even number.

Q4 Prove that the sum of any three consecutive even numbers is always a multiple of 6.

You might have to prove that a statement isn't true.
The best way to do this is to find an example that doesn't work...

Q5 Prove that these statements are wrong.
a) If the sum of two integers is even, one of the integers must be even.
b) If n is an integer and n^2 is divisible by 4, then n is also divisible by 4.

Q6 Christine says "if $a^2 = b^2$, then $a = b$". Prove that Christine is wrong.

You'll need all kinds of maths tools to tackle these last few proofs — power rules, algebraic fractions, sequences... Don't panic if you're feeling stuck — you might need to experiment a bit before you find the right method.

Q7 Prove that $5^{20} - 5^{19}$ is even, without using a calculator.

Q8 Without using a calculator, prove that $3^8 - 1$ is not a prime number.

Try thinking of this as a difference of two squares.

Q9 If $x > 0$, $y > 0$ and $x < y$, prove that $\frac{y^2 + 1}{y^2} - \frac{x^2 + 1}{x^2} < 0$.

Q10 The nth term of a sequence is given by $n^2 - 2n + 2$.
Prove that the sum of any two consecutive numbers in the sequence is an odd number.

Q11 Fay claims that $x^2 + 3 > 2x + 1$ for all values of x.
Prove that Fay is correct.

Try completing the square for this one...

Direct and Inverse Proportion

Q1 y is directly proportional to x. If $y = 5$ when x is 25, find y when x is 100.

Q2 If $y \propto x$ and $y = 132$ when $x = 10$, find the value of y when $x = 14$.

Q3 If $y = 3$ when $x = 8$ and y is inversely proportional to x, find the value of y when $x = 12$.

Q4 If $y \propto \frac{1}{x}$ and $x = 4$ when $y = 5$, find the value of x when $y = 10$.

Q5 Given that $y \propto \frac{1}{x}$, complete this table of values.

x	1	2	3	4	5	6
y					9.6	

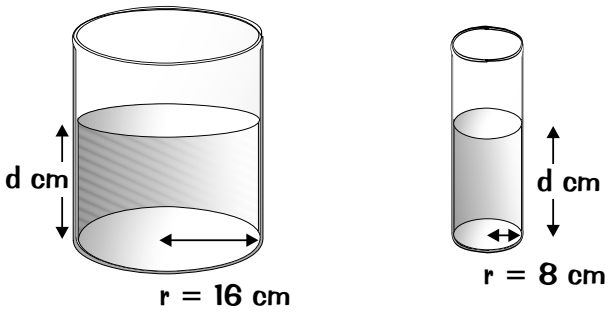
Put the numbers into the equation $y = k/x$ to find the value of k. Then you can find the rest of the ys.

Q6 Given that y varies inversely as the square of x, complete the following table of values, given that x is always positive.

x	1	2	5	
y			4	1

x	2			8
y	24	6	$2\frac{2}{3}$	

Q7 Two cylindrical containers are filled to the same depth, d cm, with water. The mass of the water in each container is proportional to the square of the radius of each container. The first container has a radius of 16 cm and the water has a mass of 16 kg. If the second container has a radius of 8 cm, find the mass of the water inside it.



Q8 Given that r varies inversely as the square of s, and $r = 24$ when $s = 10$, find the values of:

a) r when $s = 5$

b) s when $r = 150$, given that s is positive

c) r when $s = 2$

d) s when $r = 37\frac{1}{2}$, given that s is negative

Don't forget about that little joker, the "inverse square" variation — they'll expect you to know that, too.

Q9 The gravitational pull of the Earth is inversely proportional to the square of the distance from the centre of the Earth. At the Earth's surface (approx. 6371 km from the centre) the gravitational pull is around 9.8 N kg^{-1}. When launching a satellite into space, the gravitational pull helps determine the orbit. What would be the gravitational pull on a satellite at a height of 100 km above the Earth's surface (to 1 d.p)?

Coordinates

Remember — 1) x comes before y
2) x goes a-cross (get it) the page. (Ah, the old ones are the best...)

Q1 ABCD is a parallelogram. A is (-1, 3), B is (-2,-1) and C is (4,-1).
Draw axes with x from -3 to 5 and y from -2 to 4.
Plot A, B and C then find the missing coordinates for D.

Q2 Draw axes with x from -9 to 9 and y from -12 to 12.
On the same set of axes draw the following shapes and find their missing pair of coordinates.

a) ABCD is a square
A is (1, 1)
B is ?
C is (-3,-3)
D is (-3, 1)

b) ABCD is a parallelogram
A is (2, 8)
B is (6, 8)
C is ?
D is (1, 5)

c) ABCD is a rectangle
A is ?
B is (3,-8)
C is (3,-6)
D is (-5,-6)

d) ABCD is a kite
A is (-9, 3)
B is (-6, 8)
C is (-4, 8)
D is ?

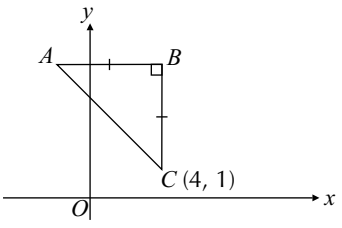
e) ABCD is a parallelogram
A is (-2,-10)
B is (4,-10)
C is (6,-12)
D is ?

f) ABCD is a parallelogram
A is (-8, 10)
B is (-6, 10)
C is ?
D is (-5, 12)

Q3 ABCD is a rectangle with the line $x = 0$ as a line of symmetry.
Draw axes with x from -3 to 3 and y from -3 to 3.
If A = (-2,-2) and B is (-2, 1), find the coordinates of C and D.

I'm afraid you'll just have to get the hang of using things like "the line x = 4", as they seem to prefer it to "a vertical line through the point 4 on the horizontal axis". Yeah, that is a bit long winded, I guess — so maybe they've got a point...

Q4 The isosceles triangle ABC shown below has two equal sides of length 6 units.
Find the coordinates of point A.



Coordinates

Q5 The coordinates of three vertices of a parallelogram are (0, –2), (3, –2) and (1, 2). Draw axes with x from -4 to 4 and y from -1 to 4. If the fourth vertex lies on the line $y = 2$, give its two possible coordinates.

Q6 Find the midpoint of the line AB, where A and B have coordinates:

a) A(2,3) B(4,5)

b) A(1,8) B(10,2)

c) A(0,11) B(11,11)

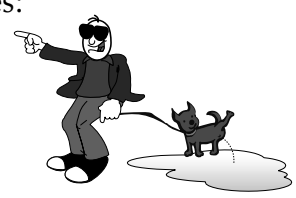
d) A(3,15) B(14,3)

e) A(6,7) B(0,0)

f) A(16,16) B(3,3)

g) A(8,33) B(32,50)

h) A(17,28) B(44,13)



Your answers should be coordinates too.

Q7 Anna is designing the plan of a kitchen using some computer aided design software. The coordinates of the room on screen are (0, 10), (220, 10), (0, 260), (220, 260). She needs to enter the coordinates of the ceiling light, which will be exactly in the centre of the room. What will the coordinates of the light be?

Q8 Find the midpoints of each of these lines:

a) Line PQ, where P has coordinates (–1,5) and Q has coordinates (5,6).

b) Line AB, where A has coordinates (–3,3) and B has coordinates (4,0).

c) Line RS, where R has coordinates (4,–5) and S has coordinates (0,0).

d) Line PQ, where P has coordinates (–1,–3) and Q has coordinates (3,1).

e) Line GH, where G has coordinates (10,13) and H has coordinates (–6,–7).

f) Line CD, where C has coordinates (–4,6) and D has coordinates (12,–7).

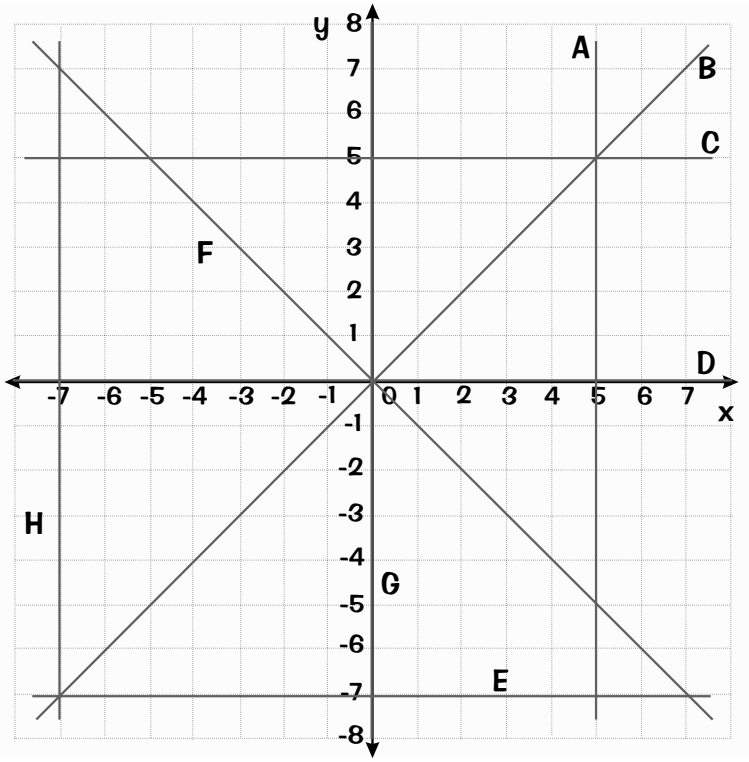
g) Line MN, where M has coordinates (–5,–8) and N has coordinates (–21,–17).

h) Line AB, where A has coordinates (–1,0) and B has coordinates (–9,–14).

Straight-Line Graphs

Q1 Which letters represent the following lines:

a) $x = y$
b) $x = 5$
c) $y = -x$
d) $x = 0$
e) $y = -7$
f) $x + y = 0$
g) $y = 5$
h) $x - y = 0$
i) $y = 0$
j) $x = -7$?



Don't get confused if you've got "x + y = ... " — just rearrange the equation to "y = -x + ..." and as if by magic, you've got a line you recognise.

Q2 Complete the following table for the line $y = 3x - 1$:

x	-4	-3	-2	-1	0	1	2	3	4
3x									
-1									
y									

Plot these points on graph paper and hence draw the graph of $y = 3x - 1$.
Use a scale of 1 cm for 2 units on the y-axis and 2 cm for 1 unit on the x-axis.

Q3 Complete the following table for the line $y = \frac{1}{2}x - 3$:

x	-6	-4	-2	0	2	4	6
½ x							
-3							
y							

Plot these points on graph paper and hence draw the graph of $y = \frac{1}{2}x - 3$.

Straight-Line Graphs

If you know it's a straight line, you only really need <u>two</u> points, but it's always a <u>good idea</u> to plot three — it's a bit of a safety net, really.

Q4 Complete this table of values for $y = 2x + 3$:

x	0	3	8
y			

Plot these points on graph paper and draw the graph of $y = 2x + 3$.
Use your graph to find:

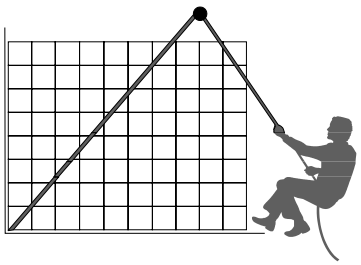
a) The value of y when $x = 5$
b) The value of y when $x = 2$
c) The value of x when $y = 11$
d) The value of x when $y = 17$

Q5 Complete this table of values for $y = \frac{1}{4}x - 3$:

x	-8	-4	8
y			

Plot these points on graph paper and draw the graph of $y = \frac{1}{4}x - 3$.
Use your graph to find:

a) The value of y when $x = 2$
b) The value of y when $x = 0$
c) The value of x when $y = -2$
d) The value of x when $y = -1.5$



Q6 The cost of electricity is calculated using the formula:
Total cost = Fixed charge + (cost per unit × number of units).
Customers can choose two different methods of payment:
Method A: Fixed charge £10, cost per unit 25p
Method B: Fixed charge £40, cost per unit 5p
Copy and complete this table:

Number of Units used	0	100	200	300
Cost using method A				
Cost using method B				

Plot these points on a graph (put the number of units on the horizontal axis, cost on the vertical axis):

a) Use your graph to find the total cost when 70 units are used for:
i) Method A
ii) Method B

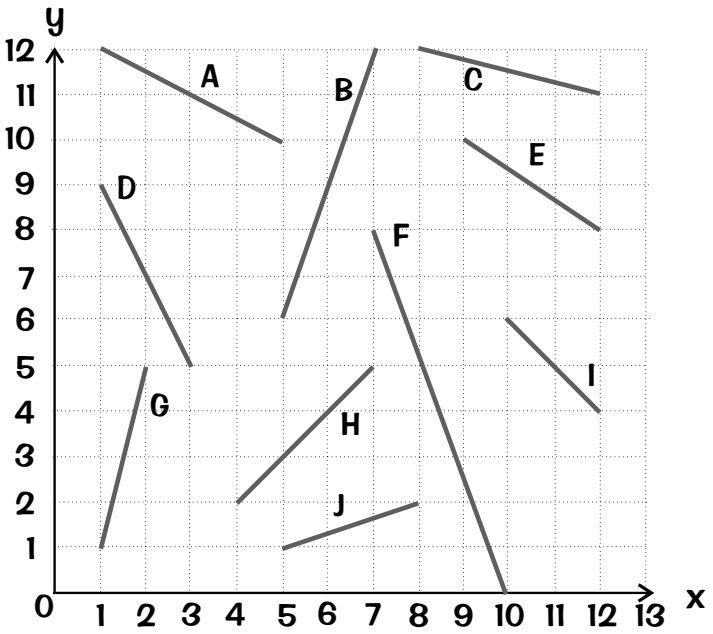
b) Miss Wright used 75 units. Which method should she use to minimize her bill, Method A or Method B?

c) Use your graph to work out how many units Miss Wright would have to use for both methods to cost the same amount.

Finding the Gradient

Q1 What is the gradient of:

a) line A
b) line B
c) line C
d) line D
e) line E
f) line F
g) line G
h) line H
i) line I
j) line J
k) a line parallel to A
l) a line parallel to B



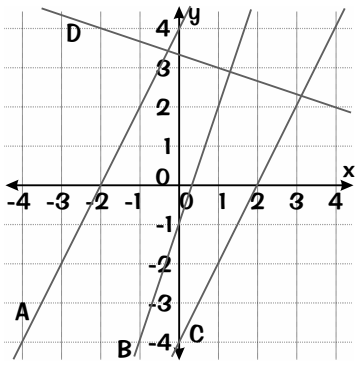
Uphill gradients are always positive, downhill always negative. Impressed? Hmmm....thought not. Can be a bit of an uphill battle, these.

Q2 What is the gradient of the lines joining the points:

a) (3, 5) and (5, 9)
b) (6, 3) and (10, 5)
c) (-6, 4) and (-3, 1)
d) (8, 2) and (4, 10)
e) (8, 5) and (6, 4)
f) (-3, -1) and (1, -4)?

Q3 Look at the graph on the right.

a) Which two lines have the same gradient?
b) Line E has a gradient of –1 and passes through the point (3, 0).
What are the coordinates of the point where it intersects with Line B?



Q4 Lauren works in a ski resort grading ski runs. A blue run has a gradient shallower than -0.2, a red is steeper than a blue, but has a gradient shallower than -0.25. Anything steeper is a black.
A run covers a horizontal distance of 1.75 km long and descends 400 meters.
What colour should Lauren grade it?

"y = mx + c"

Writing the equation of a line in the form y = mx + c gives you a nifty way of finding the gradient and y-intercept. Remember that — it'll save you loads of time. Anything for an easy life...

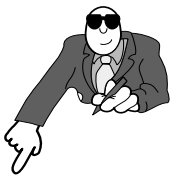
Q1 For each of the following lines, give the gradient and the coordinates of the point where the line cuts the y-axis.

a) $y = 4x + 3$
b) $y = 3x - 2$
c) $y = 2x + 1$
d) $y = -3x + 3$
e) $y = 5x$
f) $y = -2x + 3$
g) $y = -6x - 4$
h) $y = x$
i) $y = -\frac{1}{2}x + 3$
j) $y = \frac{1}{4}x + 2$
k) $3y = 4x + 6$

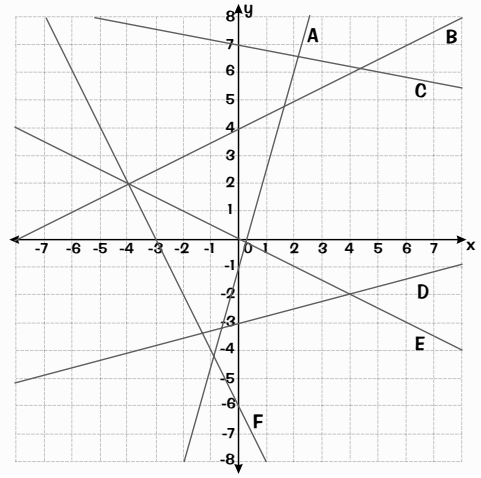
I know these are a bit algebra-ish, but don't worry, they won't bite.

Q2 Find the equations of the following lines:

a) A **b)** B **c)** C
d) D **e)** E **f)** F



Yeah, OK, this sounds a bit scary, but just work out the gradient (m) and look at the y-intercept (c) and pop them back into "y = mx + c"... easy lemons.



Q3 Find the equation of the straight line which passes through:

a) (3, 7) and has a gradient of 1
b) (2, 8) and has a gradient of 3
c) (4, –4) and has a gradient of –1
d) (–1, 7) and has a gradient of –3.

Q4 Write down the equation of the line which passes through the points:

a) (2, 2) and (5, 5)
b) (1, 3) and (4, 12)
c) (1, 0) and (5, –12)
d) (–5, 6) and (–1, –2).

Q5 What is the value of x or y if:

a) the point $(x, 13)$ is on the line $y = 3x + 1$
b) the point $(x, -2)$ is on the line $y = \frac{1}{2}x - 6$
c) the point $(4, y)$ is on the line $y = 2x - 1$
d) the point $(-3, y)$ is on the line $y = -3x$?

Q6 Which of the following points lie on the line $y = 3x - 1$?
(7, 20), (6, 15), (5, 14)

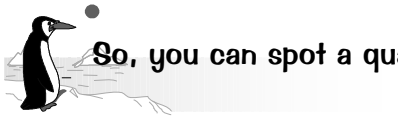
Q7 Line A has the equation $y = 3x - 4$.
Line B is parallel to line A and passes through (8, 25).

a) Find the gradient of line B.
b) Find the equation for line B.

Q8 Line C is defined by $y = \frac{1}{2}x + 12$.
Line D is perpendicular to line C and passes through (5, -13).

a) Find the gradient of line D.
b) Find the equation for line D.

Quadratic Graphs



So, you can spot a quadratic graph at ten paces, but can you draw one...

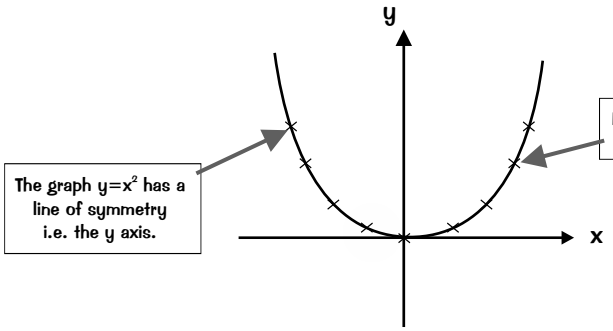
The simplest quadratic graph is $y = x^2$

x	-4	-3	-2	-1	0	1	2	3	4
y	16	9	4	1	0	1	4	9	16

$y = ax^2 + bx + c$

There will always be an x^2 term. The value of 'a' cannot be zero

Sometimes either 'b' or 'c' could be zero. Even if both were equal to zero, the graph would still be quadratic



Quadratic graphs are always shaped like a bucket. (Funny shaped bucket, but you see what I mean).

Q1 Complete this table of values for the quadratic graph $y = 2x^2$.

a) Draw axes with x from -4 to 4 and y from 0 to 32.
b) Plot these 9 points and join them with a smooth curve.
c) Label your graph.

x	-4	-3	-2	-1	0	1	2	3	4
$y=2x^2$	32	18					8		

Remember to square first then × 2

Q2 Complete this table of values for the graph $y = x^2 + x$.

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9					4		
$y=x^2+x$	12					2			

By putting more steps in your table of values, the arithmetic is easier

a) Draw axes with x from -4 to 4 and y from 0 to 20.
b) Plot the points and join them with a smooth curve.
c) Draw and label the line of symmetry for the quadratic graph $y = x^2 + x$.

If the x^2 term has a minus sign in front of it, the bucket will be turned upside down.

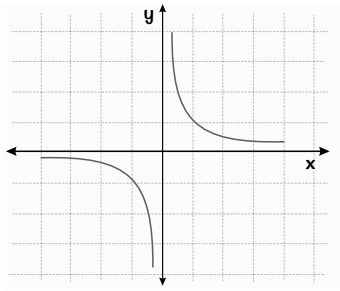
Q3 **a)** Complete this table of values for the graph $y = 3 - x^2$.
b) Draw the graph $y = 3 - x^2$ for x from -4 to 4.
c) State the maximum value of the graph $y = 3 - x^2$.

x	-4	-3	-2	-1	0	1	2	3	4
3	3	3	3	3	3	3	3	3	3
$-x^2$	-16						-4		
$y=3-x^2$	-13						-1		

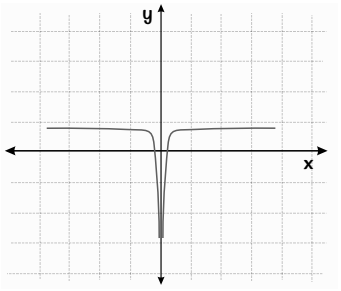
Harder Graphs

Q1 Identify the type of graph shown below.
Choose from straight line, quadratic, cubic and reciprocal:

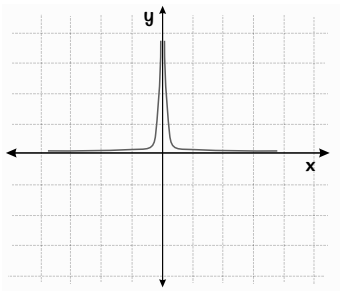
a)



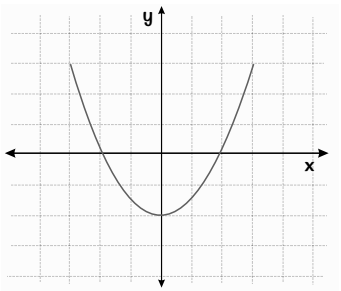
b)



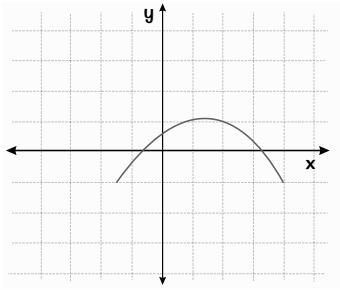
c)



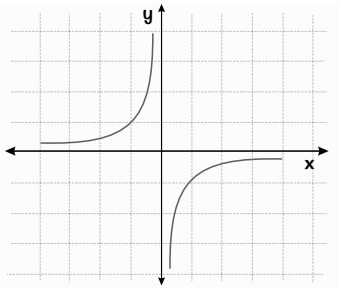
d)

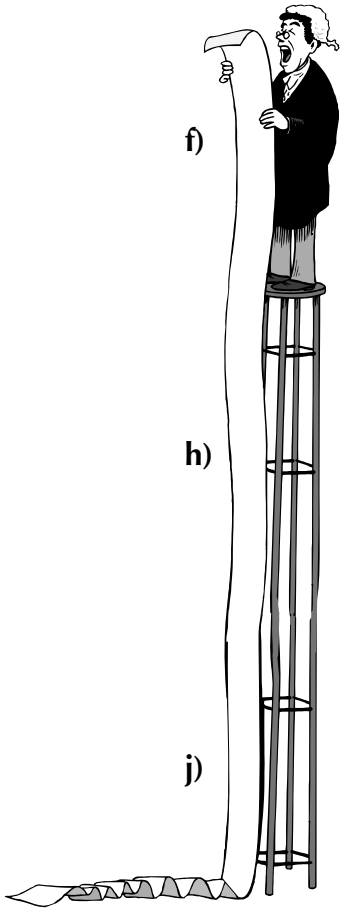


e)

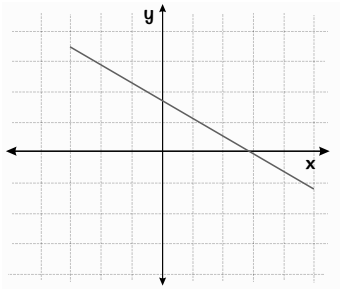


f)

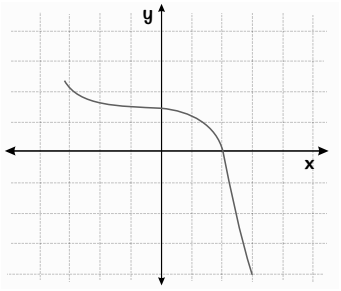




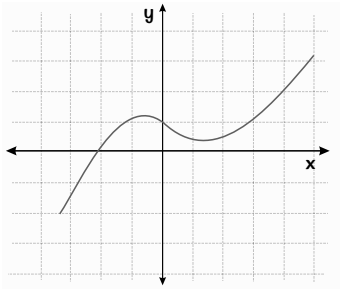
g)



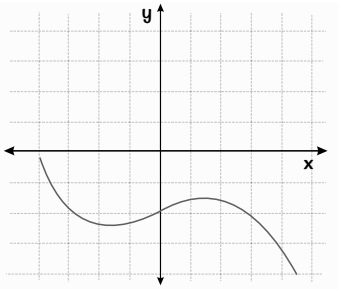
h)



i)



j)



You need to be able to <u>recognise</u> all these graphs — yeah, scary huh.
Don't worry — there aren't that many types you're expected to remember (phew).
4 types are covered here — <u>straight line</u> (easy), <u>x^2</u> (buckets), <u>x^3</u> (wiggly) and <u>1/x</u> (2 bits and "x=0" missing).

Harder Graphs

Q2 Here are some equations, and there are some curves below. Match the equations to the curves.

a) $y = -2x - 1$

b) $y = 3x$

c) $y = x^2 + 2$

d) $y = -x^2 + 3$

e) $y = x^2$

f) $y = 2x^3 - 3$

g) $y = -\frac{1}{2}x^3 + 2$

h) $y = x^3$

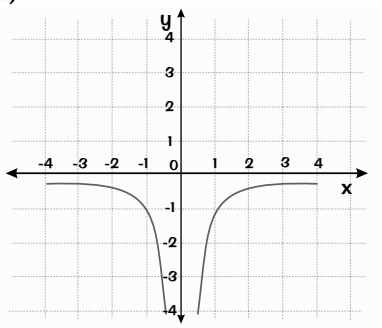
i) $y = -\frac{3}{x}$

j) $y = \frac{2}{x}$

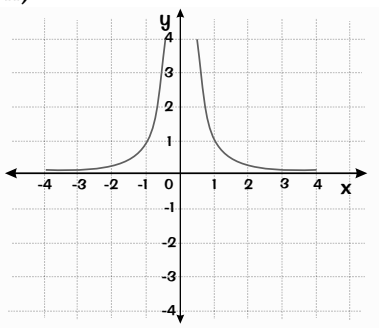
k) $y = \frac{1}{x^2}$

l) $y = -\frac{1}{x^2}$

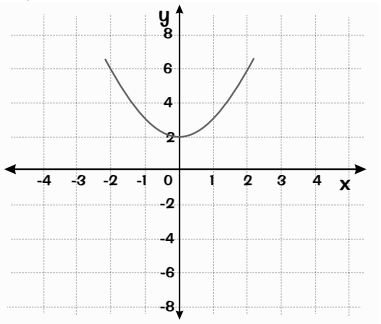
i)



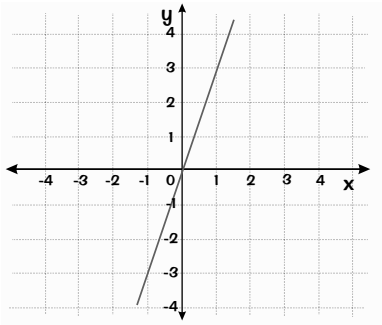
ii)



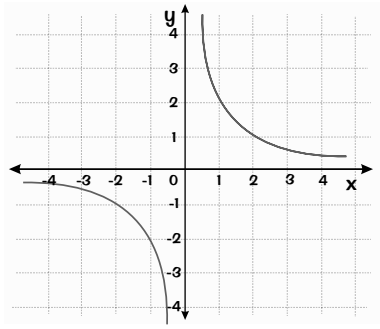
iii)



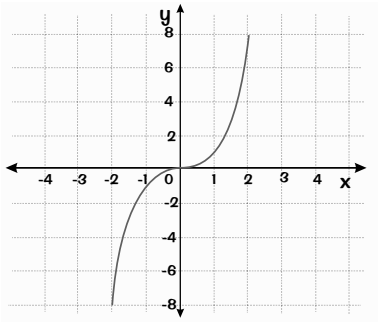
iv)



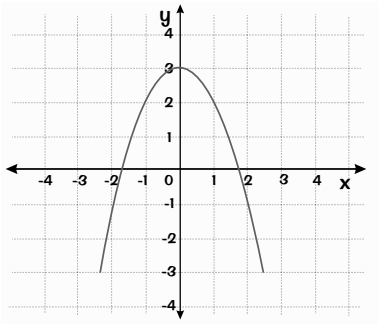
v)



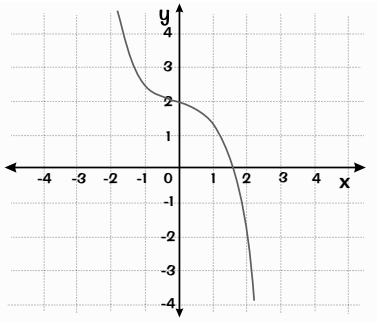
vi)



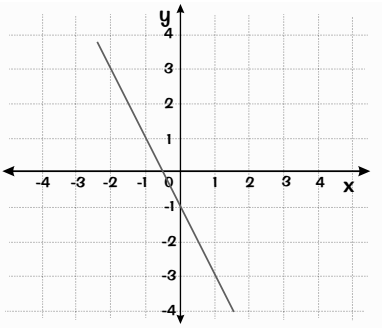
vii)



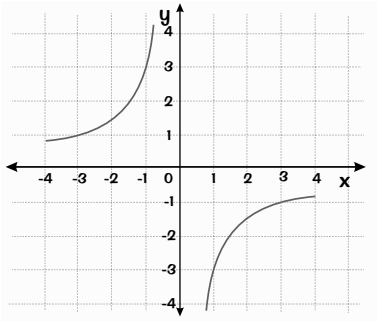
viii)



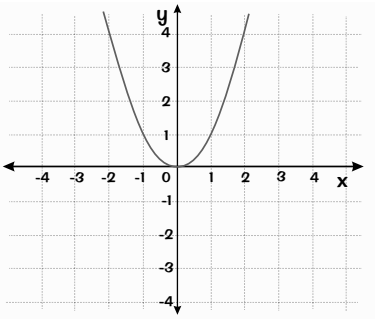
ix)



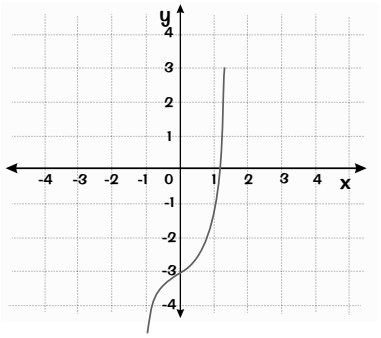
x)



xi)



xii)



Harder Graphs

You go about a cubic in the same way as you would a quadratic — but you should get a different shaped graph, of course. It's always a good idea to put lots of steps in the table of values — that way it's easier to check any points that look wrong.

Q3 Complete this table of values for $y = x^3$:

x	-3	-2	-1	0	1	2	3
$y=x^3$							

Draw the graph of $y = x^3$.

Q4 Complete this table of values for $y = -x^3$:

x	-3	-2	-1	0	1	2	3
$y=-x^3$							

Draw the graph of $y = -x^3$.

Q5 Complete this table of values for $y = x^3 + 4$:

x	-3	-2	-1	0	1	2	3
x^3							
$y=x^3+4$							

Draw the graph of $y = x^3 + 4$.

Remember — no rulers.

Q6 Complete this table of values for $y = -x^3 - 4$:

x	-3	-2	-1	0	1	2	3
$-x^3$							
$y=-x^3-4$							

Draw the graph of $y = -x^3 - 4$.

Harder Graphs

Here are some equations and shapes that are even more alien... Just go about doing them the same way as a cubic or quadratic and you'll be fine. Don't try to skip stages when you're working through it — errors will then become much harder to spot.

The two halves of a 1/x graph never touch and are symmetrical around y = x and y = -x.

Q7 Complete this table of values for the graph $y = 1/x$.

a) Draw axes with x from -4 to 4 and y from -1 to 1.
b) Plot these points and join them with a smooth curve.
c) Label your graph.

x	-4	-3	-2	-1	0	1	2	3	4
y=1/x		-0.33			n/a				0.25

The 0's just there to fool you — any equation with A/x in it never makes a graph that passes through x = 0.

A/x² graphs are similar to 1/x graphs but the two halves are next to each other.

Q8 Complete this table of values for the graph $y = 3/x^2$.

x	-4	-3	-2	-1	0	1	2	3	4
x^2			4			1		9	
$y=3/x^2$						3			

A is any number — positive or negative

a) Draw axes with x from -4 to 4 and y from 0 to 3.
b) Plot the points and join them with a smooth curve.
c) Draw and label the line of symmetry for the graph $y = 3/x^2$.

k^x graphs are always above the x-axis and pass through the point (0, 1).

Q9 **a)** Complete this table of values for the graph $y = 2^x$.
b) Draw the graph $y = 2^x$ for x from -4 to 4.
c) Why do k^x graphs always pass through (0, 1)?

k is any positive number

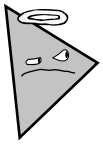
x	-4	-3	-2	-1	0	1	2	3	4
$y=2^x$				0.5	1		4		

All these types of equations can be combined — see what shape of graph you get.

Q10 **a)** Complete this table of values for the graph $y = 3^x - 6/x$.
b) Draw the graph $y = 3^x - 6/x$ for x from -3 to 3.

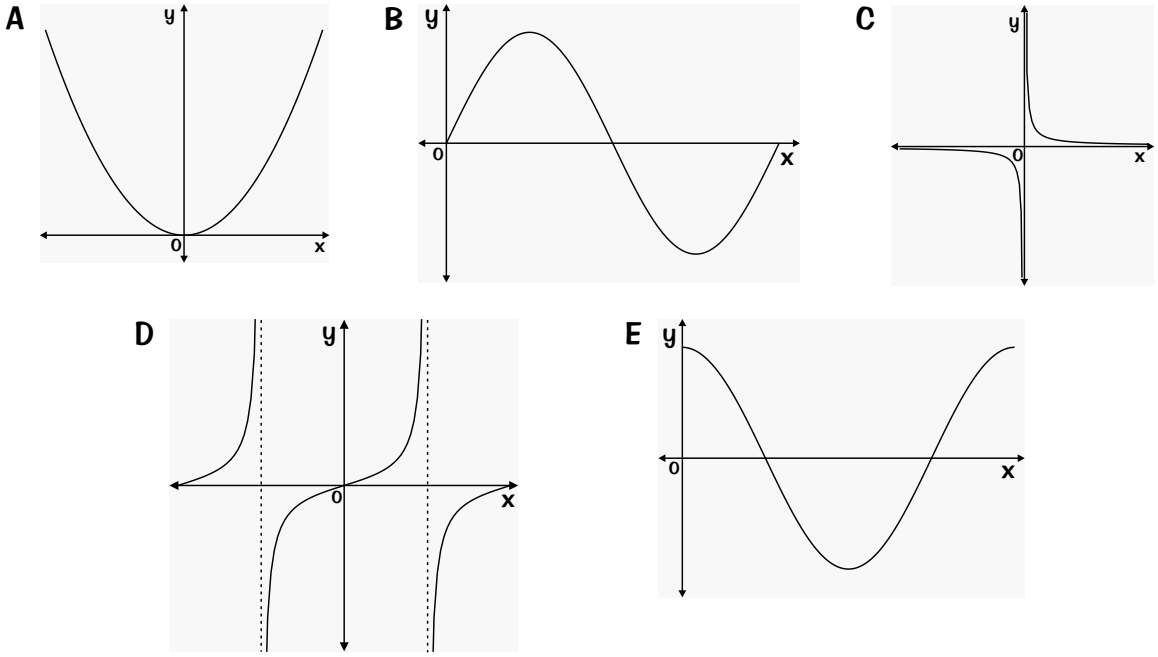
x	-3	-2	-1	0	1	2	3
3^x			0.33		3		
6/x						3	
$y=3^x - 6/x$					-3		

Harder Graphs



Just when you thought you'd mastered graphs, along comes trigonometry to really put you to the test. You'll be fine, as long as you know your angles from your angels...

Q11 Which of the diagrams below could show the graph of:
a) $y = \sin x$ **b)** $y = \cos x$ **c)** $y = \tan x$



Q12 **a)** Complete this table of values for the different angles.

x	0°	30°	60°	90°	120°	150°	180°
sin x	0					0.5	
cos x			0.5				-1
tan x			1.73	—		-0.58	

b) Draw axes with x from 0° to 180° and y from 0 to 1.
c) Use the table to draw the graph of $y = \sin x$ for values of x between 0° and 180°.

Q13 **a)** Complete this table of values for the graph $y = \cos x$.

x	360°	405°	450°	495°	540°	585°	630°	675°	720°
y=cos x	1			-0.71			0		

b) Draw the graph $y = \cos x$ for values of x between 360° and 720°.

Functions

Right, functions. Don't let appearances deceive you — treat them like the regular equations that you've seen loads of times before. Nothing to worry about here then.

Q1 Express the following equations using function notation:

a) $y = 3 + x$ **b)** $y = 790 - 41x$ **c)** $y = 3(9x^2 + 2)$

Q2 For the following functions find x when $f(x) = 2$:

a) $f(x) = 2x - 7$
b) $f(x) = x^2 + 1$
c) $f(x) = 5 - 4x \times 8x$
d) $f(x) = 3x^2 - 10$
e) $f(x) = x^2 + 7x$
f) $f(x) = 252 - 2x^3$

Q3 For each of the following functions determine which values of x are excluded from the domain of f:

a) $f(x) = \sqrt{4 - x}$
b) $f(x) = x^{-\frac{1}{2}}$
c) $f(x) = (x - 1) \div x$
d) $f(x) = \sqrt{2x + 7}$
e) $f(x) = \dfrac{10}{4x + 3}$

Q4 If $f(x) = 4x + 12$ and $g(x) = 3 - x^2$ then find:

a) $f(8)$
b) $g(3)$
c) $f(-4)$
d) $gf(x)$
e) $fg(x)$
f) $gf(2)$

Q5 If $f(x) = \dfrac{11}{x + 1}$, $g(x) = \dfrac{3x}{4}$ and $h(x) = 6 + x$ then find:

a) $h^{-1}(x)$
b) $f^{-1}(x)$
c) $g^{-1}(h(x))$
d) $f^{-1}(g(x))$
e) $h^{-1}(f(5))$
f) $h^{-1}(g(-1))$

Q6 Evaluate the following:

a) $f(-1)$ where $f(x) = \dfrac{2 + 4x}{3}$

b) $g(9)$ where $g(x) = 7x^3 - 13$

c) $kj(-3)$ where $j(x) = \dfrac{6(x - 2)}{2}$ and $k(x) = -14x$

d) $n^{-1}(2)$ where $n(x) = 32 + 4(-x + 7)$

e) $t{:}x \longrightarrow 21 - x^2$ where $x = 3$

f) $v{:}x \longrightarrow -\dfrac{3x}{12}$ where $x = 3$

Q7 Find:

a) $hi(x)$, if $h(x) = \dfrac{11}{x^2} - 8$ and $i(x) = -x + \dfrac{x^2}{2}$

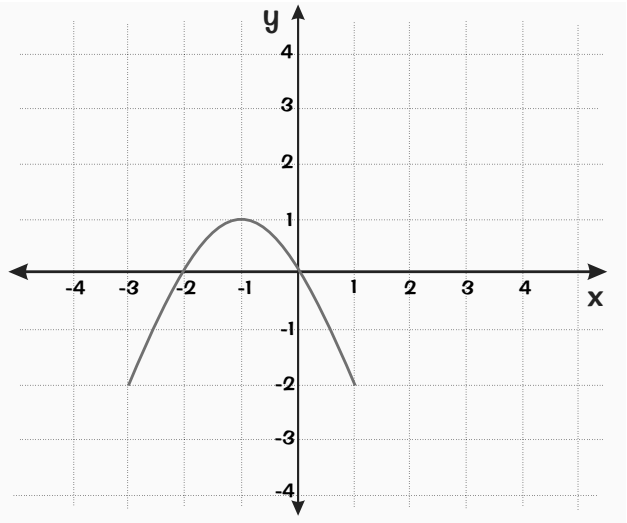
b) $m^{-1}(x)$, if $m(x) = \dfrac{18x - 12}{10} + 3$

c) $p^{-1}(q(x))$, if $p(x) = 8x + 5$ and $q(x) = \dfrac{13}{x - 2}$

Graph Transformations

You've got to learn the rules for these shifts, reflections and stretches — there are only 2 types of each, so it won't take long. Remember — $y = f(x + a)$ shifts the graph to the left and $y = f(x - a)$ shifts the graph to the right.
$y = f(ax)$ squashes the graph when $a > 1$ or $a < -1$ and stretches it when $-1 < a < 1$.

Q1 This is a graph of $y = f(x)$.



Use the graph of $y = f(x)$ to sketch:

a) $y = f(x) + 3$

b) $y = f(x) - 3$

c) $y = f(x + 2)$

d) $y = f(x - 3)$

e) $y = -f(x)$

f) $y = f(-x)$

g) $y = 2f(x)$

h) $y = \frac{1}{2}f(x)$

i) $y = f(3x)$

Q2 This is a graph of $y = f(x)$.

Use the graph of $y = f(x)$ to sketch:

a) $y = f(x) + 2$

b) $y = f(x) - 2$

c) $y = f(x + 2)$

d) $y = f(x - 2)$

e) $y = -f(x) + 2$

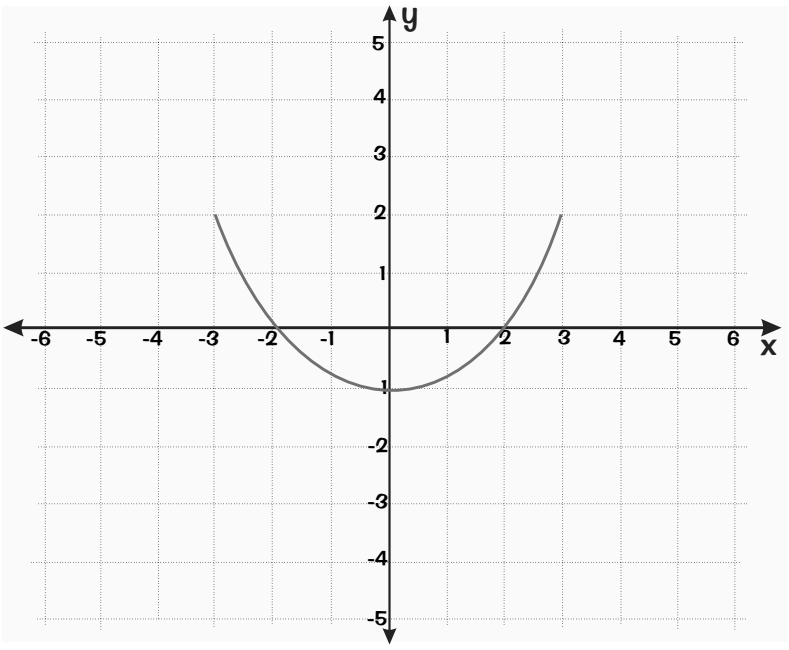
f) $y = f(-x) - 1$

g) $y = f(x + 3) - 1$

h) $y = f(x - 1) + 3$

i) $y = 2f(x)$

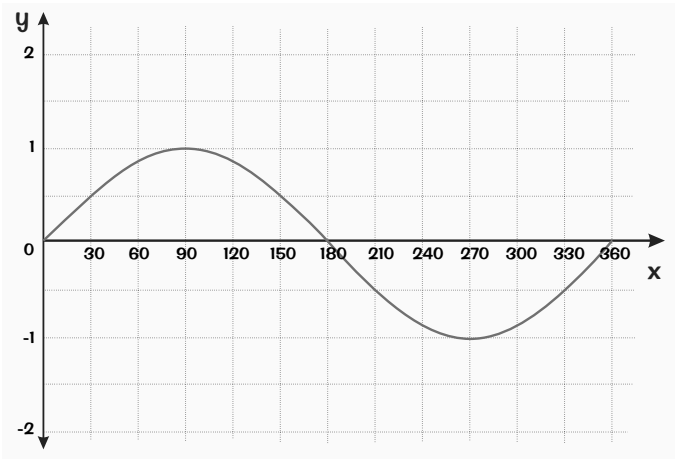
j) $y = f(\frac{1}{2}x)$



Q3 Describe how a graph of $y = f(x)$ is transformed into $y = -f(-x)$.

Graph Transformations

Q4 The graph of $y = \sin x$ is shown below.

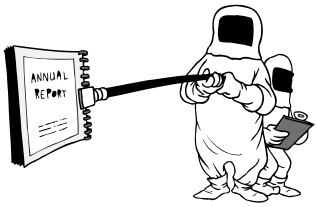


On a single set of axes, draw the graphs of:

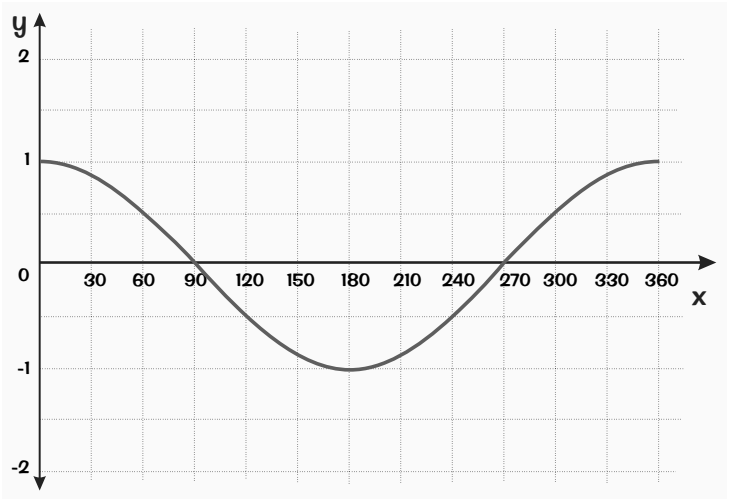
a) $y = 2 + \sin x$

b) $y = \sin(x + 60)$

c) $y = -\sin x$.



Q5 This is the graph of $y = \cos x$:

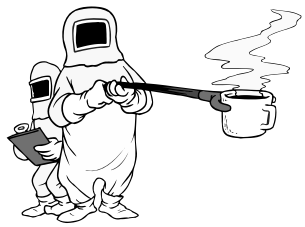


On a single set of axes, draw the graphs of:

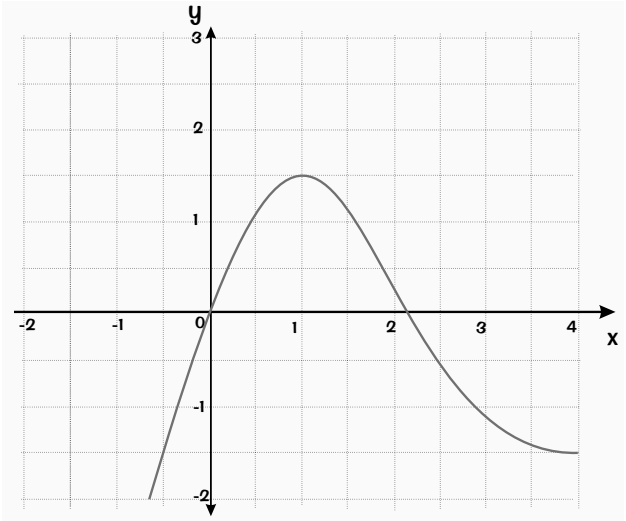
a) $y = 1 + \cos x$

b) $y = 2 \cos x$

c) $y = \cos(x - 90)$.



Q6 The graph of $y = f(x)$ is shown on the right. Write down the coordinates of the maximum point for the following graphs:



a) $y = f(x - 3)$

b) $y = f(x) + 2$

c) $y = -f(x)$

d) $y = 2f(x)$

e) $y = f(x + 1) - \frac{3}{2}$

f) $y = f(-x)$

g) $y = f(x - \frac{1}{3}) + 4$

Q7 The curve $y = x^3 + 2x + 4$ is reflected in the y-axis. Write down the equation of the new curve.

Differentiation

Differentiation — not as nightmarish as it first looks. It just takes a bit of practice and remembering that...

This bit just means 'the result of differentiating the thing in the brackets'.

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Q1 Find $\frac{dy}{dx}$ when:

a) $y = x^4$ **d)** $y = 2x^2$ **g)** $y = \frac{1}{2}x^4$ **j)** $y = 22$

b) $y = x^2$ **e)** $y = 5x^3$ **h)** $y = 33x$ **k)** $y = -3x^3$

c) $y = x^{13}$ **f)** $y = 7x^4$ **i)** $y = x$ **l)** $y = -\frac{1}{4}x^{16}$

Q2 Find:

a) $\frac{dy}{da}$ when $y = a^7$ **c)** $\frac{dy}{ds}$ when $y = s$

b) $\frac{dy}{dt}$ when $y = 10t^5$ **d)** $\frac{dy}{dw}$ when $y = -\frac{2}{3}w^6$

Q3 Find $\frac{dy}{dx}$ when:

a) $y = x^5 + 5$ **d)** $y = 2x^2 + x$ **g)** $y = 4x^8 + x^2 + 3$

b) $y = 2x^7 + 11$ **e)** $y = x^9 + 3x$ **h)** $y = 3x^5 + x^3 + x$

c) $y = x + 2$ **f)** $y = 7x^3 + 6x^2$ **i)** $y = \frac{1}{3}x^9 + x^5 + x^2 + 9$

Q4 Find $\frac{dy}{dx}$ when $y = 9x^4 + x^3 + 4x^2 + 6x + 22$

Q5 Find $\frac{dy}{dx}$ when $y = 3x^5 + 7x^4 + 8x^3 + 2x^2 + 10x + 13$

Q6 Find $\frac{dy}{dx}$ when $y = 5x^8 + 4x^6 + 12x^4 + 7x^2 + 8$

Q7 Find $\frac{dy}{dd}$ when $y = 11d^4 + 12d^3 + 9d^2 + 14d + 15$

Differentiation

Q8 Differentiate to get the gradient expression for the graphs of each of these equations.

a) $y = -4x^3 - x^2$

b) $y = 5x^3 + 3x^2 + x$

c) $y = -6x^3 - 2x^2 - 6$

d) $y = 2x^3 + x^2 - 8x + 3$

e) $y = \frac{1}{x}$

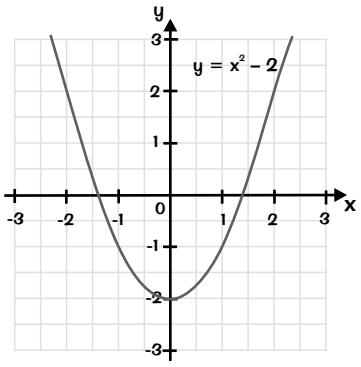
f) $y = \frac{1}{x^2}$

Q9 Find the gradient of each of these graphs at $x = 0.5$ by:

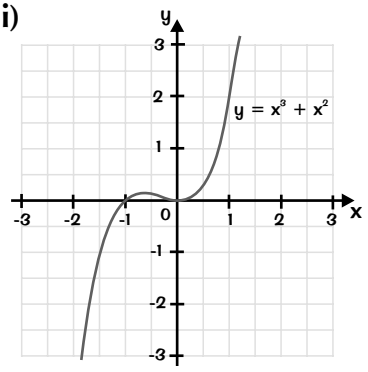
a) Drawing a tangent to the curve and finding its gradient.

b) Differentiation.

i)



ii)



Q10 Find the gradient of the graph of each of the following equations at $x = -1$ and $x = 2$.

a) $y = -2x - 1$

b) $y = 3x^3 + 2x^2$

c) $y = x^2 + 2$

d) $y = -x^2 + 3$

e) $y = x^2$

f) $y = 2x^3 - 3$

g) $y = -\frac{1}{2}x^3 + 2$

h) $y = x^3$

i) $y = -\frac{3}{x}$

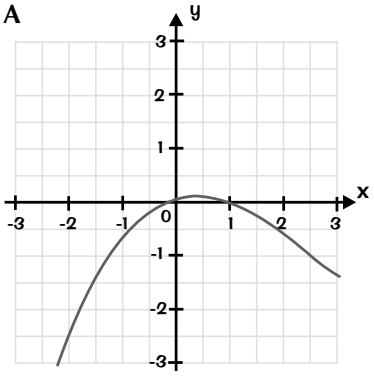
j) $y = \frac{2}{x}$

k) $y = \frac{1}{x^2}$

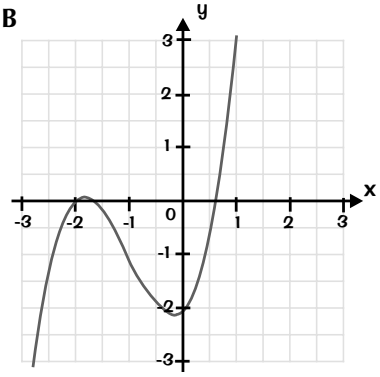
l) $y = -\frac{1}{x^2}$

Q11 Using differentiation to find gradients, decide which of the following is the graph of $y = x^3 + 3x^2 + x - 2$. Show your working.

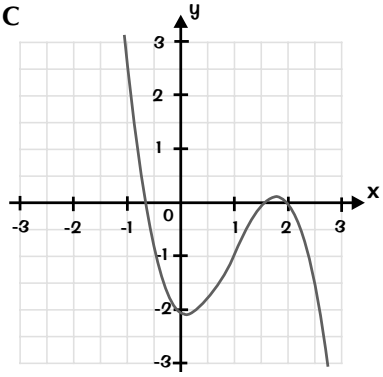
A



B



C



Differentiation

Q12 What are the coordinates of the point on the graph of the equation $y = 2x^2 + 3x$ where the gradient is +11?

Q13 What are the coordinates of the point on the graph of the equation $y = 4x^2 + 7$ where the gradient is +4?

Q14 The vertical distance, d, of a hot air balloon from the ground in km is given by the formula $d = -t^2 + 3t$, where t is the time in hours and $0 \leq t \leq 3$.

a) For how long, to the nearest minute, is the hot air balloon more than 1 km above the ground?

b) Find an expression for the vertical velocity of the balloon, v.

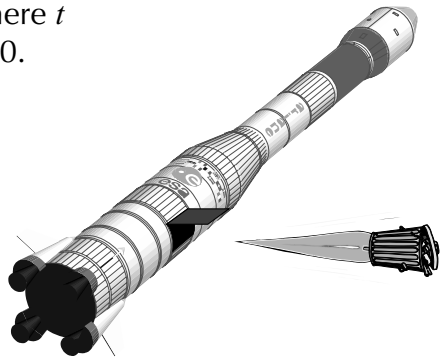
c) Find the vertical velocity of the balloon after 30 minutes and after 1 hour.

Q15 An object is fired from a space rocket. Its distance, d metres, from a point O is given by the formula $d = 2t^2(t + 1)$, where t is the number of seconds that have passed and $0 \leq t \leq 10$.

a) How far from point O is the object after 6 seconds?

b) Find the velocity of the object after 6 seconds.

c) Find the acceleration of the object after 6 seconds.



Q16 The velocity (in m/s) of a flying alien at time t seconds is given by the formula $v = 4t^2 + 2t + 3$ (for $0 \leq t \leq 300$).

a) Find the velocity of the alien after 40 seconds.

b) Find the acceleration of the alien after 40 seconds.

Differentiation

Q17 Find the coordinates of the turning points of these graphs. For each, say if the turning point is a maximum or minimum.

a) $y = 2x^2$

b) $y = 5x^2 + x$

c) $y = 3x^2 + 2x - 5$

d) $y = -x^2 + 4x - 8$

e) $y = (x + 4)(x - 8)$

Q18 Find $\frac{dy}{dx}$ for each of the following equations.

a) $y = x^4 - x^2 - 3x - 8$

b) $y = 4x^5 + 9x^4$

c) $y = \frac{2}{3}x^3 - \frac{1}{x^2} + 10x - 2$

d) $y = x^3 + 7.5x^2 + \frac{3}{x^4} + 1$

Remember that $1/x^2$ is the same as x^{-2}.

Q19 Find the coordinates of the stationary points of the graphs of each of the following equations.

a) $y = \frac{1}{3}x^3 - x^2 - 3x - 8$

b) $y = \frac{4}{3}x^3 - 16x^2 + 48x$

c) $y = \frac{2}{3}x^3 - 4.5x^2 - 5x - 2$

d) $y = x^3 + 6x^2 + 12x + 1$

Q20 The temperature (in °C) of a room is given by the formula $C = -20t^3 + 40t^2 - 10$, where t is the time in hours after a heater was turned on (for $0 \leq t \leq 2$).

a) What was the temperature of the room when the heater was turned on?

b) What was the rate of temperature change after 30 minutes?

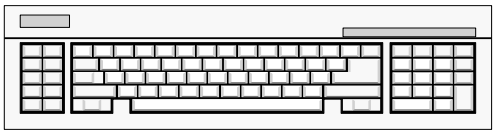
c) After some time, the heater was turned off. The room started to cool immediately. At what time was the heater switched off?

Scale Drawings

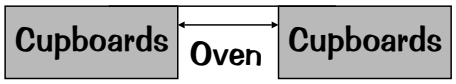
Watch out for those units... there's quite a mixture here — you'll have to convert some of them before you can go anywhere.

Q1 A rectangular room measures 20 m long and 15 m wide. Work out the measurements for a scale drawing of the room using a scale of 1 cm = 2 m.

Q2 Katie drew a scale drawing of the top of her desk. She used a scale of 1:10. This is her drawing of the computer keyboard. What are its actual dimensions?



Q3



This is a scale drawing of part of Paul's kitchen. Measure the width of the gap for the oven.

The drawing uses a scale of 1 : 60.
Work out the maximum width of oven, in cm, that Paul can buy.

Q4 A rectangular room is 4.8 m long and 3.6 m wide.

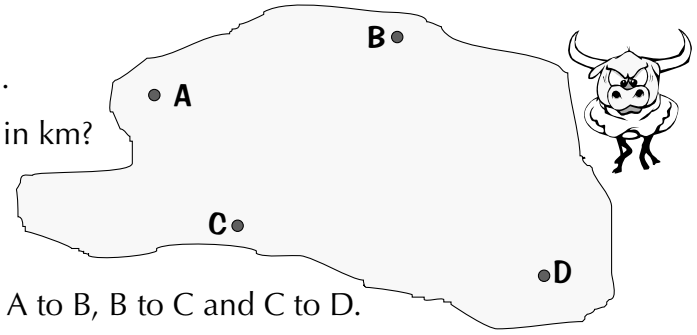
a) Make a scale drawing of it using a scale of 1 cm to 120 cm.

b) On your scale drawing mark a window which has an actual length of 2.4 m on one long wall and mark a door, actual width 90 cm, on one shorter wall.

Q5 The scale on this map is 1 cm : 4 km.

a) Measure the distance from A to B in cm.

b) What is the actual distance from A to B in km?

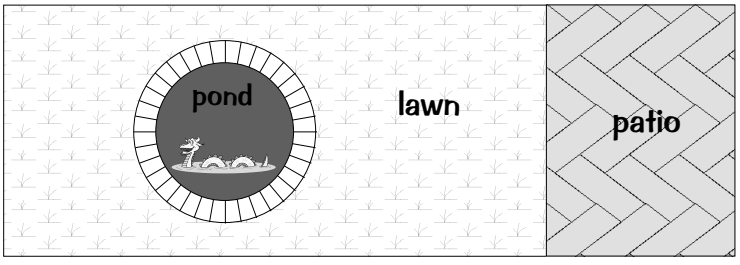


c) A helicopter flies on a direct route from A to B, B to C and C to D. What is the total distance flown in km?

Q6 Frank has made a scale drawing of his garden to help him plan some improvements. The scale on the drawing is 1 : 70.

a) Frank wants to put up a fence along the three outside edges of the lawn. How many metres of fencing does he need to buy?

b) What is the actual area of Frank's patio in m?

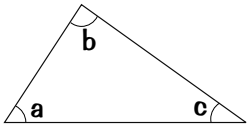


If the scale doesn't say what units it's in, it just means that both sides of the ratio are the same units — so 1 : 1000 would mean 1 cm : 1000 cm.

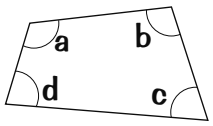
Geometry

Here are some angle rules then — just the 7 for now. You can't get away without knowing these, I'm afraid, so get learning.

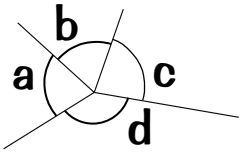
1) Angles in a triangle add up to 180°



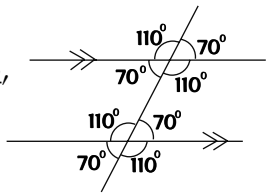
2) Angles in a quadrilateral add up to 360°



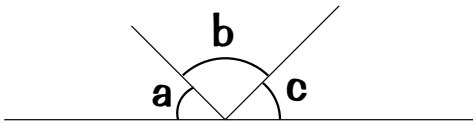
3) Angles round a point add up to 360°



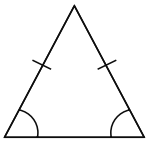
4) When a line crosses TWO PARALLEL LINES, the two bunches of angles are the same



5) Angles on a straight line add up to 180°

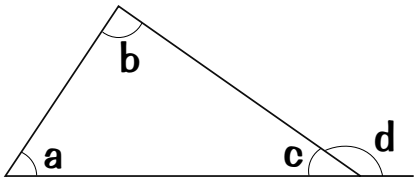


6) ISOSCELES TRIANGLES have two sides the same and two angles the same



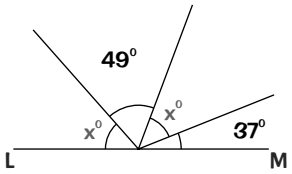
7) EXTERIOR angle of a triangle = sum of opposite INTERIOR angles.

d = a + b

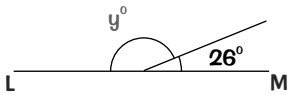


For the following diagrams, find the lettered angles. LM is a straight line.

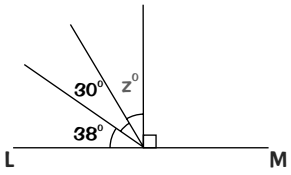
Q1 a)



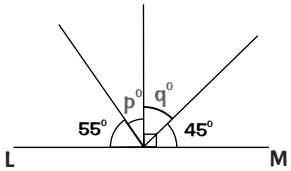
b)



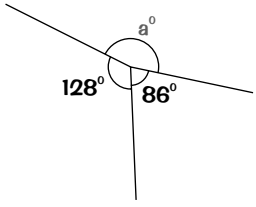
c)



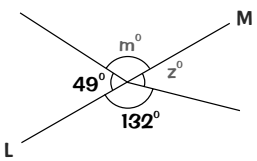
d)



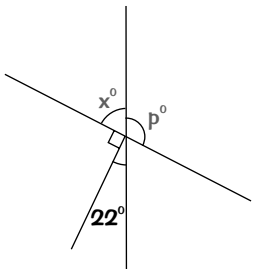
Q2 a)



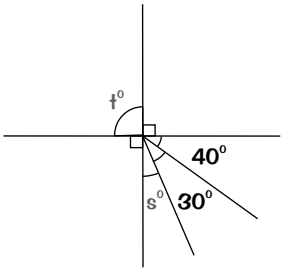
b)



c)



d)

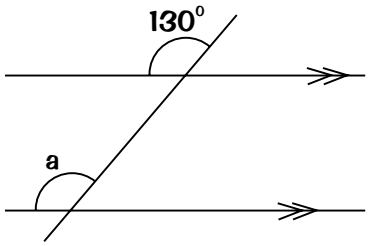


Geometry

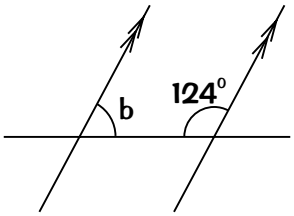
This page is a bit dull — just lots of boring angles... still, that's geometry for you. Oh and by the way, you've got to work the angles out — don't try and sneakily measure them, they're probably drawn wrong anyway...

Q3 Find the sizes of the angles marked by letters in these diagrams.

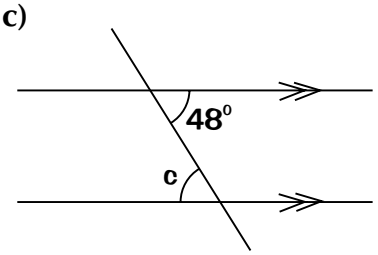
a)



b)

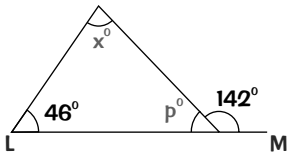


c)

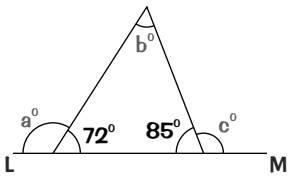


For the following diagrams, find the lettered angles. LM is a straight line.

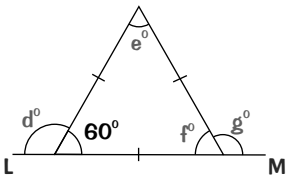
Q4 **a)**



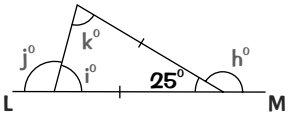
b)



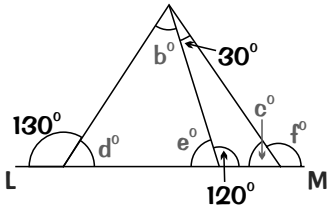
c)



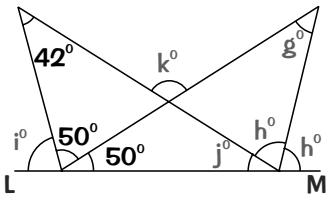
d)



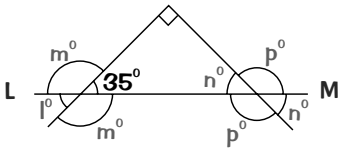
Q5 **a)**



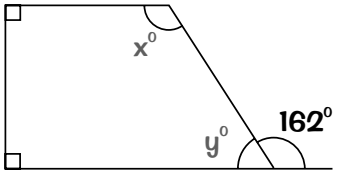
b)



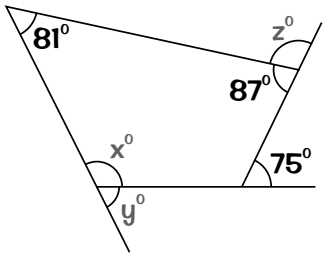
c)



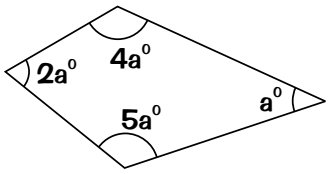
Q6 **a)**



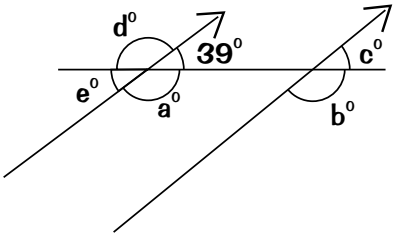
b)



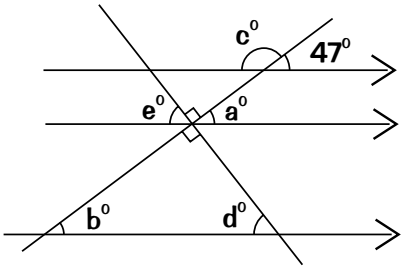
c)



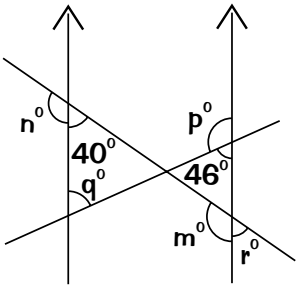
Q7 **a)**



b)



c)



Polygons

Interior and Exterior Angles could get you some easy marks in the exam — as long as you remember these formulas...

A POLYGON is a many-sided shape. A REGULAR polygon is one where ALL THE SIDES AND ANGLES ARE THE SAME.

You need to know these two formulas:

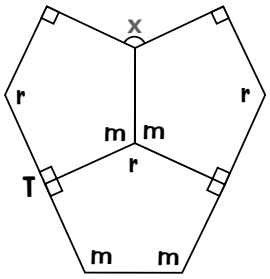
1) EXTERIOR ANGLE = 360° ÷ No. of Sides

2) INTERIOR ANGLE = 180° – EXTERIOR ANGLE

Q1 What sort of triangles occur in every regular polygon (except a hexagon), when each vertex is joined to the centre by a straight line?

Q2 Sketch a regular hexagon and draw in all its lines of symmetry.
State the order of rotational symmetry.

Q3 In each of the pentagons below, all the sides are of equal length, two of the angles are 90° and the other interior angles are m, m, and r degrees.



a) Explain in two different ways why $2m + r = 360°$.

b) What is the size of angle x?

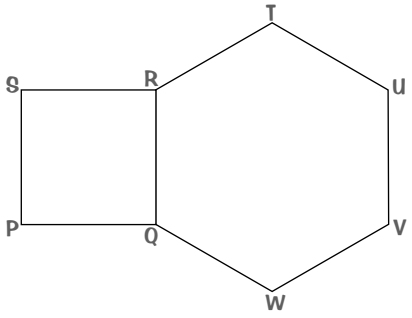
c) Copy the diagram and add two more pentagons (by tracing through) so that the point T is completely surrounded and the whole figure forms part of a tessellation. Label all the angles of the new pentagons.

Q4 A square and a regular hexagon are placed adjacent to each other.

a) What is the size of ∠PQW?

b) What is the size of ∠PRW?

c) How many sides has the regular polygon that has ∠PQW as one of its angles?



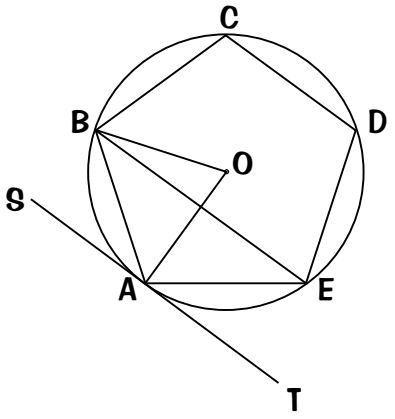
Q5 An irregular pentagon has interior angles of 100°, 104°, 120°.
If the other two angles are equal, what is their size?

Q6 **a)** The sum of the interior angles of a regular 24-sided polygon is 3960°.
Use this to calculate the size of one interior angle.

b) From your answer to part **a)** calculate one exterior angle and show that the sum of the exterior angles equals 360°.

Polygons

Q7



ABCDE is a regular pentagon. It is drawn in a circle with centre O. SAT is a tangent drawn to the circle at A.

a) Calculate the size of angle BOA.

b) Find the size of angle OBA.

c) Write down the size of angle:

i) SAO

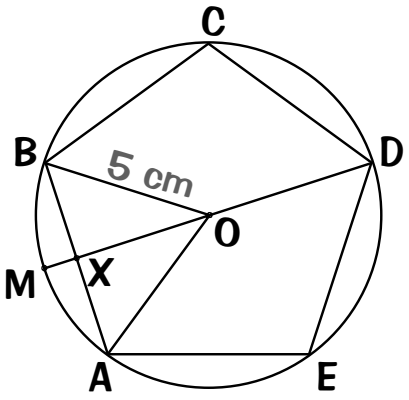
ii) BAS.

d) Hence write down the size of angle BEA, giving a reason for your answer.

Q8 The sum of the interior angles of a regular polygon is 2520°. How many sides does this regular polygon have?

Remember that formula for the sum of interior angles — it comes in handy here.

Q9



ABCDE is a regular pentagon whose vertices lie on a circle of radius 5 cm, centre O. DOM is an axis of symmetry of the pentagon and cuts the chord AB at X.

a) Calculate the size of angle BOX.

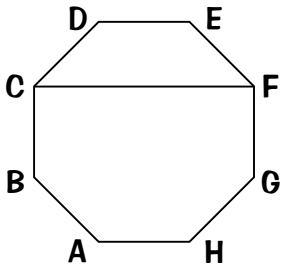
b) Find the length OX. Hence find the distance of M from the chord AB.

You need to use trigonometry to find OX — cos θ = adj / hyp.

Q10 ABCDEFGH is a regular octagon.

a) Copy the figure and mark on the axis of symmetry which maps H to A.

b) Calculate the size of angle EFC.



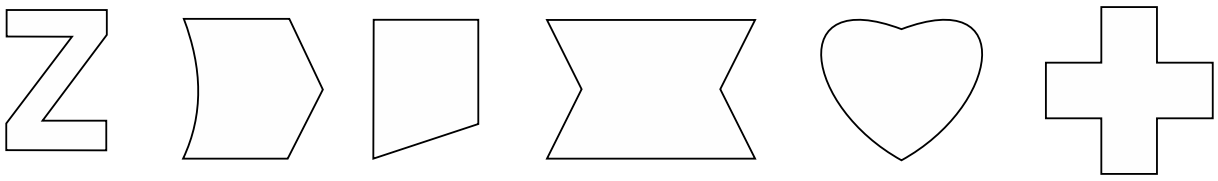
Symmetry

Two types of symmetry here — but don't worry, I reckon their names pretty much give the game away.

There are TWO types of symmetry to learn:	
1) LINE SYMMETRY	You can draw a mirror line across the object and both sides will fold together exactly.
2) ROTATIONAL SYMMETRY	You can rotate the shape or drawing into different positions that all look exactly the same.

Q1 Draw <u>all</u> the lines of symmetry for each of the following shapes.
(Some shapes may have no lines of symmetry.)

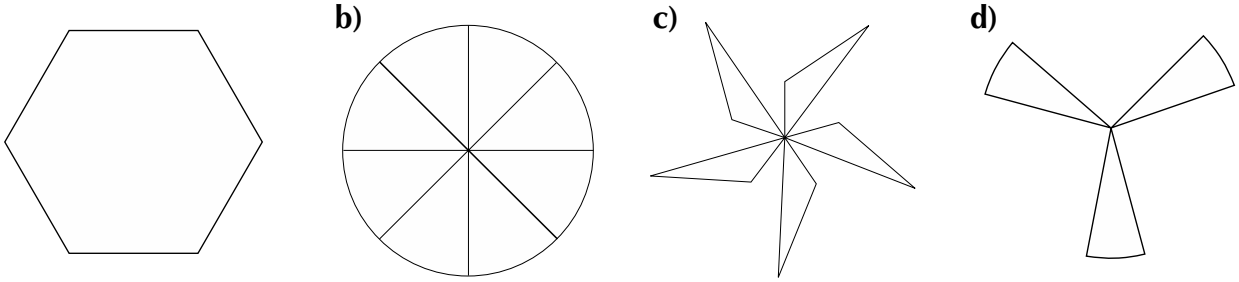
a) **b)** **c)** **d)** **e)** **f)**



These questions are a piece of cake if you use tracing paper — you can use it in the Exam too, so take some with you or ask for it.

Q2 What is the <u>order of rotational symmetry</u> for each of the following shapes?

a) **b)** **c)** **d)**



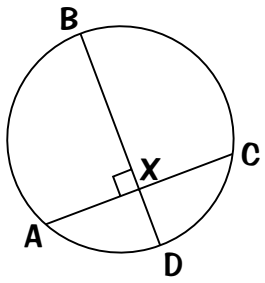
Q3 Mark in the <u>lines of symmetry</u> of the following letters.
State the <u>order</u> of rotational symmetry for each one.

Q4 Draw an example of each of the following shapes.
Put in the <u>axes of symmetry</u> and state the <u>order</u> of rotational symmetry.

a) An equilateral triangle.
b) An isosceles triangle.
c) A rhombus.
d) An isosceles trapezium.
e) A regular octagon.
f) A parallelogram.

Circle Geometry

Q1



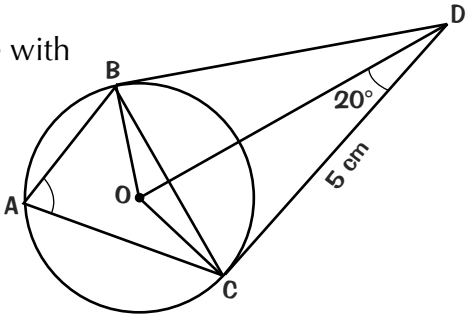
A, B, C and D are points on the circumference of a circle of radius 9 m. The chords AC and BD meet at point X, at an angle of 90°. If AC = 13 m and AX = 6.5 m, what is the length of BD? Explain your answer.

Q2 A, B and C are points on the circumference of a circle with centre O. BD and CD are tangents of the circle.

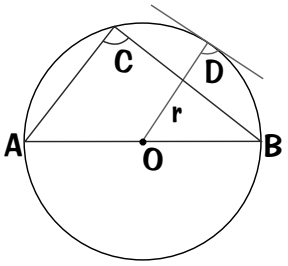
a) State the length BD.

b) Calculate the size of angle COD.

c) State the size of angle COB.



Q3

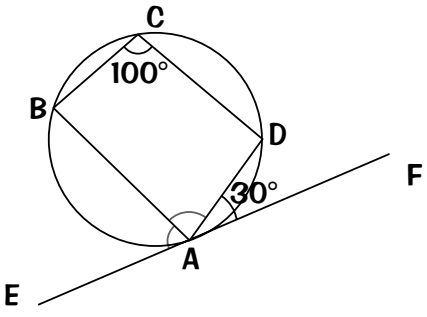


The circle in the diagram to the left contains a triangle with points (A and B) at both ends of a diameter and angle C to the edge of the circle. A tangent has been drawn which makes angle D with the radius, r. O is the centre of the circle.
What are the sizes of the angles C and D?

Q4 ABCD is a cyclic quadrilateral with angle BCD = 100°. EF is a tangent to the circle touching it at A. Angle DAF = 30°.
Write down the size of angle:

a) BAD

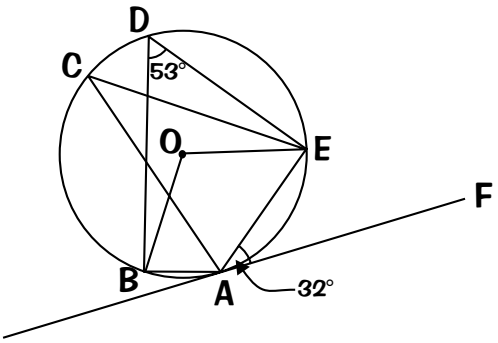
b) EAB.



Q5 A, B, C, D and E are points on the circumference of a circle with centre O. Angle BDE = 53°. The line AF is a tangent to the circle, touching it at A. Angle EAF = 32°. Find:

a) angle BOE

b) angle ACE.

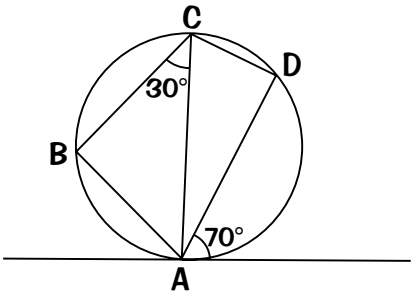


Circle Geometry

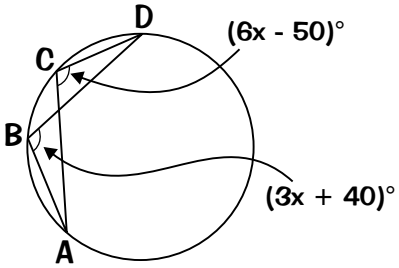
Q6 ABCD is a cyclic quadrilateral and the tangent to the circle at A makes an angle of 70° with the side AD. Angle BCA = 30°. Write down, giving a reason, the size of:

a) angle ACD

b) angle BAD.



Q7



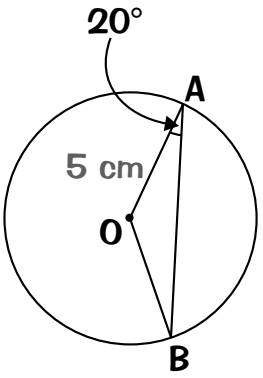
A, B, C and D are points on the circumference of a circle. Angle ABD = $(3x + 40)°$ and angle ACD = $(6x - 50)°$.

a) Give a reason why angle ABD and angle ACD are the same.

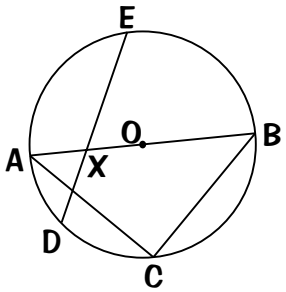
b) Form an equation in x and by solving it, find the size of angle ABD.

Q8 O is the centre of a circle and AB is a chord. The length OA = 5 cm and angle OAB = 20°. Find the length of the chord AB.

One way to do this is to work out angle AOB and use the sine rule.



Q9



A, B, C, D and E are points on the circumference of a circle with centre O. X is the point at which AB and DE intersect.

a) AC = BC and the area of triangle ABC is 64 cm^2. What is the length of AB?

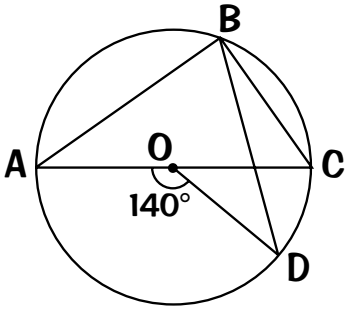
b) BX = 3AX, and DX = 6 cm. Find the lengths of EX and DE.

Q10 A, B, C and D are points on the circumference of a circle. O is the centre of the circle, AC is a diameter and angle AOD = 140°. Write down:

a) angle ABD

b) angle ABC

c) angle DBC.

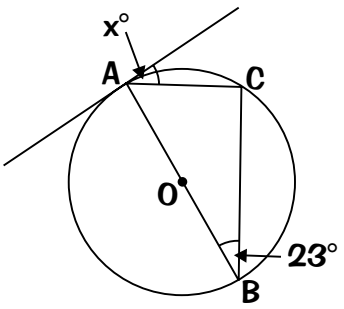


Circle Geometry

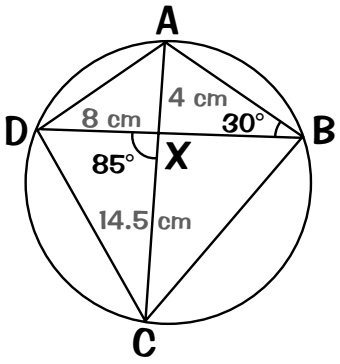
Q11 A tangent of a circle is drawn, touching it at A. C and B are two other points on the circumference and AOB is a diameter. O is the centre of the circle. Angle ABC is 23°.

a) Write down the size of angle ACB, giving a reason for your answer.

b) Find the size of the angle marked x° in the diagram.



Q12



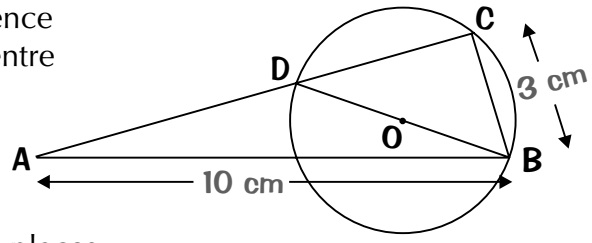
ABCD is a cyclic quadrilateral. The lines AC and BD intersect at X. Lengths AX = 4 cm, DX = 8 cm and XC = 14.5 cm. Angles DXC = 85° and ABD = 30°.

a) Show that triangles DXC and AXB are similar.

b) Find the length of XB.

c) Write down the size of angle BDC.

Q13 B, C and D are three points on the circumference of a circle with BD as a diameter. O is the centre of the circle and ADC is a straight line. AB = 10 cm and BC = 3 cm.

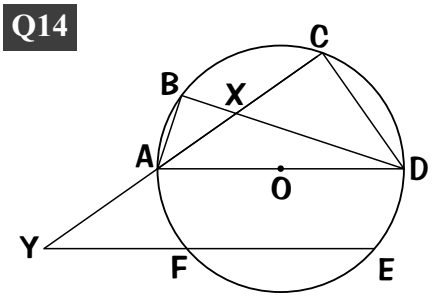


a) Write down the size of angle ACB, giving a reason for your answer.

b) Show that AC is 9.54 cm correct to 2 decimal places.

c) If AD = 5 cm, find the length of the diameter DOB correct to 2 decimal places.

Q14



A, B, C, D, E and F are points on the circumference of a circle. O is the centre of the circle. X is the point at which AC and BD intersect. AOD, CAY and EFY are all straight lines.

a) Write down the size of angles ABD and ACD, giving a reason for your answer.

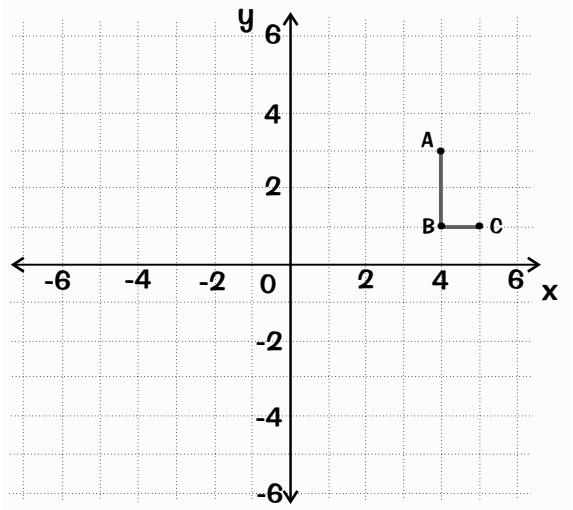
b) AX = 3 cm, XC = 6.5 cm and XD = 5 cm. Calculate BX.

c) YE = 6 cm and YA = 2.5 cm. Calculate YF.

The Four Transformations

Only 4 of these to learn... translation, reflection, rotation and enlargement. Off you go.

Q1 A is the point (4, 3), B is (4, 1) and C is (5, 1).



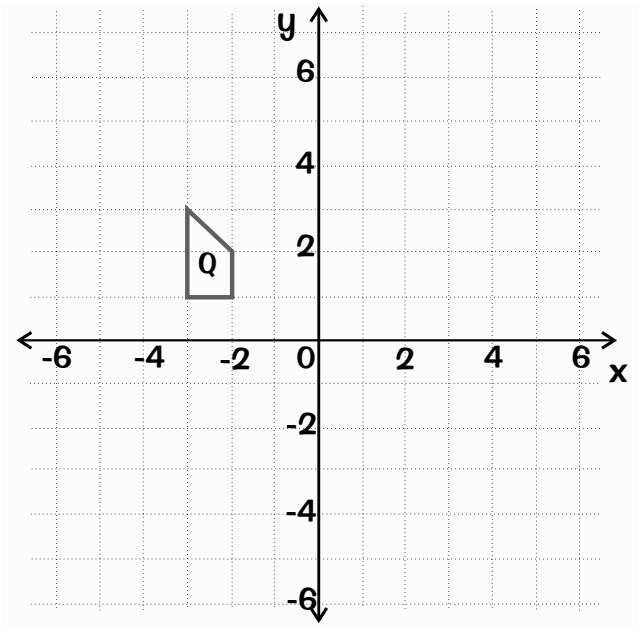
a) Using a scale of 1 cm to 1 unit draw the axes and mark on it the figure given by ABC.

b) Reflect ABC in the x-axis and label the image $A_1B_1C_1$.

c) Reflect $A_1B_1C_1$ in the y-axis and label the image $A_2B_2C_2$.

d) Describe fully the single transformation which would map ABC onto $A_2B_2C_2$.

Q2 Copy the axes using a scale of 1 cm to 1 unit. Mark on the axes a quadrilateral Q with corners (-2, 1), (-3, 1), (-3, 3) and (-2, 2).



a) Rotate Q clockwise through 90° about the point (-1, 2). Label the image R.

b) Rotate R clockwise through 90° about the point (0, 1). Label the image S.

c) Describe fully the rotation that maps Q to S.

d) Rotate Q through 180° about the point (-½, -1). Label the image T.

e) Rotate Q anticlockwise through 90° about the point (-1, -1). Label the image U.

f) Describe fully the rotation that sends U to T.

Q3 Draw axes with x and y running from 0 to 12 with a scale of 1 cm to 1 unit. O is the origin. $\overrightarrow{OP} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$, $\overrightarrow{PQ} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$, and $\overrightarrow{QR} = 2\overrightarrow{OP}$.

a) Mark P, Q and R on your axes.

b) Translate R by $\overrightarrow{QO}$. Label the image T.

c) Verify that $\overrightarrow{PQ} + \overrightarrow{QR} + \overrightarrow{RT} + \overrightarrow{TP} = O$.

Urghh — vectors... Make sure you get the coordinates the right way round — top for x direction, bottom for y direction.

The Four Transformations

Move each point separately — then check your shape hasn't done anything unexpected while you weren't looking.

Q4 Copy the axes and mark on triangle A with corners (-1, 2), (0, 4) and (-2, 4).
Use a scale of 1 cm to 1 unit.

a) Reflect A in the line $y = -x$.
Label this image B.

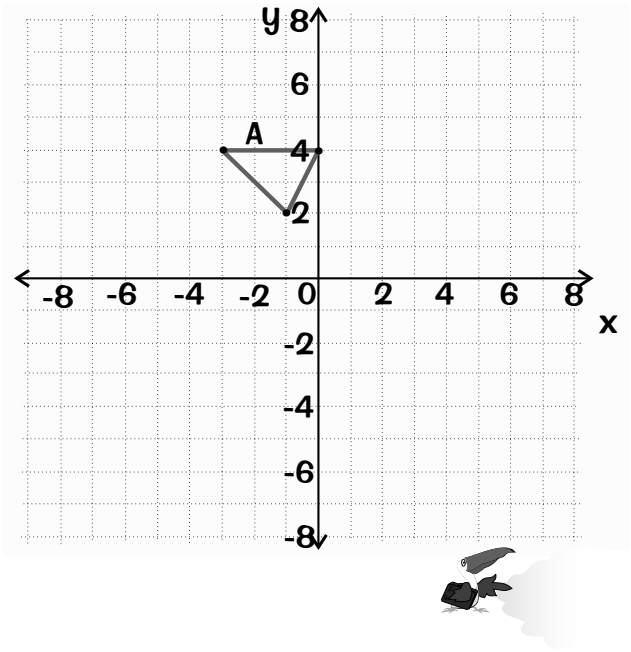
b) Reflect A in the line $x = 1$.
Label the image C.

c) Reflect A in the line $y = -1$.
Label the image D.

d) Translate triangle D with the vector $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$.
Label this image E.

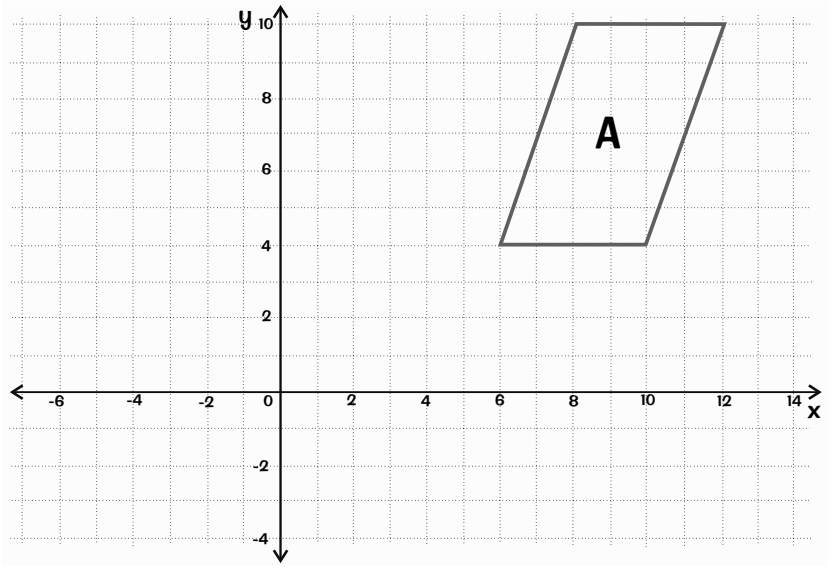
e) Translate triangle C with the vector $\begin{pmatrix} 3 \\ -3 \end{pmatrix}$.
Label this image F.

f) Describe fully the transformation that sends C to E.



Q5 Copy the axes below using a scale of 1 cm to 1 unit.

A parallelogram A has vertices at (6, 4), (10, 4), (8, 10) and (12, 10).
Draw this parallelogram onto your axes.
An enlargement of scale factor ½ and centre (0, 0) transforms parallelogram A onto its image B.



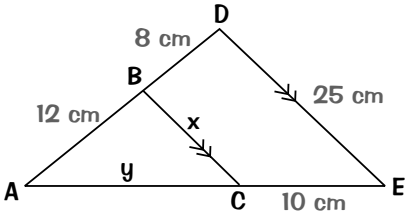
a) Draw this image B on your axes.

b) Translate B by the vector $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$ and label this image C.

c) Calculate the ratio of the area of parallelogram C to the area of parallelogram A.

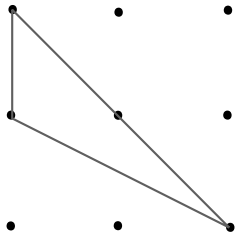
Congruence, Similarity and Enlargement

Q1 In the diagram below, BC is parallel to DE.
AB = 12 cm, BD = 8 cm, DE = 25 cm and CE = 10 cm.

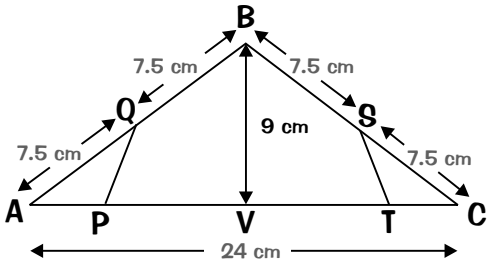


a) Explain why triangles ABC and ADE are similar.
b) Find the lengths of x and y in the diagram.

Q2 Another triangle, congruent to the triangle shown on the right, must be drawn with vertices at three of the dots. Show in how many different ways this can be done.

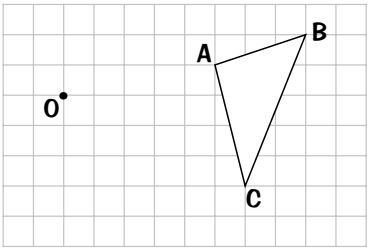


Q3 A boy made a symmetrical framework with metal rods as shown. Lengths AB = BC, ST = TC and AP = PQ. Angle BVC = 90° and length BV = 9 cm.



a) Find two triangles which are similar to triangle ABC.
b) Calculate the length of AP. Hence write down the length of PT.
c) Calculate the area of triangle ABC.
d) Find the area of triangle APQ. Give your answer correct to 3 significant figures.
e) Hence write down the area of PQBST correct to 2 significant figures.

Q4 Copy the diagram on the right onto squared paper.

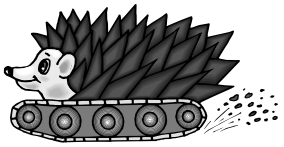


a) Draw an enlargement of the triangle ABC of scale factor 2. Use the point O as the centre of enlargement and label the image $A_1B_1C_1$.
b) Rotate triangle ABC through 180° about the point O. Label the image $A_2B_2C_2$.
c) Which image is congruent to triangle ABC?

Congruence, Similarity and Enlargement

Don't forget that when you're enlarging areas and volumes, there's a bigger scale factor — that one catches everyone out.

Q5 Sharon has a fish tank which is 42 cm wide and has a volume of 30 litres.
She sees a similar fish tank in the pet shop which is 63 cm wide.
What is the volume of the larger fish tank to the nearest litre?



Q6 A cylindrical bottle can hold 1 litre of oil. A second cylindrical bottle has twice the radius but the same height. It also contains oil.

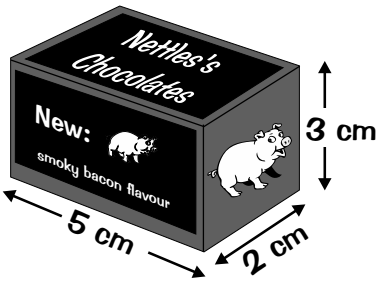
a) Explain why these bottles are not similar.

b) How much oil can the larger bottle hold?

Q7 For a graphics project, Eliza makes a model of a chocolate box in the shape of a cuboid 5 cm long, 2 cm wide and 3 cm high.

a) Calculate the area of material needed to make the model (assuming no flaps are required for glueing).

b) Eliza decides that the full size packaging will be similar to the model, but enlarged by a scale factor of 4. Calculate the area of material Eliza needs to make a full-size box.



Q8 On a holiday near the sea, children built a sandcastle in the shape of a cone. The radius of the base is 100 cm and the height is 100 cm.

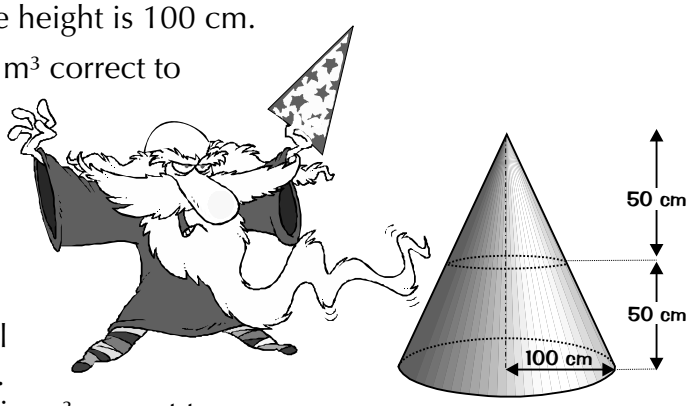
a) What is the volume of the sandcastle in m^3 correct to 3 significant figures?
The children now remove the top portion to make a similar cone but only 50 cm in height.

b) State the radius of the base of this smaller cone.

c) State the ratio of the volume of the small cone to the volume of the original cone.

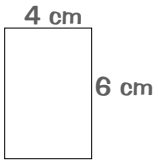
d) Calculate the volume of the small cone in m^3 correct to 3 significant figures.

e) Hence write down the ratio of the volume of the portion left of the original cone to the smaller cone in the form n:1.

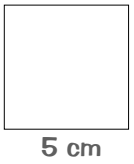


Perimeter and Area

Q1 Calculate the area and perimeter of the rectangle.



Q2 Calculate the area and perimeter of the square.

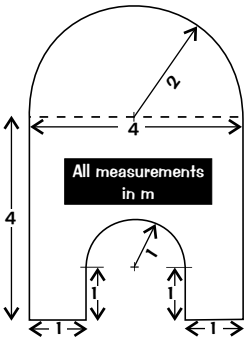


Q3 You have been asked to paint the outside wall of a building shown in the diagram opposite.

a) Find the area of wall that needs painting.

b) A 1 litre tin of paint will cover 13 m^2. How many 1 litre tins are needed to give the wall two coats of paint?

c) The wall is also going to have a tiled border around the edge. Calculate the length of the edge of the wall to be tiled.



You need the two circle formulas here — $C = \pi \times d$ and $A = \pi \times r^2$.

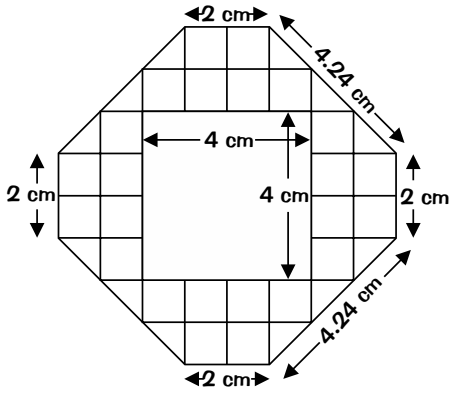
Q4 A rectangular dining room, with a width equal to half its length, needs carpet tiling.

a) Calculate the area of the floor, if its width is 12 m.

b) If carpet tiles are 50 cm by 50 cm squares, calculate how many tiles will be required.

c) If carpet tiles cost £4.99 per m^2, calculate the cost of tiling the dining room.

Q5 An attachment on a child's toy is made from plastic in the shape of an octagon with a square cut out. By counting squares or otherwise, find the area of plastic needed to make 4 of these attachments.

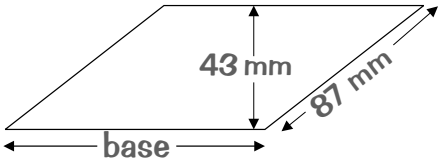


Q6 Josh is making a cube bean bag out of material for his textiles coursework. If each side of the cube is to have edges of length 60 cm, how many square metres of material will Josh need?

Q7 Natasha is training for a marathon by jogging around the outside of a square field of area 9000 m^2. One evening, Natasha completed 11 laps of the field. How far did she run? Give your answer to the nearest 100 m.

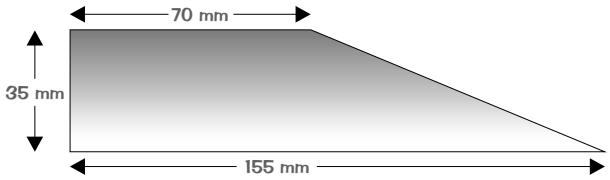
Q8 A lawn is to be made 48 m^2. If its width is 5 m, how long is it? How many rolls of turf 50 cm wide and 11 m long should be ordered to grass this area?

Q9 This parallelogram has an area of 4773 mm^2. How long is its base?



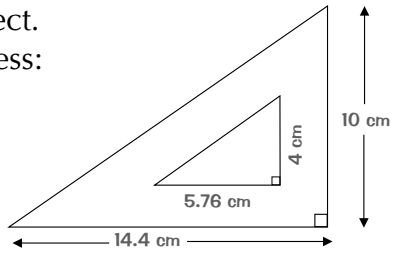
Perimeter and Area

Q10 A metal blade for a craft knife is the shape of a trapezium.
Calculate the area of the metal.

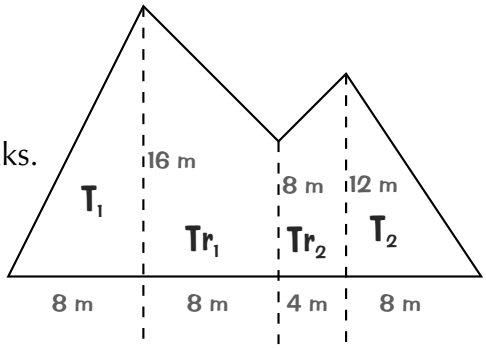


If you can't remember the formula for the area of a trapezium, you can always split the shape into a rectangle and a triangle.

Q11 Jon is making a metal bracket as part of his technology project.
The bracket is stamped out of sheet metal in a 2 phase process:
1st: The outer triangle, measuring 14.4 cm by 10 cm, is stamped out.
2nd: A smaller inner triangle measuring 5.76 cm by 4 cm is stamped out of the larger triangle.
The bracket should be made from no more than 50 cm^2 of sheet metal if the fixing is to support its weight.
Will the fixing take the weight of Jon's bracket?

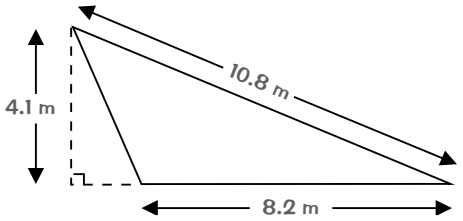


Q12 A modern glass sculpture is to be erected.
It is made from glass in the shape of two mountain peaks.
Calculate each separate area and hence find the total area of glass required.



Q13 A fighter aircraft's wing is shown on the right.
Calculate its area, and its perimeter.

Two lots of Pythagoras are needed to find the length of the third side.

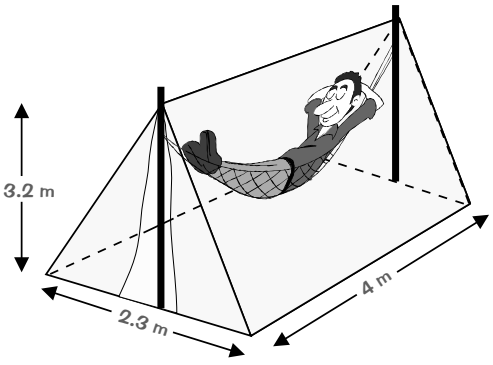


Q14 A simple tent is to be made in the shape of a triangular prism. The dimensions are shown in the diagram.

a) The two end faces are isosceles triangles. Find their areas.

b) The two sides and ground sheet are rectangles. Find their areas.

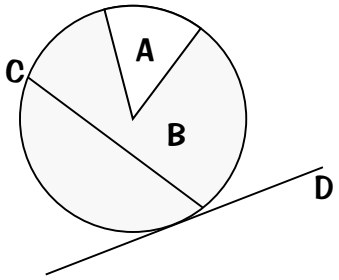
c) How much material is required to make this tent?



Q15 Calculate the area of a rhombus with diagonals 7 km by 11 km.

Perimeter and Area

Q16 A minor sector is labelled A on the diagram. Name the features labelled B, C and D.

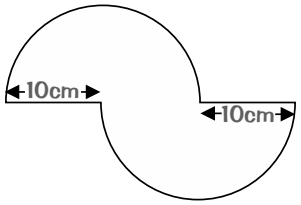


Q17 Using $\pi = 3.14$, find:

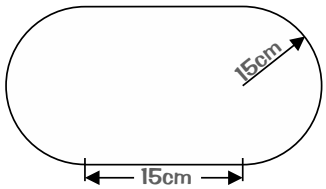
a) The area of a circle with radius = 6.12 m. Give your answer to 3 d.p.
b) The circumference of a circle with radius = 7.2 m. Give your answer to 2 s.f.
c) The circumference of a circle with diameter = 14.8 m. Give your answer to 1 d.p.
d) The area of a circle with diameter = 4.246 cm. Give your answer to 3 d.p.
e) The area of a semicircle with radius = 2.77 cm. Give your answer to 3 s.f.
f) The perimeter of a semicircle with diameter = 10.11 cm. Give your answer to 2 d.p.

Q18 Find the area and the perimeter of each of the shapes drawn here. Use $\pi = 3.14$.

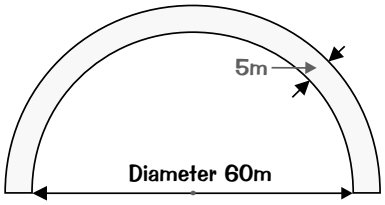
a)



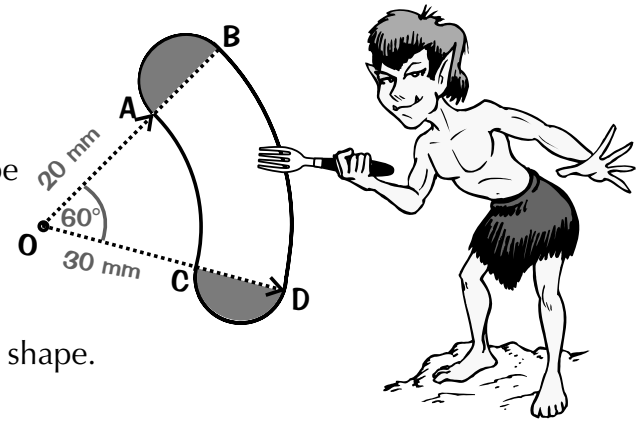
b)



c)

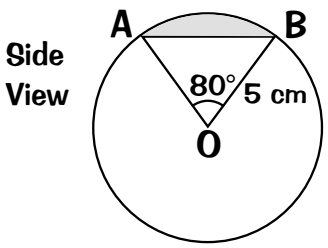


Q19 A plastic strip is made in the shape shown. The curves AC and BD are both arcs of circles with centre O. The larger circle has radius 30 mm and the smaller circle has radius 20 mm. The shaded ends of the shape are both semicircles.



a) Find the area of the shape ABDC.
b) Find the area of the two semicircular ends. Hence write down the area of the complete shape.

Q20



A washing powder ball looks from the side like a circle with the shaded area removed. The circle has radius 5 cm and the angle AOB = 80°.

a) Find the area of the sector OAB.
b) Find the area of triangle AOB and hence the area of the shaded area.

The formula you need for part b) is area = ½absin(C).

Surface Area

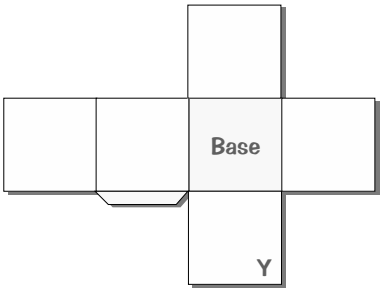
Before you go any further — make sure you know these 3 facts...

Surface Area and Nets

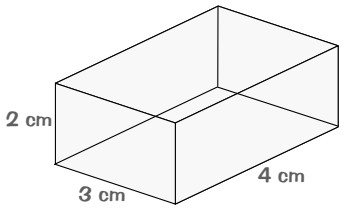
1) SURFACE AREA only applies to solid 3-D objects. It's the TOTAL AREA of all the OUTER SURFACES added together.
2) A NET is just A SOLID SHAPE folded out FLAT.
3) SURFACE AREA OF SOLID = AREA OF NET.

Q1 The net shown will fold to make a cube. Only one flap is shown. Copy the diagram.

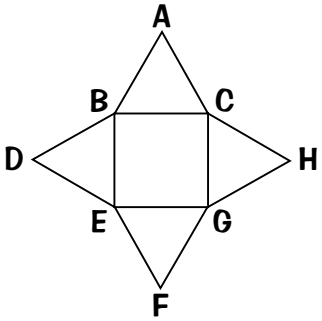
a) Put an X in each corner that touches Y when the cube is made up.
b) Put an F where the flap will join one face to another when the cube is made up.
c) Put on the other flaps necessary to glue the cube together.



Q2 Draw an accurate net that would fold to make the 3-D cuboid shown (diagram is not full size).
It is not necessary to include flaps.



Q3

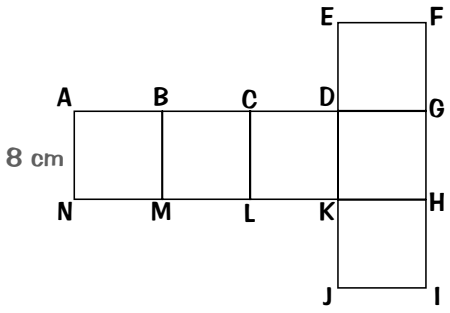


The diagram shows the net of a solid in which ABC is an equilateral triangle and BCGE is a square.

a) Which points will coincide with A when the net is folded up to make the solid?
b) Describe the symmetry of the net.
c) How many faces, edges and vertices does it have when in solid form?

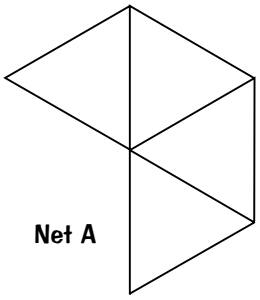
Q4 The diagram shows the net of a cube of edge 8 cm.

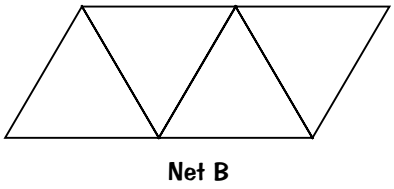
a) Which point coincides with M when the net is folded to make the cube?
b) Find the area of the face DGHK.
c) What is the total surface area of the cube?
d) Use isometric paper to draw a 3-D scale drawing of the completed cube.



Surface Area

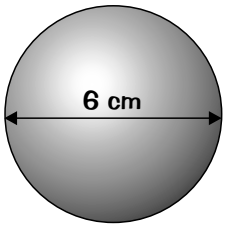
Q5 Which of these two nets will form a pyramid on a triangular base with all four faces equilateral triangles?



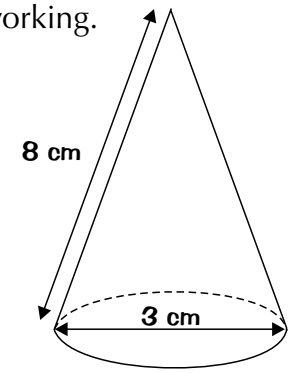


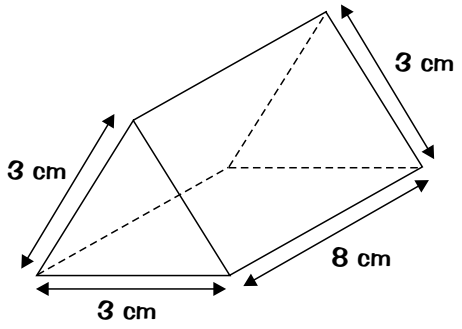
Q6 Hannah bought a bottle of perfume as a present in a cylindrical container, which was 20 cm tall and had a radius of 4 cm. She wanted to wrap the container in shiny red paper. She has enough paper to cover an area of 600 cm^2. Is that enough to wrap the container?

Q7 Find the surface area of a sphere with a diameter of 6 cm.



Q8 Which of these two shapes has the greatest surface area? Show your working.



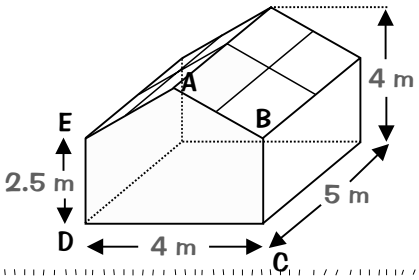


Q9 A hemisphere has a surface area of 75π. Find the radius, in cm, of the hemisphere.

Q10 Bill has a greenhouse with dimensions as shown. The roof is made up of eight panels of equal size.

A storm breaks all of the glass in the shaded area on the diagram.

Calculate the area of glass which Bill must buy to repair his greenhouse.



Look for a right-angled triangle to calculate AB.

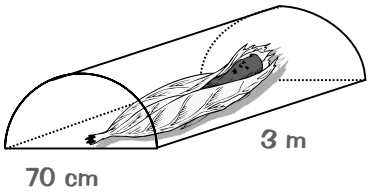
Volume

Make sure you know the five main volume formulas — for cuboids, spheres, prisms, pyramids and cones.

Q1 Joe buys a polythene tunnel to protect his plants from frost. It has a semicircular diameter of 70 cm and a length of 3 m.

a) Find the cross-sectional area.

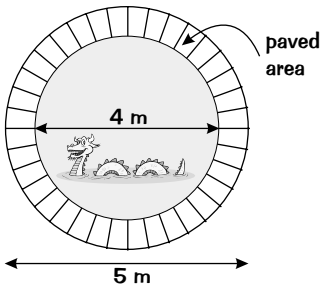
b) Hence find the volume of the tunnel.



Q2 I am planning to build a circular pond in my garden surrounded by a ring shaped paved area. The pond will be 50 cm deep and filled with water.

a) Calculate the approximate cost of paving the area around the pond with slabs costing £16 per m^2. Give your answer to the nearest £10.

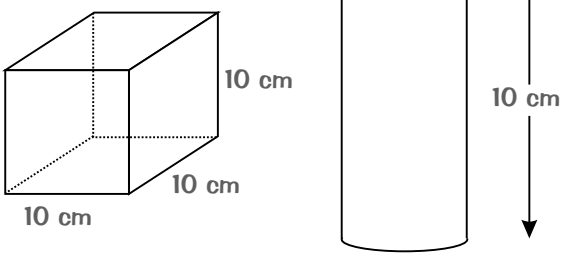
b) I need to add 15 ml of liquid pond treatment for every m^3 of water in the pond. Find the volume of treatment I will need to add to the pond. Give your answer to the nearest ml.



Q3 A solid metal cube, each of whose sides is 10 cm long, is melted down and made into a solid cylinder 10 cm high.

a) What is the radius of this cylinder?

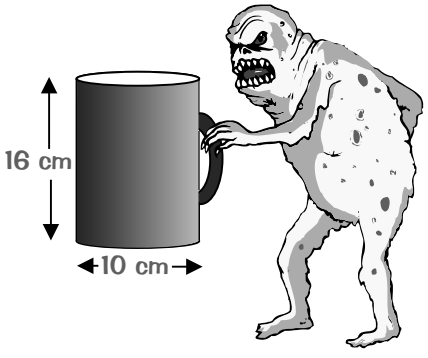
b) Find the surface area of the cylinder.



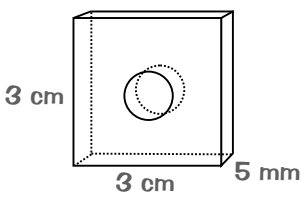
Q4 A tin mug has the dimensions shown.

a) What is the greatest volume of milk the mug can hold?

b) In fact, 600 cm^3 of milk is poured in. How high will it go up the mug?



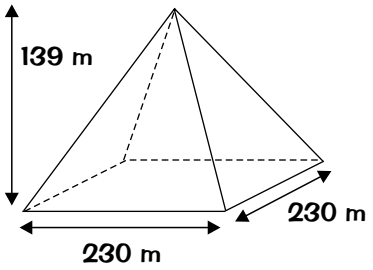
Q5



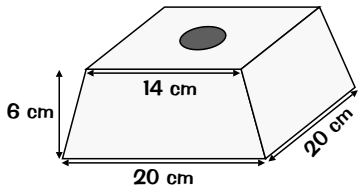
A nut has the cross-section illustrated. The circular hole has a diameter of 1.4 cm and the nut is 5 mm thick. Find the volume of the nut in cm^3.

Volume

Q6 The largest Egyptian pyramid has a square base of 230 m and a height of 139 m.
What is the volume of this pyramid?
Give your answer to the nearest m^3.

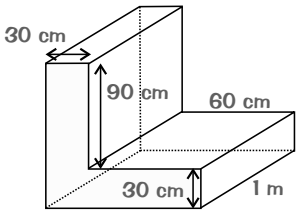


Q7 Steve has bought a pair of speaker stands. The base of each stand is a hollow prism with the dimensions shown. A hollow tube of diameter 4 cm and height 110 cm screws into the top of each base to form the stand.
Steve is filling the stands with sand to improve stability. Find the volume of sand Steve needs to use to fill both stands (the bases and the tubes).
Give your answer in litres to 2 d.p.



1 litre = 1000 cm^3

Q8



Jill buys a bookshelf with the dimensions shown in the diagram.

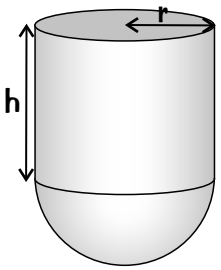
a) Find the cross-sectional area.

b) Find the volume of the bookshelf in m^3.

Q9 Davey is pumping up a basketball. He knows the diameter of a fully-inflated basketball should be no more than 30 cm. Calculate the maximum volume of air that Davey should use to inflate his basketball.

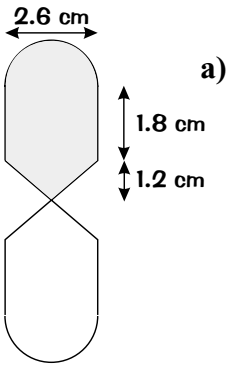
Q10 An earplug is made up of a rubber hemisphere joined to a rubber cylinder. The radius of the cylinder base is equal to the radius of the hemisphere base.

If $\frac{\text{volume of cylinder}}{\text{volume of hemisphere}} = 3$, find an expression for h in terms of r.



Volume

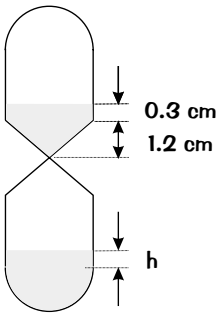
Q11 An egg timer is symmetrical and consists of hemispheres, cylinders and cones joined together as shown to the left.



a) Calculate the volume of sand in the upper container.

You need to find three volumes and add them together.

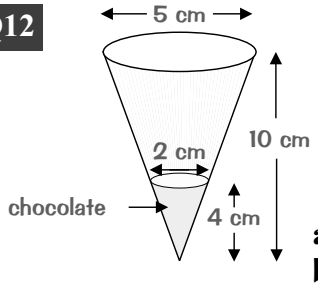
Sand runs into the bottom container at a constant rate of 0.05 cm³ per second. At the end of a certain time period the sand has fallen through into the bottom container as shown to the right.



b) How high (h) has it risen up the cylindrical part of the bottom container?

c) How long has it taken the sand to fall through until it is at this height?

Q12

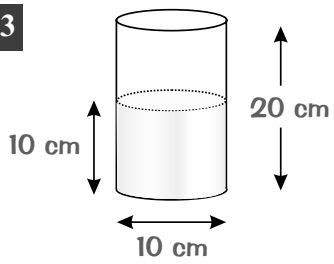


An ice-cream cone is 10 cm deep and has a base diameter of 5 cm. The bottom 4 cm of the cone is filled with solid chocolate as shown. The rest of the cone is filled with ice cream and a hemisphere of ice cream is mounted on top so that the base of the hemisphere coincides with the base of the cone.

a) Calculate the volume of ice cream required to make one ice cream.

b) Calculate the outer surface area of the cone.

Q13



Mike and Shelly are doing an experiment to find the radius of a marble. They fill a cylindrical container of diameter 10 cm and height 20 cm with water to a depth of 10 cm. 200 identical marbles are now submerged in the water. The depth increases to 14.5 cm. Calculate the radius of one marble.

The volume increase is a cylinder and you're told the height.

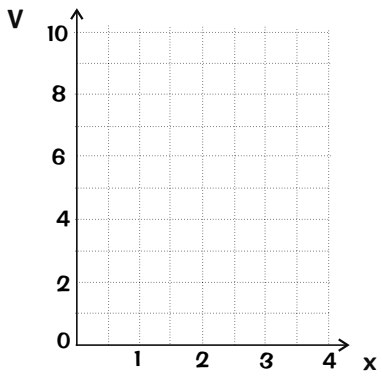
Q14 A cuboid has a height of x m, a width of $(3 - x)$ m and a length of $(5 - x)$ m.

a) Write down an expression for the volume of the cuboid.

b) Complete the table of values using your expression for the volume of the cuboid.

x	0	1	2	3
V			6	

c) Draw a graph of V against x for $0 \leqslant x \leqslant 3$.



d) Use your graph to estimate the maximum volume of the cuboid.

e) Estimate the surface area of the cuboid when the volume is at its maximum.

f) A particular cuboid has a volume of 6 m³. By using your graph to find the two possible values of x, estimate the maximum total surface area of the cuboid for this volume.

Time

Q1 The times below are given using a 24-hour system.
Using am or pm, give the equivalent time for a 12-hour clock.

a) 0500 **c)** 0316 **e)** 2230
b) 1448 **d)** 1558 **f)** 0001

Q2 The times below are taken from a 12-hour clock. Give the equivalent 24-hour readings.

a) 11.30 pm **c)** 12.15 am **e)** 8.30 am
b) 10.22 am **d)** 12.15 pm **f)** 4.45 pm

Q3 Joe is watching a film. It starts at 19.55 and ends at 22.20.
How many minutes does the film last?

Q4 Find the time elapsed between the following pairs of times:

a) 0820 on 1 October 2010 and 1620 on the same day
b) 10.22 pm on 1 October 2010 and 8.22 am the next day
c) 2.18 am on 1 October 2010 and 2.14 pm later the same day
d) 0310 on 1 October 2010 and 0258 on 3 October 2010.

Q5 Convert the following into hours and minutes:

a) 3.25 hours **b)** 0.4 hours **c)** 7.3 hours **d)** 1.2 hours.

Q6 Convert the following into just hours:

a) 2 hours and 20 minutes
b) 3 hours and 6 minutes
c) 20 minutes.

Q7 This timetable refers to three trains that travel from Asham to Derton.
a) Which train is quickest from Asham to Derton?
b) Which train is quickest from Cottingham to Derton?

Asham – Derton			
	Train 1	Train 2	Train 3
Asham	0832	1135	1336
Bordhouse	0914	1216	1414
Cottingham	1002	1259	1456
Derton	1101	1404	1602

c) I live in Bordhouse. It takes me 8 minutes to walk to the train station.
At what time must I leave the house by to arrive in Derton before 2.30 pm?

Speed, Density and Pressure

Speed, density and pressure — just learn the formulas and keep an eye on the units and everything'll be fine and dandy.

Q1 An athlete can run 100 m in 11 seconds.
Calculate the athlete's speed in:

a) m/s
b) km/h.

Q2 A plane flies over city A at 09.55 and over city B at 10.02.
What is its average speed if these cities are 63 km apart? Give your answer in km/h.

Q3 The distance from Kendal (Oxenholme) to London (Euston) is 420 km. The train travels at an average speed of 115 km/h. Pete needs to be in London by 10.30. If he catches the 07.05 from Kendal, will he be in London on time? Show all your working.

Q4 In a speed trial a sand yacht travelled a measured kilometre in 36.4 seconds.
a) Calculate this speed in km/h.
On the return kilometre it took 36.16 seconds.
b) Find the total time for the two runs.
c) Calculate the average speed of the two runs in km/h.

Remember, for the average speed, you use the total time and total distance.

Q5 A motorist drives from Manchester to London. 290 km is on motorway where he averages 100 km/h. 90 km is on city roads where he averages 45 km/h, and 25 km is on country roads where he averages 40 km/h.
a) Calculate the total time taken for the journey.
b) How far did he travel altogether?
c) Calculate the average speed for the journey.

Q6 Victor walks at an average speed of 4 km/h. He needs to walk to Askam-in-Furness, 3 km away. He needs to be there at 3.00pm. What time should he set off?

Q7 The distance between two railway stations is 145 km.
a) How long does a train travelling at 65 km/h on average take to travel this distance?
b) Another train travels at an average speed of 80 km/h, not including a 10 minute stop during the journey. How long does this second train take?
c) If both arrive at 1600, what time did each leave?

Q8 Two athletes run a road race. One ran at an average speed of 16 km/h, the other at 4 m/s. Which was the fastest? How long would each take to run 10 km?

Q9 A plane leaves Amsterdam at 0715 and flies at an average speed of 650 km/h to Paris, arriving at 0800. It takes off again at 0840 and flies at the same average speed to Nice arriving at 1005.
a) How far is it from Amsterdam to Paris?
b) How far is it from Paris to Nice?
c) What was the average speed for the whole journey, including the stop at Paris?

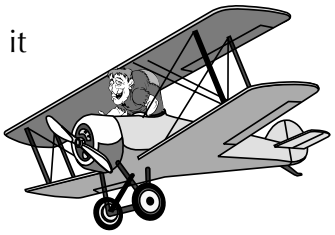
Speed, Density and Pressure

Q10 A runner covered the first 100 m of a 200 m race in 12.3 seconds.

a) What was his average speed for the first 100 m?

b) The second 100 m took 15.1 seconds. What was his average speed for the 200 m?

Q11 A military plane can achieve a speed of 1100 km/h. At this speed it passes over town A at 1205 and town B at 1217.

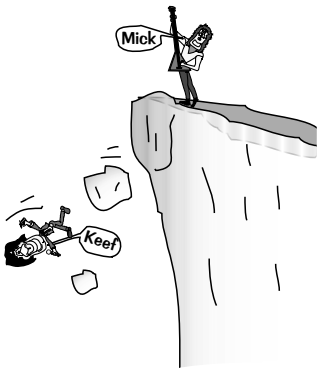


a) How far apart are towns A and B?

b) The plane then flies over village C which is 93 km from B. To the nearest minute, how long does it take from B to C?

Q12 Two cars set off on 290 km journeys. One travels mostly on A roads and manages an average speed of 68 km/h. The other car travels mostly on the motorway and achieves an average speed of 103 km/h when the car is moving, but the driver stops for a break. If they both take the same time over the journey, for how long does the second car stop?

Q13 A stone is dropped from a cliff top. After 1 second it has fallen 4.9 m, after 2 seconds a total of 19.6 m and after 3 seconds a total of 44.1 m. Calculate its average speed:



a) in the first second;

b) in the third second;

c) for all 3 seconds.

d) Change all the m/s speeds to km/h.

Q14 Find the density of each of these pieces of wood, giving your answer in g/cm^3:

a) Mass 3 g, volume 4 cm^3

b) Mass 12 kg, volume 20,000 cm^3

c) Mass 20 g, volume 25 cm^3

d) Mass 14 kg, volume 0.02 m^3.

Q15 Calculate the mass of each of these objects:

a) a small marble statue of density 2.6 g/cm^3 and volume 24 cm^3

b) a plastic cube of volume 64 cm^3 and density 1.5 g/cm^3

c) a gold ingot measuring 12 cm by 4 cm by 4 cm with density 19.5 g/cm^3

d) a pebble with volume 30 cm^3 and density 2.5 g/cm^3.

Q16 Work out the volume of each of these items:

a) a bag of sugar of mass 1 kg and density 0.85 g/cm^3

b) a packet of margarine with density 0.9 g/cm^3 and mass 250 g

c) a box of cereal with density 0.2 g/cm^3 and mass 500 g.

d) a 50 kg sack of coal with density 1.1 g/cm^3

Speed, Density and Pressure

Q17 My copper bracelet has a volume of 3.9 cm^3. The density of copper is 8.9 g/cm^3. Work out the mass of my bracelet.

Q18 Ice has a density of 0.93 g/cm^3. If the mass of a block of ice is 19.5 kg, what is its volume?

Q19 Some petrol in a can has a mass of 4 kg. The density of the petrol is 0.8 g/cm^3. How many litres of petrol are in the can?

Q20 A jug holds 1.9 litres of lemonade. The mass of the lemonade is 2 kg. Find the density of the lemonade in g/cm^3.

Q21 A 1.5 kg box full of self raising flour measures 12 cm by 18 cm by 6 cm. A 1 kg box of granary flour measures 10 cm by 14 cm by 6 cm.

a) Find the density of each sort of flour in g/cm^3.

b) Jake needs to measure out 450 g of granary flour but his scales are broken. Use your answer to part **a)** to work out how much flour he should measure out in his measuring jug. Give your answer in ml.

Q22 A solid rubber ball has a diameter of 7 cm. The density of the rubber is 920 kg/m^3. What is the mass of the rubber ball? Give your answer in grams to 2 d.p.

The formula for the volume of a sphere is: $V = \frac{4}{3}\pi r^3$

Q23 A mattress weighing 450 N sits on the floor, which is horizontal. The side of the mattress resting on the floor measures 1.5 m by 2 m. Work out the pressure exerted by the mattress on the floor.

Q24 When a chest of drawers of weight 680 N rests on horizontal ground, it exerts a pressure of 850 N/m^2. Work out the area of the base of the chest of drawers.

Q25 A large speaker sitting on horizontal ground exerts 640 N/m^2 of pressure on the ground. The side of the speaker that rests on the ground has an area of 0.7 m^2. What is the weight of the speaker?

Q26 A chair with a circular base exerts a pressure of 500 N/m^2 onto the floor it rests on. If the chair weighs 147 N, what is the radius of its base? Give your answer in cm to 2 d.p.

D/T and S/T Graphs

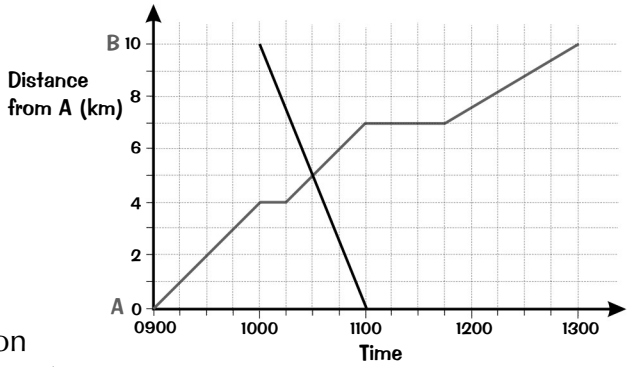
You need to remember what the different bits of a travel graph mean — what it looks like when <u>stopped</u>, <u>changing speed</u> and <u>coming back</u> to the starting point.

Q1 Peter set out from A at 0900 to walk to B.

a) How far did he walk in the 1st hour?

b) He stopped twice; how long was each stop?

c) What was his speed after the second stop?



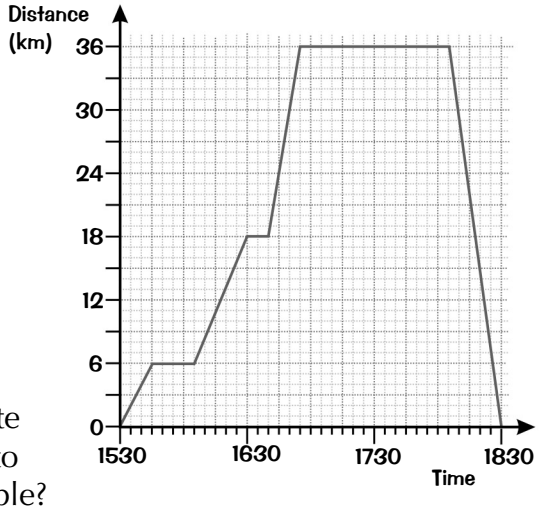
At 1000 Sarah set out on her bike to ride from B to A.

d) What time did she arrive at A?

e) What was her average speed?

f) At what time did Peter and Sarah pass each other?

Q2 Dave drives a bus from Kendal to Ingleton and back again. The bus company graphed the journey to help them organise their bus schedules.



a) How long did it take to get to Ingleton?

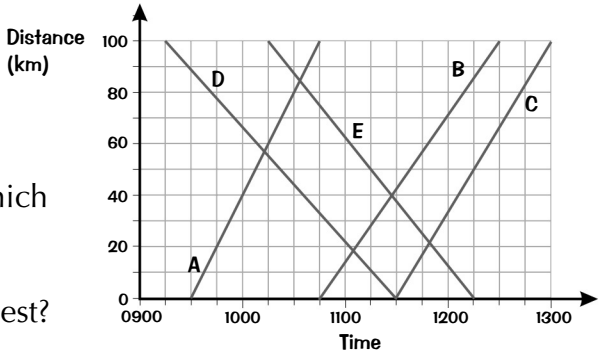
b) How much time was spent driving to and from Ingleton excluding stops?

c) What was the average speed for the journey from Kendal to Ingleton?

d) What was Dave's fastest speed?

e) The transport manager wants Dave to reduce the duration of the stops on the Kendal to Ingleton route so that he can make another journey from Kendal to Windermere starting at 1630. Would this be possible?

Q3 A train operator plans to purchase a new engine. They've graphed the journeys made by five engines over a 100 km stretch of track to help them decide which one is best.



a) Calculate the speed of each train and state which one was the fastest.

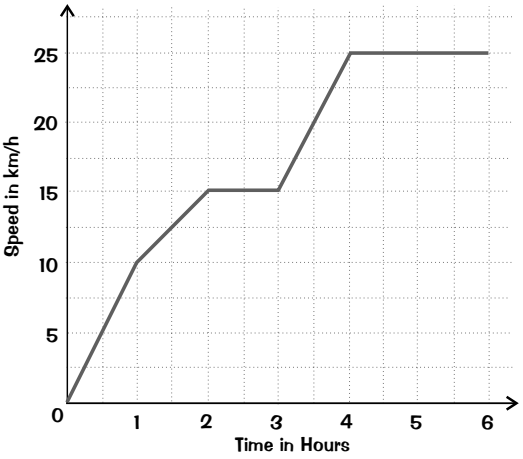
b) How could you tell by looking at the diagram which was the fastest and which was the slowest?

D/T and S/T Graphs

Q4 Mr Smith leaves home at 0730 to go to work. He walks at a steady 6 km/h for 2 km. He catches the 0755 train, which takes 35 mins to travel 50 km. He then walks 3 km to work and arrives at 0900.

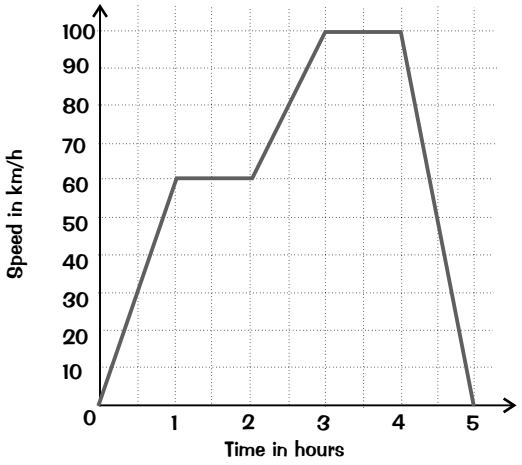
Draw a graph to show this. How long did he wait at the station for the train?

Q5 This is a speed-time graph for part of a cycle ride.



a) How long in total was the cyclist accelerating for?
b) When was the cyclist travelling the fastest?
c) What does the graph show the cyclist was doing in the third hour of the ride?

Q6 This is a speed-time graph of a train journey.



a) Calculate the acceleration of the train in the third hour of the journey.
b) In which of the first four hours of the journey was the train's acceleration greatest?

Q7 Two cars start a journey at midday (1200) — one travels from town A to village B, and the other from village B to town A. A and B are 80 km apart. The car from town A travels at an average speed of 48 km/h and the other car, from village B, at 60 km/h.

a) Draw a graph to show these journeys.
b) At what time do the cars pass? (approx.)
c) Approximately how far from A are they when they pass?

Use the speeds given to work out the time it takes for each car to travel the 80 km.

Q8 A girl set off on an all-day walk. She started at 0915 and walked at a steady speed for 9 km before stopping at 1100 for a 20 minute break. She then set off again at a steady speed and walked 8 km, stopping at 1300 for 45 minutes. After lunch she walked at 3½ km/h for 2½ hours to her destination.

a) Draw a graph to show this walk.
b) How far did she walk altogether?
c) What was the average speed for the whole walk?
d) What was her fastest walking speed?

Unit Conversions

You've got to know all the metric conversions — there's no way out of it, you'll just have to sit down and learn them, sorry and all that...

Q1 Express the given quantity in the unit(s) in brackets:

a) 2 m [cm]
b) 3.3 cm [mm]
c) 4 kg [g]
d) 600 g [kg]
e) 650 m [km]
f) 9 kg [g]
g) 7 g [kg]
h) 950 g [kg]
i) 6 m [mm]
j) 2 tonnes [kg]
k) 3000 g [kg]
l) 8 cm 6 mm [mm]
m) 550 kg [tonnes]
n) 3 m 54 cm [cm]
o) 0.7 cm [mm]
p) 4200 ml [l]

Q2 A seamstress needs to cut a 47 cm strip of finest Chinese silk.

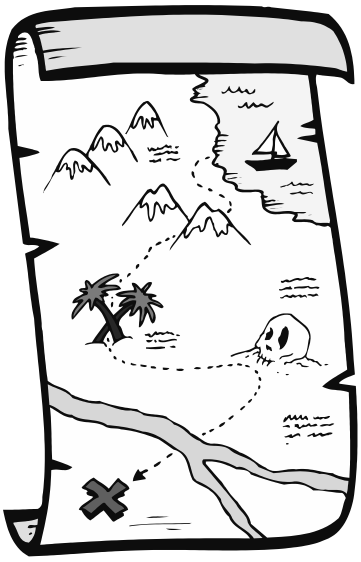
a) How many m is this?
b) How many mm is this?

Q3 The priceless Greek statue in my garden is 3 metres tall.

a) How many cm is this?
b) How many mm is this?
c) How many km is this?

Q4 The scale on a map is 1:10 000. How big are the following in real life? Give your answers in km.

a) a distance of 2 cm on the map
b) a distance of 20 cm on the map
c) a distance of 70 cm on the map
d) an area of 2 cm^2 on the map?



Maps can be tricky. Best thing is to keep the units the same when you do the initial conversion, then do another conversion to the appropriate units.

Q5 Another map has a scale of 1:3000. What size on this map are the following? Give your answers in cm.

a) a distance of 5 km in real life
b) a distance of 1 km in real life
c) an area of 100 m^2 in real life
d) an area of 50 m^2 in real life?

Just remember — the distance on the map is unlikely to be larger than the real-life distance.

Q6 Justin is shopping online. He looks up the following exchange rates:

1.60 US dollars ($) to £1 sterling.

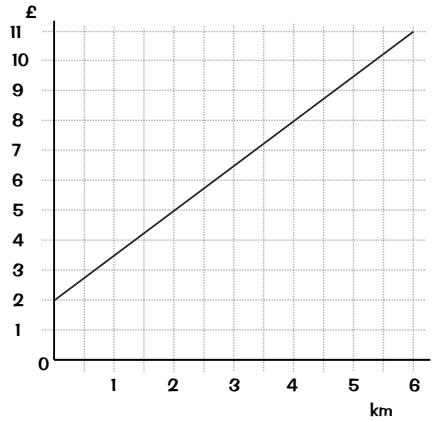
150 Japanese yen (¥) to £1 sterling.

Calculate to the nearest penny the cost in pounds sterling of each of Justin's purchases:

a) A book costing $7.50
b) An MP3 player costing ¥7660

Conversion Graphs

Q1 This graph can be used to convert the distance (km) travelled in a taxi to the fare payable (£).



a) How much will the fare be if you travel:
i) 2 km?
ii) 5 km?
iii) 10 km?

b) Mike lives 4.5 km away from his friend. Is £16 enough money for Mike to get a taxi to his friend's house and back?

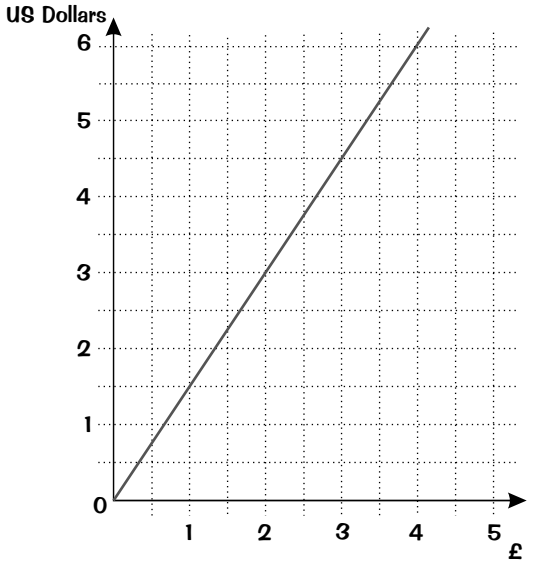
Q2 The graph on the right can be used to give a rough conversion between US dollars ($) and British pounds (£). Use the graph to estimate:

a) how many dollars you would get for £3

b) how many dollars you would get for £1.50

c) how many pounds you would get for $3

d) how many pounds you would get for $5.50.

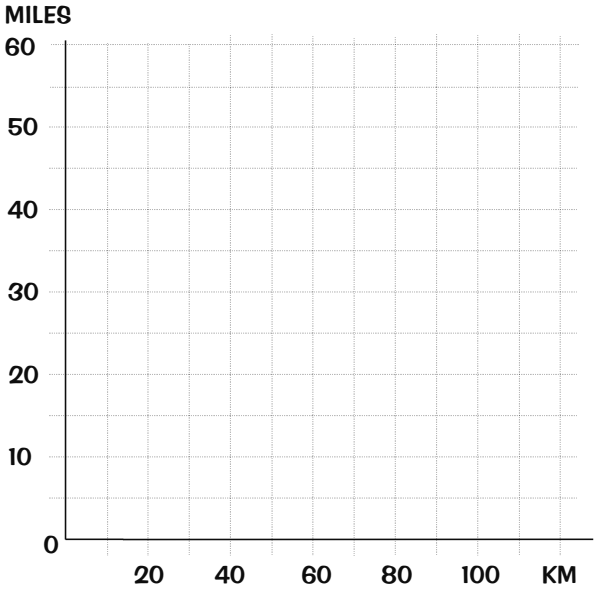


Q3 80 km is roughly equal to 50 miles. Use this information to draw a conversion graph on the grid.

When you've got to draw your own conversion graph, your best bet is to work out a few different values, and mark them on the graph first.

a) Use the graph to estimate the number of miles equal to:
i) 20 km
ii) 70 km
iii) 90 km

b) Use the graph to estimate how many km are equal to:
i) 40 miles
ii) 10 miles
iii) 30 miles



Constructions

Work through these questions bit by bit, and remember...

BISECTOR — a line splitting an angle or line exactly in two.

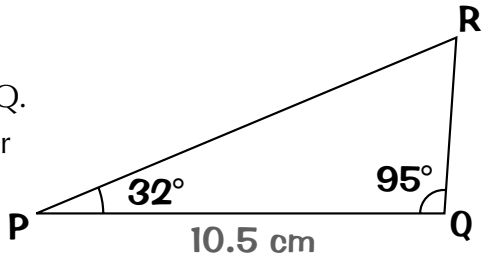
Q1 **a)** Construct a triangle ABC with AB = 4 cm, BC = 5 cm, AC = 3 cm.

b) Construct the perpendicular bisector of AB and where this line meets BC, label the new point D.

Q2 Construct triangle PQR accurately with length PQ = 10.5 cm, angle PQR = 95° and angle RPQ = 32°.

a) Construct the perpendicular bisector of the line PR. Draw in point A where the bisector crosses the line PQ.

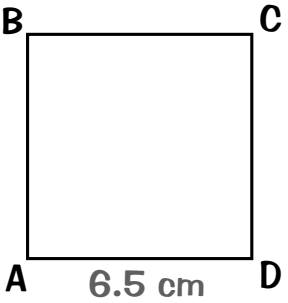
b) Bisect angle PRQ. Draw in point B where the bisector crosses the line PQ. Measure the length BA.



Q3 Use a ruler and compasses to construct square ABCD accurately with length AB = 6.5 cm.

a) Construct the bisectors of angles BAD and ADC. Mark the point O where the 2 bisectors cross.

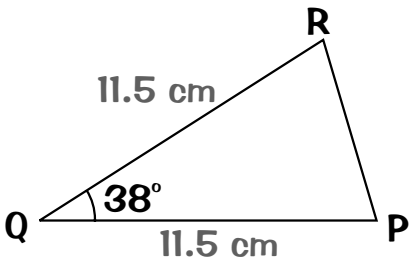
b) Bisect the angle OAD.



Q4 Construct triangle PQR with length PQ = QR = 11.5 cm and angle PQR = 38°.

a) Construct the bisectors of angles QPR and QRP. Mark the point O where the 2 bisectors cross.

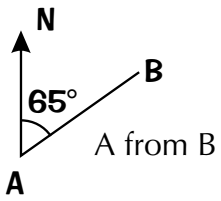
b) With centre O draw the circle which just touches the sides PQ, PR and QR of the triangle. What is the radius of this circle?



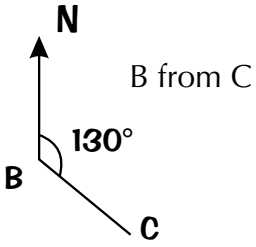
Bearings

Q1 Calculate the bearings required in these diagrams.

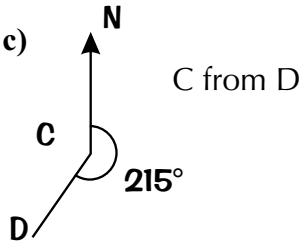
a)



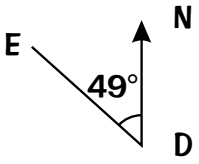
b)



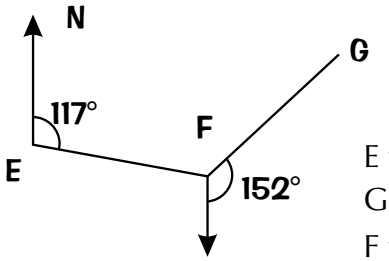
c)



d)



e)



E from F
G from F
F from G

It's easy to get lost if you don't follow the easy rule: always measure bearings from the north line.

Q2 A coastguard spots a boat on a bearing of 040° and at a distance of 350 m. He can also see a tree due east of him. The tree is due south of the boat.

a) Draw a scale diagram and measure accurately the distances from the:

i) boat to the tree

ii) coastguard to the tree

b) Check by Pythagoras to see if your answers are reasonable.

The word "from" is the most important word in a bearings question, so look out for it — it tells you where to start from.

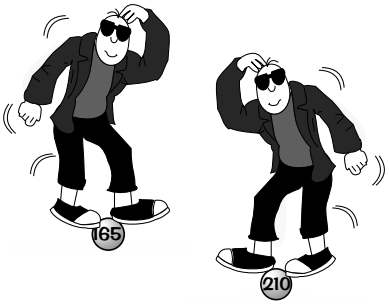
Q3 Four towns W, X, Y and Z are situated as follows:
W is 90 km north of X, Y is on a bearing 175° and 165 km from X, X is on a bearing 129° and 123 km from Z. Draw an accurate scale diagram to represent the situation.

From your drawing measure the distances:

a) WZ **b)** WY **c)** ZY.

Measure the bearings:

d) Y from Z **e)** W from Z **f)** Y from W.

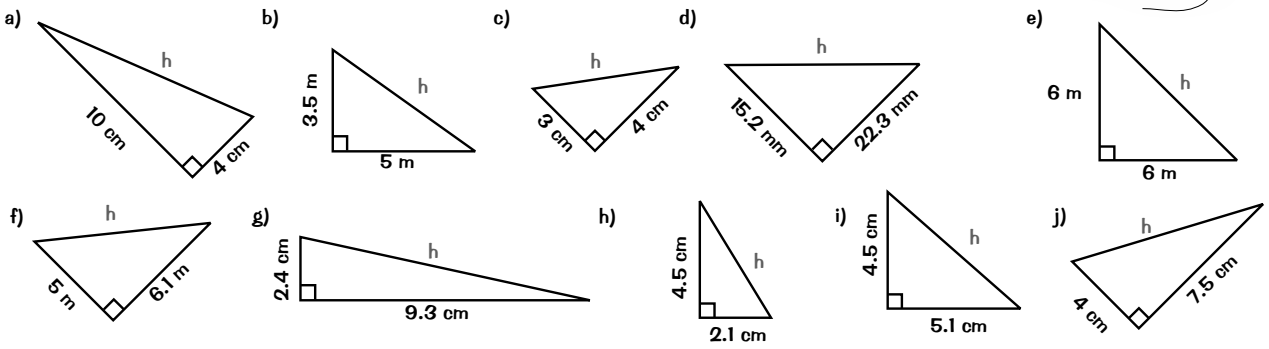


Q4 A walker travels 1200 m on a bearing of 165° and then another 1500 m on a bearing of 210°. By accurate measurement, find how far she is now from her starting point. What bearing must she walk on to return to base?

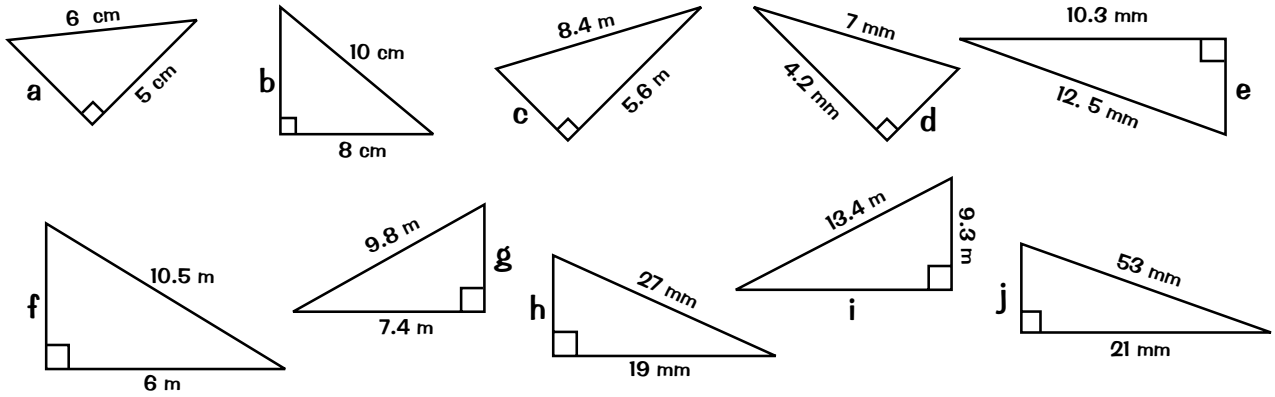
Pythagoras' Theorem

Don't try and do it all in your head — you've got to label the sides or you're bound to mess it up. Go on, get your pen out...

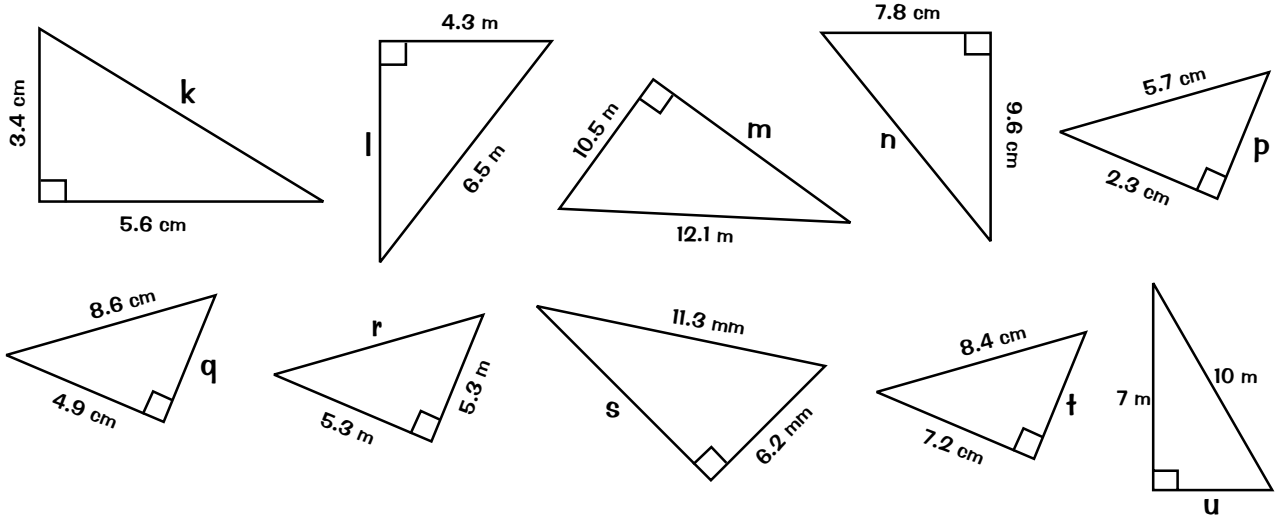
Q1 Find the length of the hypotenuse in each of the following triangles.



Q2 Find the length of the shorter side in each of the following triangles.

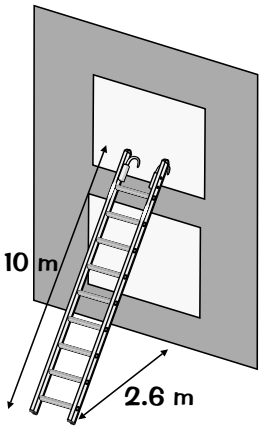


Q3 Find the unknown length in each of the following triangles.

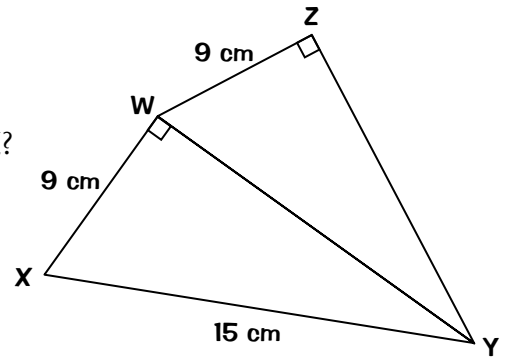


Pythagoras' Theorem

Q4 A window cleaner wants to clean the upstairs windows of an office. To meet safety regulations, his 10 m long ladder needs to be angled so that the bottom of the ladder is at least 2.6 m away from the wall. What is the maximum height that the top of the ladder can reach when used safely? Give your answer to 1 decimal place.



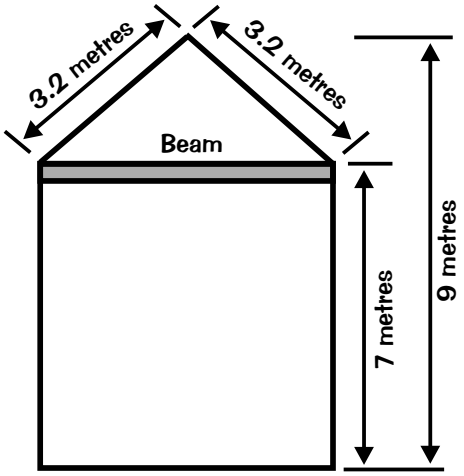
Q5 **a)** Calculate the lengths WY and ZY.
b) What is the total distance WXYZW?
c) What is the area of quadrilateral WXYZ?



Q6 A rectangular field is 250 m by 190 m. How far is it across diagonally?

Q7 A square tablecloth has a diagonal measurement of 130 cm. What is the length of one side?

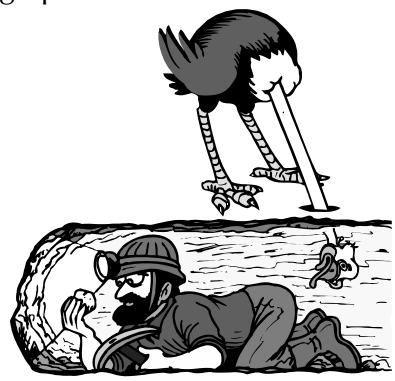
Q8 A builder is replacing a roof beam and wants to work out how long it needs to be. It has to be the same width as the house. The measurements he already knows are shown on the diagram. How long should the beam be?

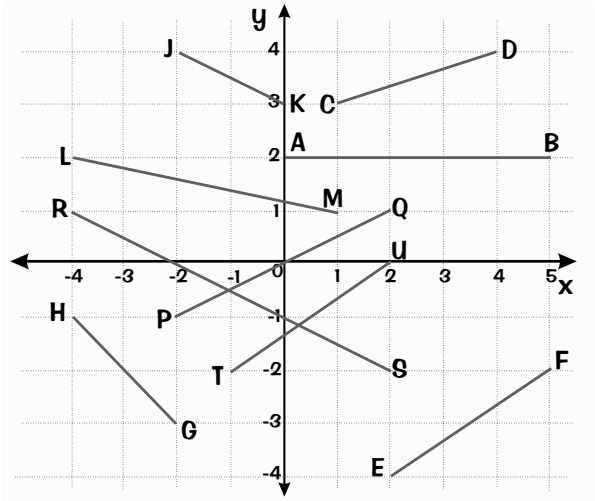


Q9 A flagpole 10 m high is supported by metal wires each 11 m long. How far from the foot of the pole must the wires be fastened to the ground if the other end is attached to the top of the pole?

Pythagoras' Theorem

Q10 Find the length of each of the lines on this graph.





Q11 The coordinates of four points are A(2,1), B(6,4), C(7,0) and D(3,–3).

a) Calculate the distances:
i) AB **ii)** BC **iii)** CD **iv)** BD **v)** AC

b) What shape is ABCD?

Q12 Find the length of line MN, where M and N have coordinates:

a) M(6,3) N(2,8)
b) M(1,5) N(8,12)
c) M(0,1) N(7,3)
d) M(9,5) N(4,8)
e) M(10,4) N(10,0)
f) M(12,6) N(13,0)

Q13 Find the length of line PQ, where P and Q have coordinates:

a) P(2,–3) Q(3,0)
b) P(1,–8) Q(4,3)
c) P(0,–1) Q(2,–3)
d) P(–6,–1) Q(7,–9)
e) P(12,–3) Q(–5,5)
f) P(–10,–2) Q(–2,–8)

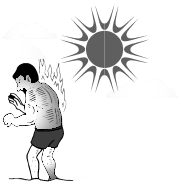
OK, so there's a few negative numbers creeping in here, but just do them in the same way.

Q14 A plane flies due east for 153 km then turns and flies due north for 116 km. How far is it now from where it started?

Q15 A fishing boat travels at 12 km/h for an hour due north. It then turns due west and travels at 7 km/h for an hour. How far is it from its starting point now? What bearing must it travel on to return to base?

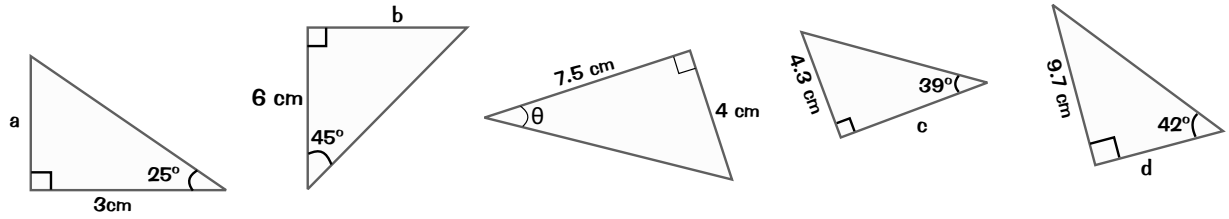
Trigonometry — Sin, Cos, Tan

Before you start a trigonometry question, write down the formulas using SOH CAH TOA (Sockatoa!) — it'll help you pick your formula.

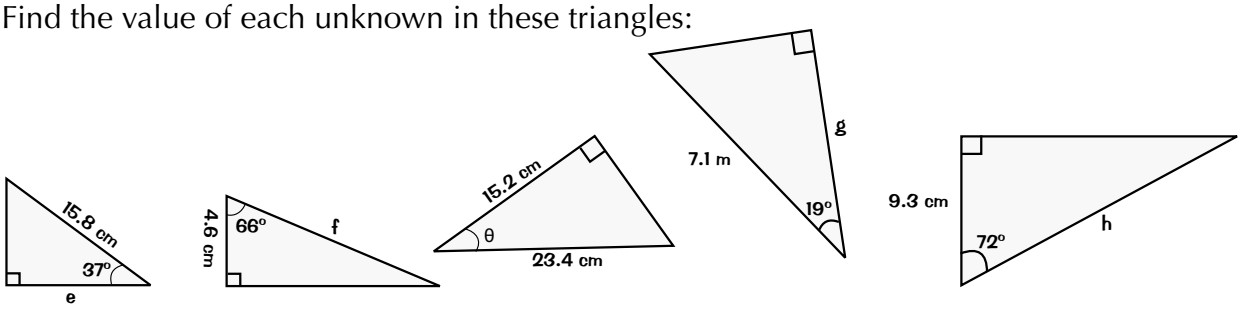


Q1 Calculate the tan, sin and cos of each of these angles:
a) 17° **b)** 83° **c)** 5° **d)** 28° **e)** 45°.

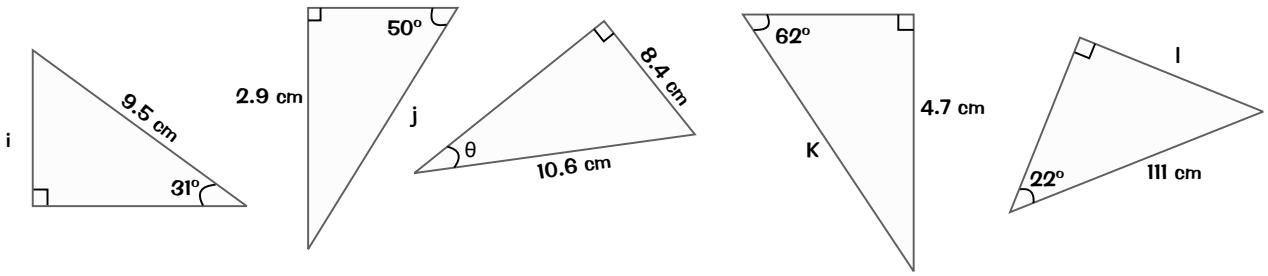
Q2 Find the unknowns in each of these triangles:



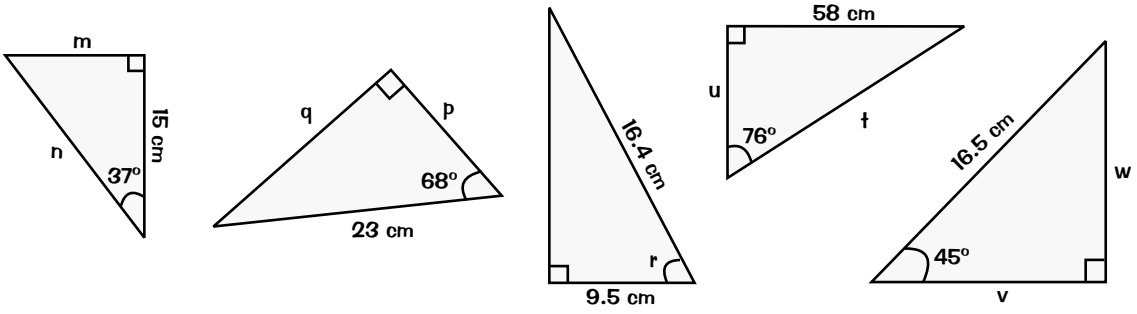
Q3 Find the value of each unknown in these triangles:



Q4 Calculate the unknowns in the following:



Q5 Find the unknowns in each of these triangles:



Trigonometry — Sin, Cos, Tan

Q6 A right-angled triangle has sides measuring 30 m, 40 m and 50 m.

a) Draw a rough sketch of the triangle, clearly labelling the hypotenuse.

b) Calculate the size of the smallest angle.

Make sure you've got the hang of the inverse SIN, COS and TAN functions on your calc... and check it's in DEG mode or you'll get nowhere fast.

Q7 The points P(1, 2), Q(4, 2) and R(4, -3) when joined together form a right-angled triangle.

a) Draw a rough sketch of the triangle, labelling the length of each side.

b) Without measuring, calculate the angle RPQ.

c) Deduce angle PRQ.

Q8 The points A(1, -2), B(4, -1) and C(1, 3) are the vertices of the triangle ABC.

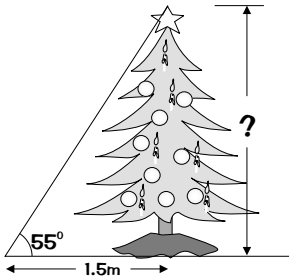
a) On graph paper, plot the points A, B and C.

b) By adding a suitable horizontal line, or otherwise, calculate the angle CAB.

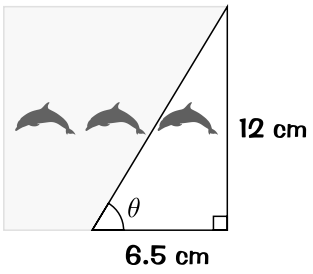
c) Similarly calculate the angle ACB.

d) By using the fact that the interior angles of a triangle add up to 180°, work out the angle ABC.

Q9 Mary was lying on the floor looking up at the star on top of her Christmas tree. She looked up through an angle of 55° when she was 1.5 m from the base of the tree. How high was the star?

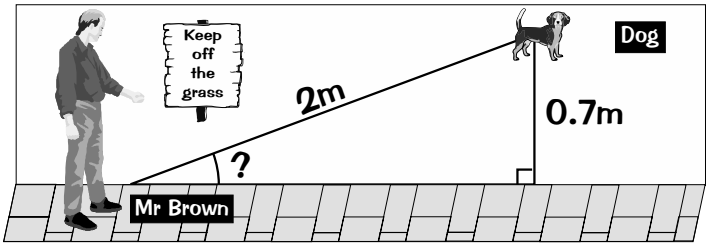


Q10 Geoff is tiling his bathroom. He needs to cut off the right-angled triangle shown so that the tiles will fit nicely on his wall. Calculate the angle, θ, he needs to cut the tile at. Give your answer to the nearest degree.



Q11 Mr Brown took his dog for a walk in the park. The dog's lead was 2 m long. The dog ran 0.7 m from the path Mr Brown was walking on, and Mr Brown dropped its lead.

What angle did the lead make with the path?

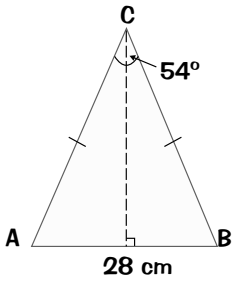


Q12 A boat travels 9 km due south and then 7 km due east. What bearing must it travel on to return directly to base?

Trigonometry — Sin, Cos and Tan

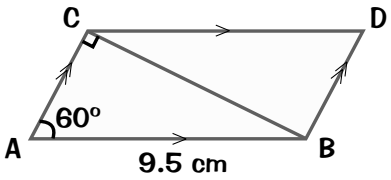
Q13 This isosceles triangle has a base of 28 cm and a top angle of 54°. Calculate:

a) the length of sides AC and BC
b) the perpendicular height to C
c) the area of the triangle.



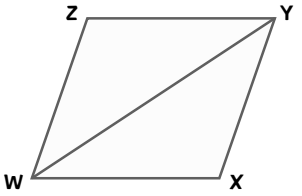
Q14 An isosceles triangle has two equal sides of 7 cm and an angle between them of 65°. Calculate the area of the triangle.

Q15 In this parallelogram the diagonal CB is at right angles to AC. AB is 9.5 cm and ∠CAB is 60°.
Calculate:
a) CB **b)** BD **c)** the area of the parallelogram.



Q16 This rhombus WXYZ has sides of length 15 cm and diagonal WY of 28 cm. Calculate the:

a) length of diagonal XZ
b) area of the rhombus
c) angle WY makes with WX.



Q17 Two mountains are 1020 m and 1235 m high. Standing on the summit of the lower one I look up through an angle of elevation of 16° to see the summit of the higher one. Calculate the horizontal distance between the two mountains.

Q18 A girl is flying a kite. She holds the string, which is 45 m long, at a height of 1.3 m above the ground. The string of the kite makes an angle of 33° with the horizontal. What is the vertical height of the kite from the ground?

Q19 I am standing on top of an 80 m high tower. I look due north and see two cars with angles of depression of 38° and 49°. Calculate:

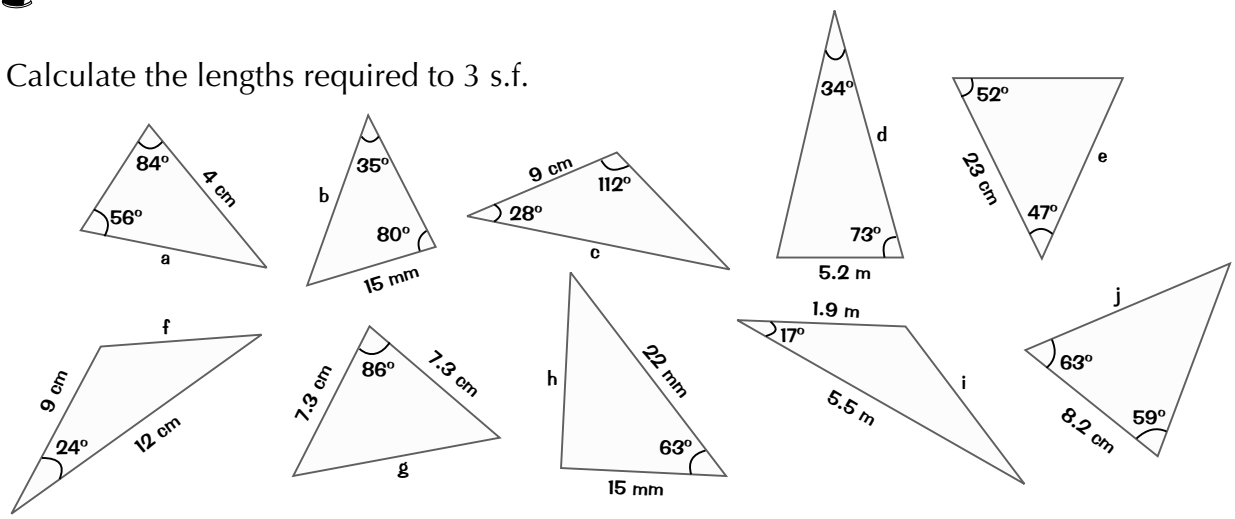
a) how far each car is from the base of the tower
b) how far apart the cars are.

Q20 A ship sails on a bearing of 300° for 100 km. The captain can then see a lighthouse due south of him that he knows is due west of his starting point. Calculate how far west the lighthouse is from the ship's starting point.

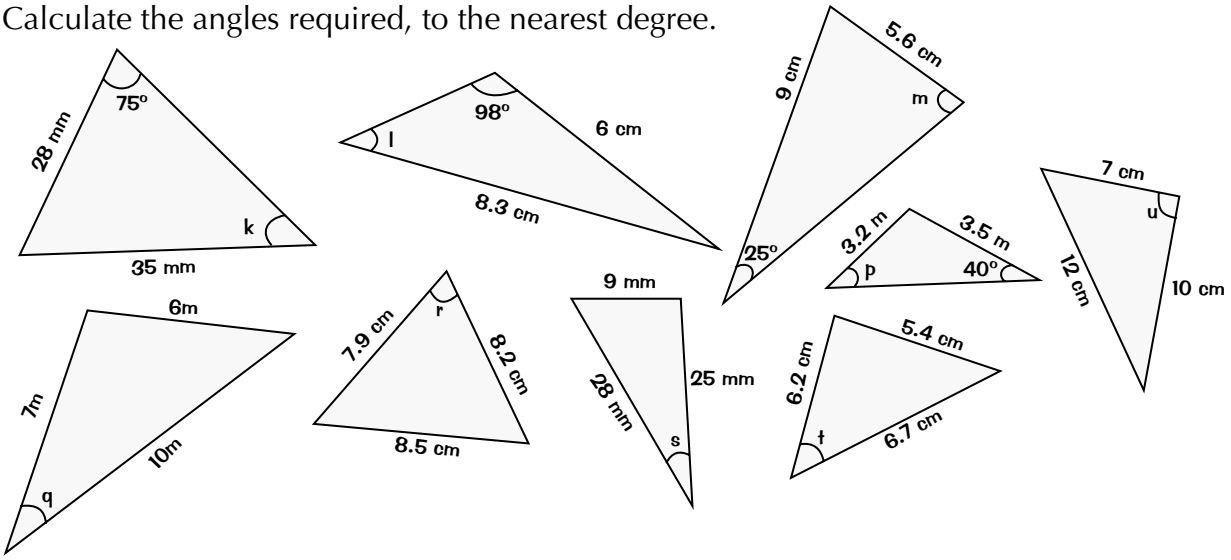
The Sine and Cosine Rules

Make sure you know the Sine Rule and <u>both forms</u> of the Cosine Rule. The one to use depends on which angles and sides you're given.

Q1 Calculate the lengths required to 3 s.f.



Q2 Calculate the angles required, to the nearest degree.

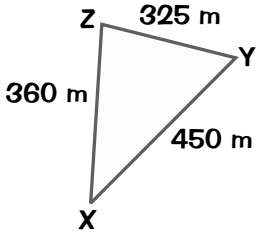


Q3 This field has measurements as shown. Calculate:

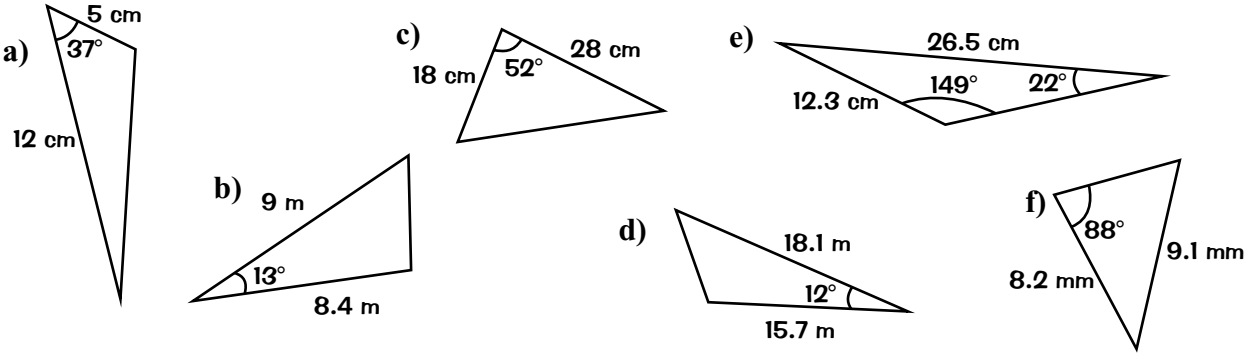
a) ∠ZXY

b) ∠XYZ

c) ∠YZX.



Q4 Find the areas of the following triangles. Give your answers to 1 decimal place.

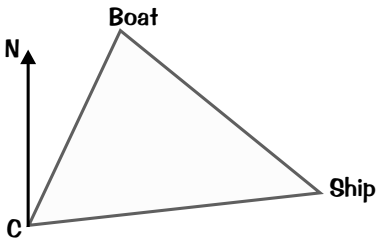


The Sine and Cosine Rules

Q5 A coastguard sees a boat on a bearing of 038° from him and 25 km away. He also sees a ship 42 km away and on a bearing of 080°. Calculate:

a) the distance of the boat from the ship

b) the bearing of the boat from the ship.



Q6 A parallelogram has sides of length 8 cm and 4.5 cm. One angle of the parallelogram is 124°. Calculate the lengths of the two diagonals.

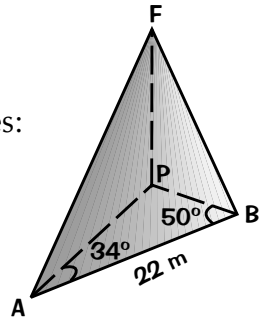
Q7 Air traffic control are testing the reliability of their computer software by monitoring two aeroplanes and checking the computer's calculations with their own. If the horizontal distance between the planes drops to 3 miles or less, an alarm should be triggered on the computer. One of the test planes is at a distance of 5 miles from the tower, and on a bearing of 020° from the tower. The second is at a distance of 4.6 miles on a bearing of 034° and the alarm is ringing. Calculate the horizontal distance between the planes and comment on the reliability of the software.

Q8 A vertical flagpole FP has two stay wires to the ground at A and B. They cannot be equidistant from P, as the ground is uneven. AB is 22 m, ∠PAB is 34° and ∠PBA is 50°. Calculate the distances:

a) PA **b)** PB.

If A is level with P and the angle of elevation of F from A is 49°, calculate:

c) FA **d)** PF.



Q9 An aircraft leaves A and flies 257 km to B on a bearing of 257°. It then flies on to C, 215 km away on a bearing of 163° from B. Calculate:

a) ∠ABC

b) distance CA

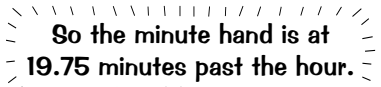
c) the bearing needed to fly from A direct to C.

Q10 On my clock the hour hand is 5.5 cm, the minute hand 8 cm and the second hand 7 cm, measured from the centre. Calculate the distance between the tips of the:

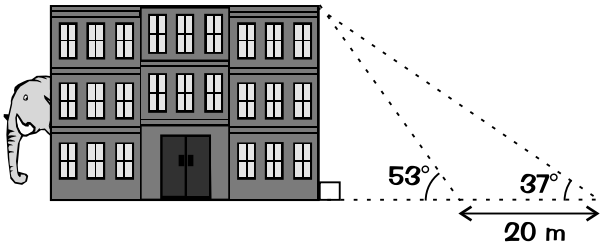
a) hour and minute hands at 10 o'clock

b) minute and second hands 15 seconds before 20 past the hour

c) hour and minute hands at 1020.



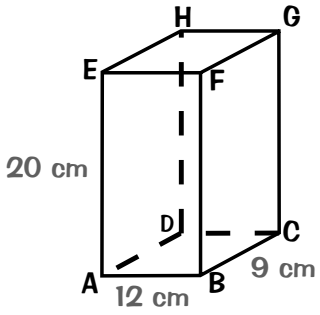
Q11 A surveyor wants to measure the height of a building. She measures the angle of elevation of the top of the building from the two different positions shown. Calculate the height of the building to the nearest metre.



Q12 Mary and Jane were standing one behind the other, 2.3 m apart, each holding one of the two strings of a kite flying directly in front of them. The angles of elevation of the kite from the girls were 65° and 48° respectively. Assuming the ends of both strings are held at the same height above the ground, calculate the length of each string.

3D Pythagoras and Trigonometry

Q1

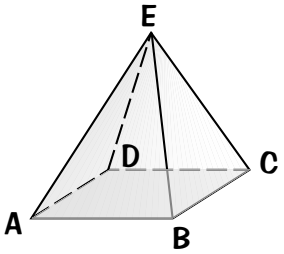


This rectangular box is 20 cm by 12 cm by 9 cm. Calculate:

a) angle ABE
b) length AF
c) length DF
d) angle EBH.

Q2 This pyramid is on a square base of side 56 cm. Its vertical height is 32 cm. Calculate the length of:

a) the line from E to the mid-point of BC
b) the sloping edge BE.

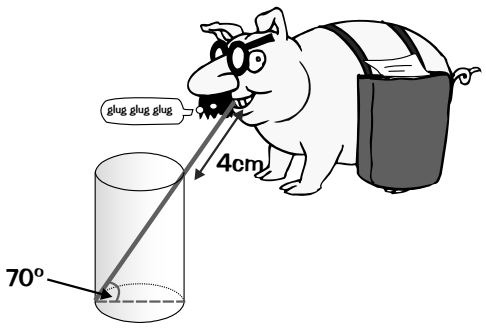


Q3 A rectangular box measures 20 cm by 30 cm by 8 cm. Calculate the lengths of:

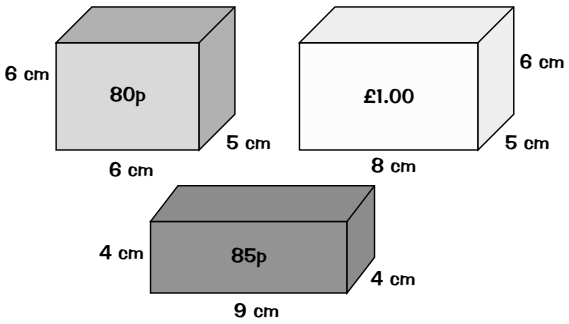
a) the diagonal of each rectangular face
b) the diagonal through the centre of the box.

Q4 This glass has a radius of 2.8 cm. The straw in the glass makes an angle of 70° with the base and protrudes 4 cm above the rim.

a) How tall is the glass?
b) How long is the straw?

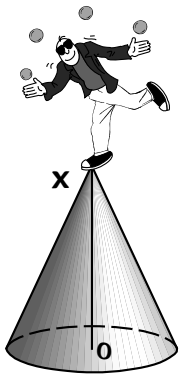


Q5 A shop sells the three different gift boxes shown on the right. Katie wants to buy the cheapest box that will fit a pen that is 10 cm long. Which box should she buy?



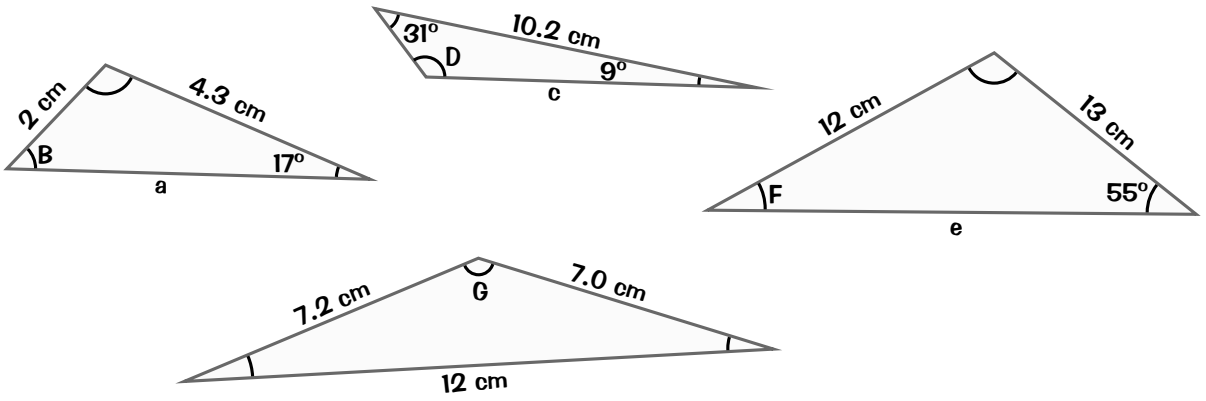
Q6 This cone has a perpendicular height of 9 cm. The centre of the base is O. The slant line from X makes an angle of 23° with the central axis. Calculate:

a) the radius of the base
b) the area of the base
c) the volume of the cone.



Sin, Cos and Tan for Larger Angles

Q1 Calculate the lettered sides and angles.



Q2 If $90° < x < 180°$, find x when:

a) $\sin x = 0.84$ **b)** $\sin x = 0.173$ **c)** $\tan x = -1$ **d)** $\tan x = -14.3$

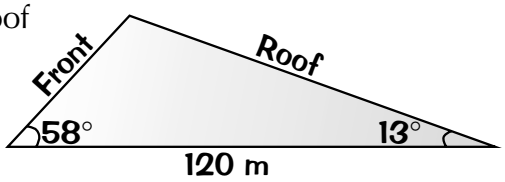
Q3 Fayrmin looks upon a mystical glade. In the glade is a unicorn being guarded by 2 minotaurs. One minotaur is 23 metres from the unicorn and the other one is 14 metres from the unicorn. They stand either side of the unicorn at an angle of 118°. How far apart are the minotaurs?

Q4 Adam surveys a building from inside. He finds the angle between the floor and the front is 58°. He walks to the other end of the building, where the roof meets the floor, and finds the angle between them to be 13°. The distance from one end of the building to the other is 120 m. Calculate:

a) the angle between the front of the building and the roof

b) the length of the building front and roof

c) the height of the highest point of the building.



Q5 A sailor sees a storm in the distance. The sailor knows that an island is 7 kilometres away from the boat and calculates that the angle between this island and the storm is 94° and that there is 26 kilometres of water between the storm and the island.
Calculate the proximity of the ship to the storm.

Q6 PQR is a triangle. The size of angle P is 25°.
Side r is 12 m long and side p is 7.5 m long.
What are the two possible values of the size of angle R?

Q7 A spacecraft (S) is travelling to the moon. When it is 210 000 km from mission control on Earth (E) it is found to be 170 000 km away from the landing site at the moon base (M). The straight-line distance between mission control and the moon base is 370 000 km. Calculate the angle ESM. Give your answer to 1 d.p.

Vectors

If you've got some squared or graph paper handy, it just might be useful for the first few questions on this page...

Q1 ABCDE is a pentagon.

$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$ $\overrightarrow{AD} = \begin{pmatrix} -2 \\ 6 \end{pmatrix}$ $\overrightarrow{AE} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

a) Draw this pentagon accurately.

b) Write down the vectors:

i) $\overrightarrow{DE}$ **ii)** $\overrightarrow{DC}$ **iii)** $\overrightarrow{EC}$

c) What sort of triangle is ACD?

Q2 ABC is a triangle.

$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$

a) Draw this triangle accurately.

b) Find $|\overrightarrow{AC}|$.

$|\overrightarrow{AC}|$ is the modulus (or size) of vector $\overrightarrow{AC}$. Use Pythagoras to find a vector's modulus.

Q3 $\boldsymbol{p} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\boldsymbol{q} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$, $\boldsymbol{r} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$, $\boldsymbol{s} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$

Calculate then draw:

a) $\boldsymbol{p} + \boldsymbol{q}$ **c)** $2\boldsymbol{r}$ **e)** $2\boldsymbol{p} - 2\boldsymbol{s}$ **g)** $2\boldsymbol{r} - \boldsymbol{q}$ **i)** $\boldsymbol{p} + 2\boldsymbol{s}$

b) $\boldsymbol{p} - \boldsymbol{q}$ **d)** $\boldsymbol{s} + \boldsymbol{p}$ **f)** $3\boldsymbol{q} + \boldsymbol{s}$ **h)** $\frac{1}{2}\boldsymbol{q} + 2\boldsymbol{r}$ **j)** $\boldsymbol{q} - 2\boldsymbol{r}$

Q4 Find the magnitude of the following vectors:

a) $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ **c)** $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ **e)** $\begin{pmatrix} 1 \\ 6 \end{pmatrix}$ **g)** $\begin{pmatrix} 5 \\ 7 \end{pmatrix}$ **i)** $\begin{pmatrix} -8 \\ 5 \end{pmatrix}$

b) $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ **d)** $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ **f)** $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ **h)** $\begin{pmatrix} 6 \\ 6 \end{pmatrix}$ **j)** $\begin{pmatrix} -10 \\ -5 \end{pmatrix}$

Q5 $\boldsymbol{e} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$, $\boldsymbol{f} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$, $\boldsymbol{g} = \begin{pmatrix} 7 \\ -3 \end{pmatrix}$, $\boldsymbol{h} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$

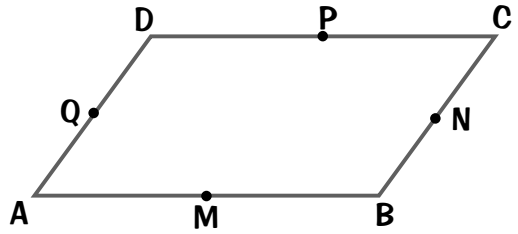
Find:

a) $\boldsymbol{e} + \boldsymbol{f}$ **c)** $\boldsymbol{f} + \boldsymbol{g}$ **e)** $\boldsymbol{e} + \boldsymbol{h}$

b) $|\boldsymbol{e} + \boldsymbol{f}|$ **d)** $|\boldsymbol{f} + \boldsymbol{g}|$ **f)** $|\boldsymbol{e} + \boldsymbol{h}|$

Vectors

Q6



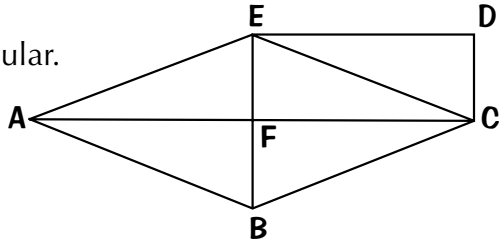
ABCD is a parallelogram. M, N, P and Q are the mid-points of the sides, as shown. $\overrightarrow{MQ} = \boldsymbol{x}$ and $\overrightarrow{AM} = \boldsymbol{y}$.

Express in terms of $\boldsymbol{x}$ and $\boldsymbol{y}$:

a) $\overrightarrow{AB}$ **c)** $\overrightarrow{NB}$ **e)** $\overrightarrow{AC}$

b) $\overrightarrow{AQ}$ **d)** $\overrightarrow{BC}$ **f)** $\overrightarrow{BD}$

Q7 In the diagram on the right, EB and AC are perpendicular. ABCE is a parallelogram. ∠EDC is a right angle.



a) Name a vector equal to:

i) $\overrightarrow{FC}$ **iii)** $\overrightarrow{BC}$ **v)** $2\overrightarrow{CD}$ **vii)** $\overrightarrow{EF} - \overrightarrow{CF}$

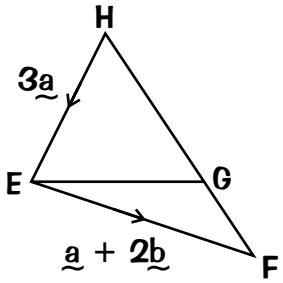
ii) $\overrightarrow{FB}$ **iv)** $\overrightarrow{CE}$ **vi)** $\overrightarrow{AE} + \overrightarrow{EC}$ **viii)** $\overrightarrow{ED} + \overrightarrow{DC} + \overrightarrow{CB}$

b) If AC = 16 cm and EB = 6 cm:

i) what is the area of ABCE?

ii) what is the area of ABCDE?

Q8 In the diagram to the right, HGF is a straight line where HG : GF = 3 : 2. Find vector $\overrightarrow{EG}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

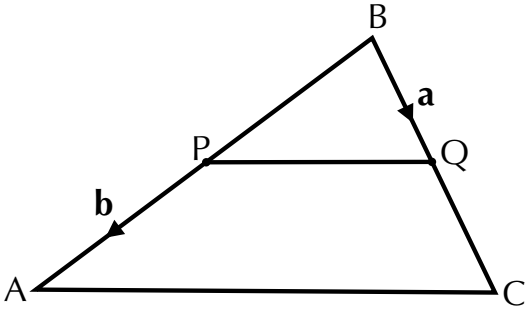


Q9 ABC is a triangle.

P is the point on BA such that BA = 2PA.

Q is the point on BC such that BC = 2BQ.

$\overrightarrow{BQ} = \mathbf{a}$, $\overrightarrow{PA} = \mathbf{b}$.



a) Express in terms of **a** and **b**:

i) $\overrightarrow{BC}$ **ii)** $\overrightarrow{CP}$ **iii)** $\overrightarrow{PQ}$

b) Prove that $\overrightarrow{PQ}$ is parallel to $\overrightarrow{AC}$.

Mean, Median, Mode and Range

For finding the mode and median put the data in order of size — it's much easier to find the most frequent and middle values.

The mean involves a bit more calculation, but hey, you're doing maths...

Q1 The local rugby team scored the following number of tries in their first 10 matches of the season:

3	5	4	2	0	1	3	0	3	4

Find their modal number of tries.

Q2 Find the mean, median, mode and range of these numbers:

1	2	−2	0	1	8	3	−3	2	4	−2	2

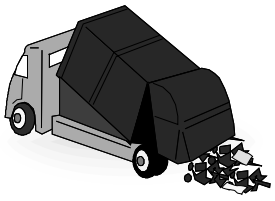
Q3 A company has 9 employees in the sales department who earn commission. They are advertising for another salesperson and want to say in the advert how much commission their staff earn on average. The amount of commission the 9 existing salespeople earned last year is as follows:

£13,000	£9,000	£7,500
£18,000	£12,000	£7,500
£23,000	£15,000	£11,500

a) Find the mean, median and mode of their earnings.
b) Which one does not give a good indication of their average commission?
c) Which should the company put in the advert, and why?

Q4 Molly is writing a letter of complaint to the bus company because she thinks her bus to school is regularly late. Over 3 weeks, Molly kept a record of how many minutes her bus was either early or late, and put this in her letter. (She used + for late and – for early.)

+2	–1	0	+5	–4
–7	0	–8	0	+4
–4	–3	+14	+2	0



a) Calculate the mean lateness/earliness of the bus.
b) Calculate the median.
c) What is the mode?
d) The bus company use the answers to **a)**, **b)** and **c)** to claim they are always on time. Is this true?

Careful with this — you have to use the averages to find the total weight, then divide to find the new average.

Q5 The average weight of the 11 players in a football team was 72.5 kg. The average weight of the 5 reserve players was 75.6 kg. What was the average weight of the whole squad? (Give your answer to 3 s.f.)

Q6 The mean daily weight of potatoes sold in a greengrocer's from Monday to Friday was 14 kg. The mean daily weight of potatoes sold from Monday to Saturday was 15 kg. How many kg of potatoes were sold on Saturday?

Mean, Median, Mode and Range

Q7 Colin averaged 83% over 3 exams. His average for the first two exams was 76%. What was Colin's score in the final exam?

Q8 The range for a certain list of numbers is 26. One of the numbers in the list is 48.

a) What is the lowest possible value a number in the list could be?

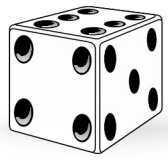
b) What is the highest possible value that could be in the list?

Q9 An ordinary dice is rolled 6 times, landing on a different number each time.

a) What is the mean score?

b) What is the median score?

c) What is the range of scores?

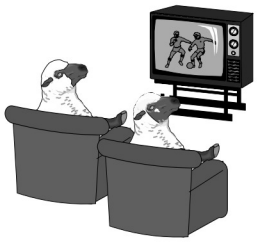


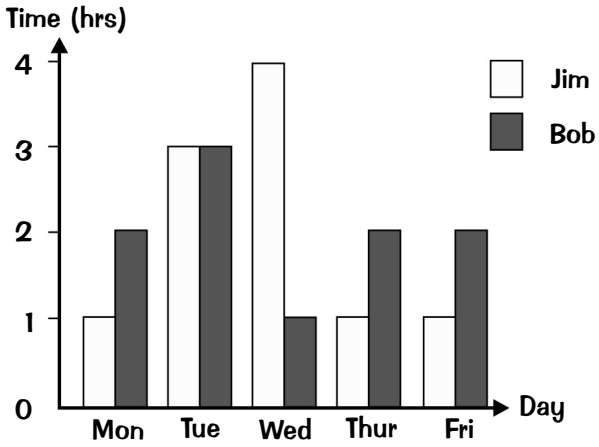
Q10 The bar graph shows the amount of time Jim and Bob spend watching TV during the week.

a) Find the mean amount of time per day each spends watching TV.

b) Find the range of times for each of them.

c) Using your answers from **a)** and **b)**, comment on what you notice about the way they watch TV.





Q11 Mr Jones posted 88 Christmas cards first class on Monday. His friends received them over the week: 40 on Tuesday, 28 on Wednesday, 9 on Thursday, 6 on Friday and the remainder on Saturday.

a) Find the modal number of days it took for the cards to arrive.

b) Find the median number of days it took for the cards to arrive.

c) "The majority of first class post arrives within 2 days." Is the above statement true or false in the light of the data?

Q12 In each of the following cases, decide which average is referred to:

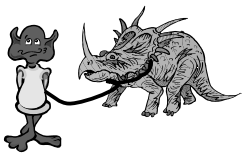
a) this average is least appropriate when the total number of values is small

b) this average is least affected if one of the values is removed at random

c) this average is most affected by the presence of extreme values.

Quartiles and Comparing Distributions

Remember to put the data in ascending order before you work out where the quartiles come in a list.



Q1 The weights (in g) of 29 eggs are:

60	72	58	60	68	69	59	72	54	56	65	68	63	70	71
67	64	63	69	62	63	67	59	72	61	66	65	67	70	

a) What is the median?
b) Which quartile is equivalent to the median?

Q2 The following table shows the number of cars parked in a multi-storey car park at midday on each day in December:

690	720	580	590	210	650	640	710
700	750	790	220	790	840	830	820
900	880	480	1000	990	1020	1010	1000
80	240	370	510	460	600	580	

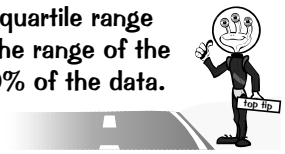
a) What is the range?
b) What is the lower quartile, Q_1?
c) What is the median?
d) What is the upper quartile, Q_3?

Quartiles divide the data into 4 equal groups.

Q3 For all the whole numbers from 1 to 399 inclusive:

What is the interquartile range?

The interquartile range tells you the range of the middle 50% of the data.



Q4 The range of 99 different integers is 98, and the median is 350.

a) What is the lower quartile, Q_1?
b) What is the interquartile range?

Q5 A dentist is about to employ a dental hygienist. She wishes to know if having a dental hygienist has an effect on the number of fillings she has to perform each year. So, prior to appointing him, the dentist takes some data from the record cards. Here it is:

No. of fillings	0	1	2	3	4	5
No. of children	1	2	8	30	60	12

Three years after appointing the dental hygienist, the dentist takes another set of data from the record cards. Here it is:

No. of fillings	0	1	2	3	4	5
No. of children	11	16	40	32	4	2

Using any statistical average you need, state what you see from the data, assuming that these records are for new patients.

Frequency Tables — Finding Averages

You've got to be able to do these in both row and column form, because they could give you either one. There's no real difference, and the rules are still the same.

Q1 To monitor their annual performance, a travel company logs all calls to their sales desk. The number of calls per day received by the sales desk over a given year are shown here.

No. of Calls	10	11	12	13	14	15	16 and over
No. of Days	110	70	120	27	18	12	8

a) Find the median number of calls.
b) Find the modal number of calls.
c) Find the interquartile range.

Q2 A student has classes in Mathematics (M), English (E), French (F), Art (A) and Science (S). Her timetable is shown opposite.

Monday	S S E E A
Tuesday	E M M A A
Wednesday	S M E F F
Thursday	F E E A S
Friday	M M E S S

a) Complete the following frequency table for a week's lessons:

Subject	M	E	F	A	S
Frequency					

b) Calculate the number of French lessons that the student will attend during a 12-week term.

c) What is the modal lesson?

Q3 20 pupils are asked to estimate the length (to the nearest m) of their gardens.
Here are the results: 10, 8, 6, 4, 10, 8, 0, 14, 12, 8, 10, 6, 1, 6, 10, 8, 6, 6, 8, 8
Copy the frequency table below and put the estimates in.

a) Find the mode of the data.
b) Find the median of the data.
c) State the range of the data.

Length (m)	4 and under	6	8	10	12	14 and over
Frequency						

Frequency Tables — Finding Averages

Q4 130 female bus drivers were weighed to the nearest kg.

a) Find the median weight.
b) Find the interquartile range of the weights.
c) Find the modal weight.
d) Calculate the mean weight, by first completing the table.

Weight (kg)	Frequency	Weight × Frequency
51	40	
52	30	
53	45	
54	10	
55	5	

Q5 A football magazine rates teams according to how many goals they're likely to score in a match, based on their last 20 matches. The table below shows the number of goals scored by Spark Bridge Wanderers over this period.

No. of goals	0	1	2	3	4	5	6
Frequency	0	1	1	7	6	3	2

Find the mean, mode and median of the data.

Q6 A tornado has struck the hamlet of Moose-on-the-Wold. Many houses have had windows broken. The frequency table shows the devastating effects.

No. of windows broken per house	0	1	2	3	4	5	6
Frequency	5	3	4	11	13	7	2

a) Find the modal number of broken windows.
b) Find the median number of broken windows.
c) Calculate the mean number of broken windows.

Q7 Using the computerised till in a shoe shop, the manager can predict what stock to order from the previous week's sales.
Opposite is the tabulated printout for last week for men's shoes.

Shoe size	5	6	7	8	9	10	11
frequency	9	28	56	70	56	28	9

a) The mean, mode and median for this data can be compared. For each of the following statements decide whether it is true or false.
i) The mode for this data is 70.
ii) The mean is greater than the median for this distribution.
iii) The mean, median and mode are all equal in this distribution.

b) What percentage of customers bought shoes of the mean size from last week's sales data:

i) 30% **ii)** 70% **iii)** 0.273% or **iv)** 27.3%?

Grouped Frequency Tables

Q1 The speeds of 32 skiers at a certain corner of a downhill course are tabulated below.

Speed (km/h)	$40 \leq s < 45$	$45 \leq s < 50$	$50 \leq s < 55$	$55 \leq s < 60$	$60 \leq s < 65$
Frequency	4	8	10	7	3
Mid-Interval					
Frequency × Mid-Interval					

a) By completing the frequency table, estimate the mean speed.
b) How many skiers were travelling at less than 55 km/h?
c) How many skiers were travelling at 50 km/h or faster?

Q2 The weights in kg of 18 newly felled trees are noted below:

272.7 333.2 251.0 246.5 328.0 259.6 200.2 312.8 344.3
226.8 362.0 348.3 256.1 232.9 309.7 398.0 284.5 327.4

a) Complete the frequency table.

Weight (kg)	Tally	Frequency	Mid-Interval	Frequency × Mid-Interval
$200 \leq w < 250$				
$250 \leq w < 300$				
$300 \leq w < 350$				
$350 \leq w < 400$				

b) Estimate the mean weight using the frequency table.
c) What is the modal group?

Q3 48 numbers are recorded below:

0.057 0.805 0.056 0.979 0.419 0.160 0.534 0.763
0.642 0.569 0.773 0.055 0.349 0.892 0.664 0.136
0.528 0.792 0.085 0.546 0.549 0.908 0.639 0.000
0.614 0.478 0.421 0.472 0.292 0.579 0.542 0.356
0.070 0.890 0.883 0.333 0.033 0.323 0.544 0.668
0.094 0.049 0.049 0.999 0.632 0.700 0.983 0.356

a) Transfer the data into the frequency table.

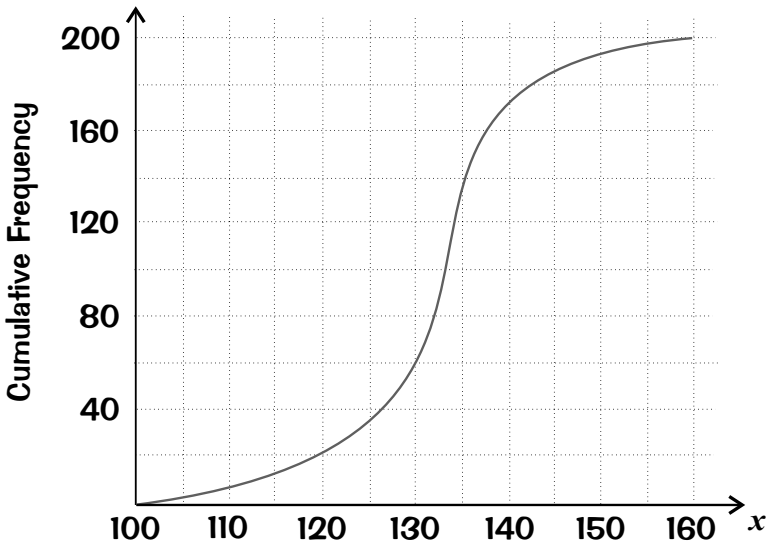
Number	$0 \leqslant n < 0.2$	$0.2 \leqslant n < 0.4$	$0.4 \leqslant n < 0.6$	$0.6 \leqslant n < 0.8$	$0.8 \leqslant n < 1$
Tally					
Frequency					
Mid-Interval					
Frequency × Mid-Interval					

b) Write down the modal class(es).
c) Which group contains the median?
d) Estimate the mean value.

Cumulative Frequency

Q1 Using the cumulative frequency curve, read off the:

a) median
b) lower quartile
c) upper quartile
d) interquartile range.



Q2 The number of passengers using a bus service each day has been recorded over a 4-week period. The data is presented in the table below:

No. passengers	$0 \leq n < 50$	$50 \leq n < 100$	$100 \leq n < 150$	$150 \leq n < 200$	$200 \leq n < 250$	$250 \leq n < 300$
Frequency	2	7	10	5	3	1
Cumulative Frequency						
Mid-Interval						
Frequency × Mid-Interval						

a) By completing the table, estimate the mean number of passengers.
b) By plotting a cumulative frequency curve, determine the median value.
c) What is the modal group?

With cumulative frequency you always plot the highest value from each class.


A mean passenger

Q3 40 pupils have taken an exam and their marks are recorded in a frequency table.

Mark (%)	$0 \leqslant m < 20$	$20 \leqslant m < 40$	$40 \leqslant m < 60$	$60 \leqslant m < 80$	$80 \leqslant m < 100$
Frequency	2	12	18	5	3
Cumulative Frequency					

a) Complete the table and plot the cumulative frequency curve.
b) What is the value of the lower quartile?
c) What is the interquartile range?
d) What is the median mark?

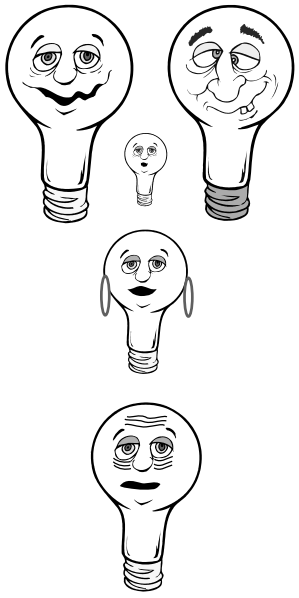
Cumulative Frequency

Q4 One hundred scores for a board game are presented in the table below.

Score	$31 \leq s < 41$	$41 \leq s < 51$	$51 \leq s < 61$	$61 \leq s < 71$	$71 \leq s < 81$	$81 \leq s < 91$	$91 \leq s < 101$
Frequency	4	12	21	32	19	8	4
Cumulative Frequency							

a) What is the modal group?
b) Which group contains the median score?
c) By plotting the cumulative frequency curve determine the actual value of the median score.
d) Find the interquartile range.

Q5 The following frequency table gives the distribution of the lives of electric bulbs.



a) Complete the frequency table.

Life (hours)	Frequency	Cumulative Frequency
$900 \leq L < 1000$	10	
$1000 \leq L < 1100$	12	
$1100 \leq L < 1200$	15	
$1200 \leq L < 1300$	18	
$1300 \leq L < 1400$	22	
$1400 \leq L < 1500$	17	
$1500 \leq L < 1600$	14	
$1600 \leq L < 1700$	9	

b) Which group contains the median value?
c) By drawing the cumulative frequency curve, find the actual value of the median.
d) Determine values for the upper and lower quartiles.

Q6 30 pupils recorded the time taken (minutes : seconds) to boil some water.
Here are their results:

2:37 2:37 3:17 3:30 2:45 2:13 3:18 3:12 3:38 3:29
3:04 3:24 4:13 3:01 3:11 2:33 3:37 4:24 3:59 3:11
3:22 3:13 2:57 3:12 3:07 4:17 3:31 3:42 3:51 3:24

a) By using a tally, transfer the data into the frequency table.
b) Draw the cumulative frequency curve.

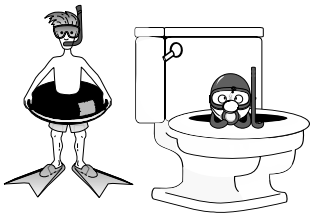
Time	$2{:}00 \leqslant t < 2{:}30$	$2{:}30 \leqslant t < 3{:}00$	$3{:}00 \leqslant t < 3{:}30$	$3{:}30 \leqslant t < 4{:}00$	$4{:}00 \leqslant t < 4{:}30$
Tally					
Frequency					
Cumulative Frequency					

c) Using your graph, read off the median and the upper and lower quartiles.
d) What is the interquartile range?

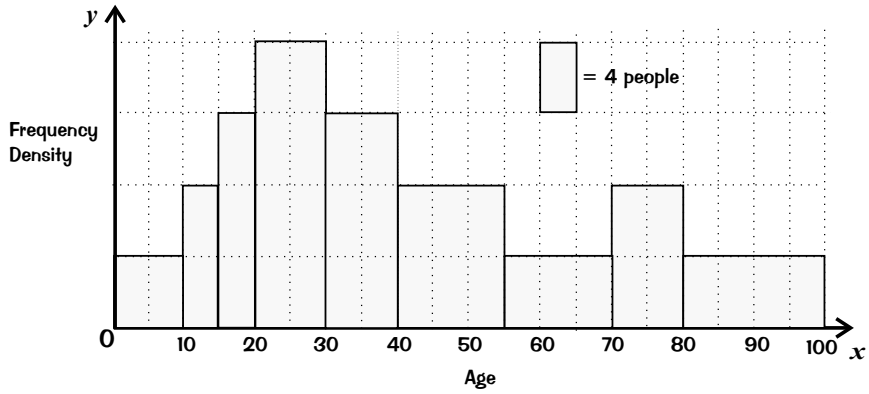
Histograms and Frequency Density

It's the <u>size that counts</u>... You've got to look at the <u>area</u> of the bars to find the frequency. That means looking at the <u>width</u> as well as the height.

Q1 The Bog Snorkelling Appreciation Society conducts a survey on the ages of all their members. The histogram below shows the age distribution of the people surveyed. The Society organises a 'Seniors' bog snorkelling event for members aged 60 or older. Use the graph to estimate the maximum number of people that might take part.



Use the key to work out the frequencies.



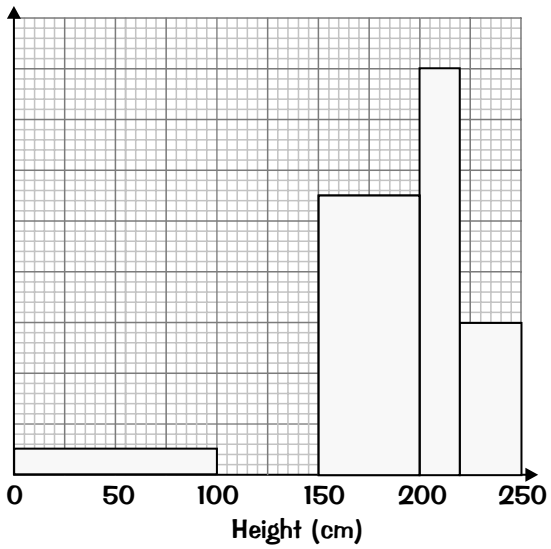
Q2 The table below shows the results of a maths test for some Year 11 students. Draw a histogram to represent this information.

Score (%)	$0 < x \leq 40$	$40 < x \leq 60$	$60 < x \leq 70$	$70 < x \leq 80$	$80 < x \leq 100$
Frequency	8	14	36	32	10

Q3 A farmer measures the heights of the sunflowers growing in his field. His results are shown in the table and the histogram below.

a) Use the histogram to complete the table.
b) Use the table to add the missing bar to the histogram.

Height (cm)	Frequency
$0 < x \leq 100$	50
$100 < x \leq 150$	150
$150 < x \leq 200$	
$200 < x \leq 220$	160
$220 < x \leq 250$	90



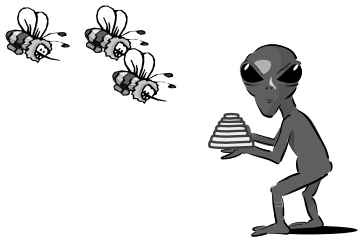
Histograms and Frequency Density

Q4 The weight of honey collected from several beehives is tabulated below.

a) Complete the frequency table by calculating the frequency densities.

b) Draw a histogram to represent this data.

c) Use your histogram to estimate the number of beehives that produced more than 6 kg of honey.



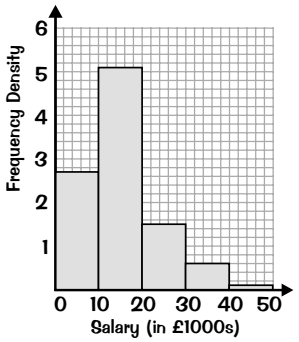
Weight (kg)	$0 \leq w < 2$	$2 \leq w < 4$	$4 \leq w < 7$	$7 \leq w < 9$	$9 \leq w < 15$
Frequency	3	2	6	9	12
Frequency density					

Q5 A local newspaper employee has collected data on the salaries of 100 people living in the area. His data is shown in the table below.

Salary (£1000s)	$0 \leq s < 10$	$10 \leq s < 20$	$20 \leq s < 30$	$30 \leq s < 40$	$40 \leq s < 50$
Frequency	10	25	42	20	3
Frequency Density					

a) Complete the table and draw a histogram to show the data.

b) The newspaper prints this histogram alongside the one shown on the right. It represents data from an identical survey done 10 years earlier. Write a comment comparing current salaries and those from 10 years ago.



Q6 A farmer keeps track of the amount of milk produced by his cows each day.

Amount of Milk (Litres)	Frequency	Frequency Density	Mid-Interval	Frequency × Mid-Interval
$0 \leqslant C < 1$	6			
$1 \leqslant C < 5$	6			
$5 \leqslant C < 8$	6			
$8 \leqslant C < 10$	6			
$10 \leqslant C < 15$	6			
$15 \leqslant C < 20$	6			

a) Complete the frequency table.

b) Use the mid-interval technique to estimate the mean.

c) Draw a histogram to show the data.

d) On how many days is less than 8 litres produced?

Other Graphs and Charts

Everyone loves a pie chart. Oh, no, sorry, that's pies...

When constructing a pie chart, follow the three steps:

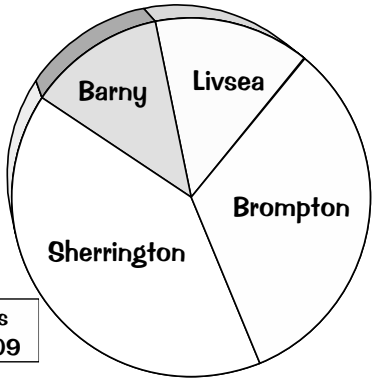
1) Add up the numbers in each sector to get the TOTAL.
2) Divide 360° by the TOTAL to get the MULTIPLIER.
3) Multiply EVERY number by the MULTIPLIER to get the ANGLE of each SECTOR.

Q1 A pie company wants to add a nutritional information diagram to their packaging. Construct a pie chart to show the following nutritional data for one of their pies:

Contents of Pie	Amount per 100 g
Carbohydrate	35 g
Protein	15 g
Fat	10 g
Magical fairy dust	40 g

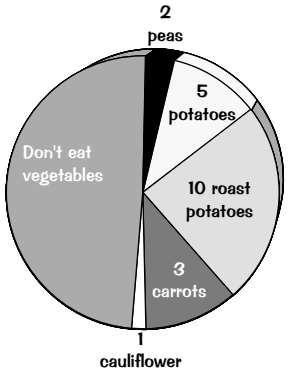
Q2 According to the tourist board for the Hindle Isles, 380,000 people visited the biggest island in the group, Sherrington, in 2009. The distribution of tourists for the whole group of islands is shown in the pie chart. Use a protractor on the diagram to find the number of tourists visiting the other islands in 2009 (rounded to the nearest 10,000).

Use the info you're given to find the number of tourists represented by 1°.



The distribution of visitors to the Hindle Isles in 2009

Q3 The pie chart shows the results of a survey of forty 11-year-olds when asked what their favourite vegetable is with Sunday lunch. Which one of the following may be deduced from the information in the pie chart?

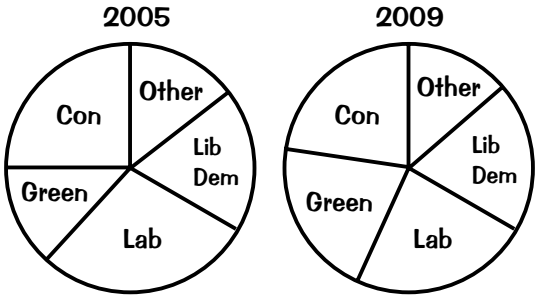


a) Potatoes are the least popular vegetable.
b) 3/4 of the children like potatoes of some type.
c) 1/10 of the children like carrots or cauliflower.
d) 11/40 of the children asked what their favourite vegetable is, replied "Don't eat vegetables."

Other Graphs and Charts

Q4 The pie charts opposite appear in a newspaper article about a local election. Nicki says that more people voted for the Green party in 2009 than in 2005.

Comment on whether it's possible to tell this from the pie charts.



Q5 This pictogram shows the favourite drinks of a group of pupils.

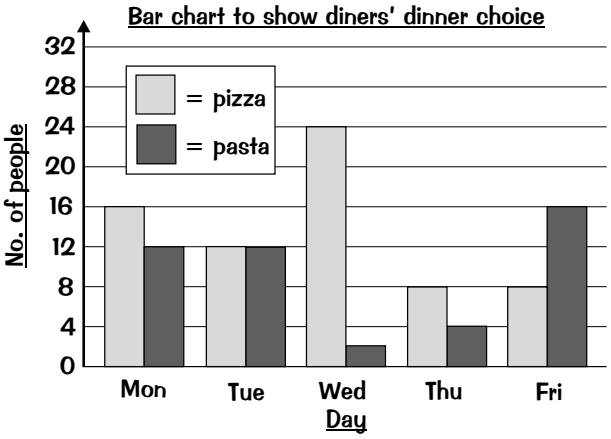
Favourite Drinks	Number of Pupils
Lemonade	✦ ✦ ✦ ✦ ✦ ✦ ✦ ✦ ✦
Cola	✦ ✦ ✦ ✦ ✦ ✦ ✦ ✦ ✦ ✦ ✦
Cherryade	✦ ✦ ✦ ✦ ✦ ✦
Orange Squash	✦ ✦ ✦
Milk	✦

✦ Represents 2 pupils

a) How many pupils were questioned?
b) How many pupils prefer either milk or orange squash?
c) 18 pupils liked lemonade best. How many more liked cola best?
d) Comment on the popularity of cola compared with milk.

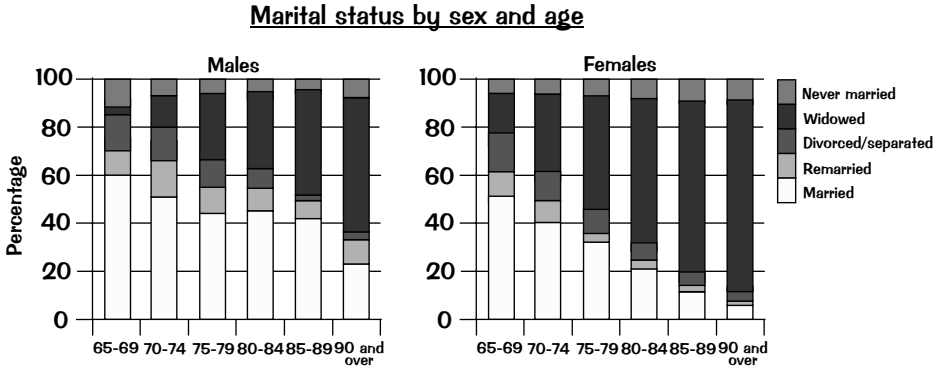
Q6 Megan records whether people choose pizza or pasta in her Italian restaurant. The dual bar chart below shows the results over one week.

a) On which days was pizza more popular with Megan's customers?
b) On which day did she have the most customers?



Other Graphs and Charts

Q7 The graphs below show statistics on marital status for a group of people aged over 65 years old.



a) What proportion of males aged 65-69 are married?

b) What proportion of females aged 70-74 have remarried?

Q8 One hundred vehicles on a road were recorded as part of a traffic study. Copy and complete this two-way table, and use it to answer these questions.

	Van	Motor-bike	Car	Total
Travelling North	15			48
Travelling South	20		23	
Total		21		100

a) How many vans were recorded?

b) How many vehicles in the survey were travelling south?

c) How many motorbikes were travelling south?

d) How many cars were travelling north?

Q9 Scott is collecting data about insects in his local park. For each insect he catches, he records in a table whether or not it has wings, and whether or not it has antennae. His results are summarised below:

- He caught 10 insects with antennae.
- 9 of the insects he caught had neither wings nor antennae.
- Of the 12 insects that had wings, half had antennae as well.

a) Draw and complete a two-way table with this information.

b) How many insects had neither wings nor antennae?

c) How many insects did he catch in total?

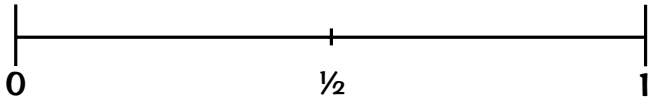
Probability

Probability can be a bit of a struggle — here's a quick reminder of the basics...

PROBABILITIES are always between 0 and 1

1) You should express probabilities as a fraction, decimal or percentage.
2) A probability of ZERO means that it will definitely not happen.
3) A probability of ONE means it will definitely happen.

Q1 The number line opposite is a probability scale. Place the letters where you think the following statements lie, in terms of the chance of the event happening.



a) The probability of getting a head on a toss of a 10p piece.
b) The probability of choosing a red ball from a bag containing 2 red balls and 1 green ball.
c) The probability of shaking a five on an ordinary dice.
d) The probability of choosing a Guatemalan stamp from a bag containing 60 British stamps and 40 French stamps.

Q2 Debbie's employer organises a weekly prize draw, where the winning employee is selected at random. Debbie only joins in if her chance of winning is at least 0.1. If there are 8 other people playing this week, will Debbie choose to play?

SHORTHAND NOTATION

1) P(x) = 0.25 simply means "the probability of event x happening is 0.25".
2) E.g. if you roll a dice, the probability of rolling a 6 will be written as P(rolls a 6).

Q3 After 49 tosses of an unbiased coin, 24 have been heads and 25 have been tails. What is P(50th toss will be a head)?

Q4 If the probability of picking a banana from a fruit bowl is 0.27, what is the probability of picking something which is not a banana?

Q5 A bag contains 3 red balls, 4 blue balls and 5 green balls. A ball is chosen at random from the bag. What is the probability that:

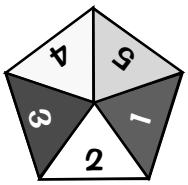
a) it is green
b) it is blue
c) it is red
d) it is not red?

Q6 Students at school conduct a survey of the colours of parents' cars, where every parent owns one car. The table shows the results.

Red	Blue	Yellow	White	Green	Other
40	29	13	20	16	14

a) What is the probability of a parent owning a red car?
b) What is the probability of a parent owning a car that is not blue or green?

Q7 Draw a sample space diagram to show all the possible outcomes of throwing a standard dice and spinning this spinner:



Probability

Q8 Charlton is making a bet with his friend before the local cricket team play a match. He thinks the match will end in a draw. A local newspaper prints the team's results over their last 20 matches, as shown.

W	W	L	D	D	W	W	L	W	L
D	L	L	D	W	D	W	W	L	L

Outcome	Frequency
W	
D	
L	

a) Complete the frequency table.

b) Charlton reasons that since there are 3 possible results for any match, the probability that the next match will be drawn (D) is $\frac{1}{3}$. Explain why Charlton is wrong.

c) Suggest a value for the probability of a draw based on the team's past performance.

d) Based on their past performance, are the team most likely to win, lose, or draw?

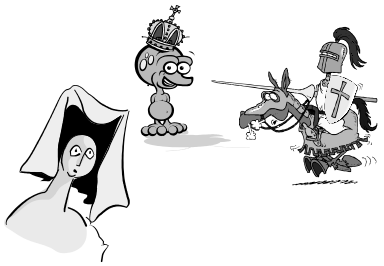
e) Based on these results, estimate how many of the next 40 matches you'd expect the team to lose.

Q9 **a)** What is the probability of randomly selecting either a black Ace or black King from an ordinary pack of playing cards?

Remember the OR rule — P(A or B) = P(A) + P(B).

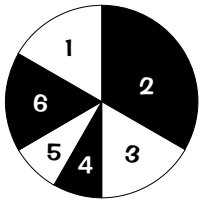
b) If the entire suit of clubs is removed from a pack of cards, what is the probability of randomly selecting a red 7 from the remaining cards?

c) If all the 7s are also removed from the pack of cards, what is the probability of randomly selecting the 4 of diamonds?



Q10 For the roulette wheel shown, the probability of the ball landing on each of the numbers is listed in the table below.

Number	1	2	3	4	5	6
Probability	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{6}$



a) Find the probability of landing on an even number.

b) What is the probability of landing on black?

c) Why is the probability of landing on a white or a 3 not $\frac{5}{12} + \frac{1}{6}$?

Q11 The notepad below shows orders for 4 different sorts of rice at a certain Indian restaurant. Based on this data, what is the probability that the next order of rice is:

a) for pilau rice?

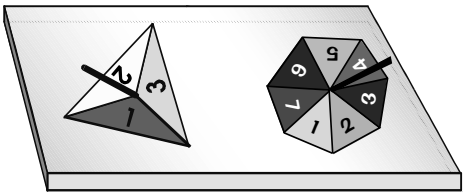
b) for spicy mushroom or special fried rice?

c) not for boiled rice?

boiled	20
pilau	24
spicy mushroom	10
special fried	6

If you're asked to work out probabilities based on some data, it's a relative frequency question.

Probability

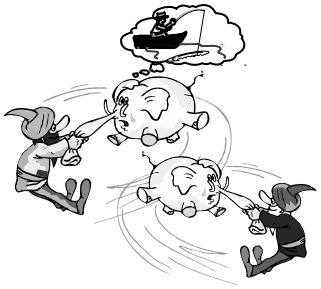


Q12 There are 2 spinners: one with 3 sides numbered 1, 2, 3, and the other with 7 sides numbered 1, 2, 3, 4, 5, 6, 7.

a) If both are spun together, list all the possible outcomes.

b) Complete the following table showing the sum of the 2 numbers for each outcome.

	1	2	3	4	5	6	7
1							
2							
3							



c) What is the probability that the sum is 6?

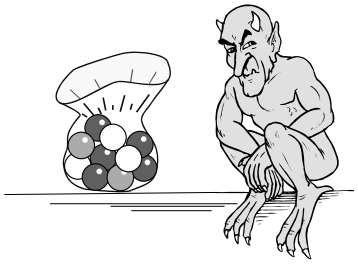
d) What is the probability that the sum is even?

e) What is the probability that the sum is greater than or equal to 8?

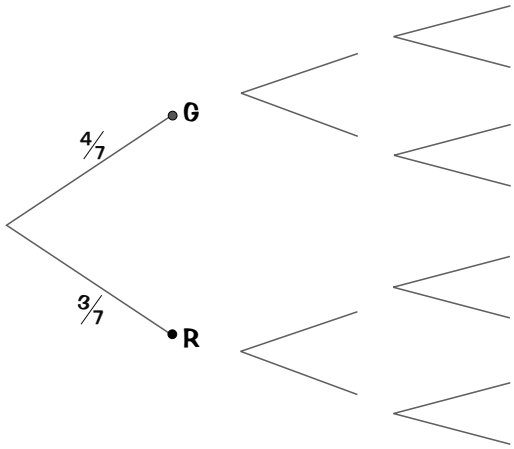
f) What is the probability that the sum is less than 8?

g) Explain how you can work out the probability in part **f)** without using the table.

Q13 3 balls are drawn at random, without replacement, from a bag containing 4 green balls and 3 red balls.



a) Complete the tree diagram below showing all the possible outcomes and their probabilities.



For AND you MULTIPLY along the branches.
For OR you ADD the end results.

b) What is the probability that exactly 2 green balls are drawn?

c) What is the probability that the last ball drawn is the same colour as the first?

Q14 a) A biased dice is rolled 40 times. A six came up 14 times. Calculate the relative frequency that a six was rolled.

b) The same dice is rolled another 60 times. From this, a six came up 24 times. Calculate the relative frequency that a six was rolled.

c) Use the data from **a)** and **b)** to make the best estimate you can of the probability of rolling a six with the dice.

Probability

Q15 How many times must you roll an ordinary 6-sided dice for the probability of getting at least one 6 to be more than 0.5?

Don't forget the "at least" trick — P(at least 1 six) = 1 – P(no sixes).

Q16 An unbiased dice in the shape of a tetrahedron has vertices numbered 1, 2, 3, 4. To win a game with this dice, you must throw a 4. At each go you have a maximum of 3 attempts.

a) Using a tree diagram, calculate the probability of winning with the second throw of the first go.

b) What is the probability of winning on the first go?

Q17 3 coins are drawn at random, without replacement, from a piggy bank containing 7 pound coins and 4 twenty-pence pieces.

a) Draw a tree diagram showing all possible outcomes and their probabilities.

b) Find the probability that the first coin selected is different in value from the third.

c) Find the probability that less than £1.50 is drawn altogether.

Q18 Fabrizio is practising taking penalties. The probability that he misses the goal completely is $\frac{1}{8}$. The probability that the goalkeeper saves the penalty is $\frac{3}{8}$. The probability that he scores is $\frac{1}{2}$. Fabrizio takes two penalties.

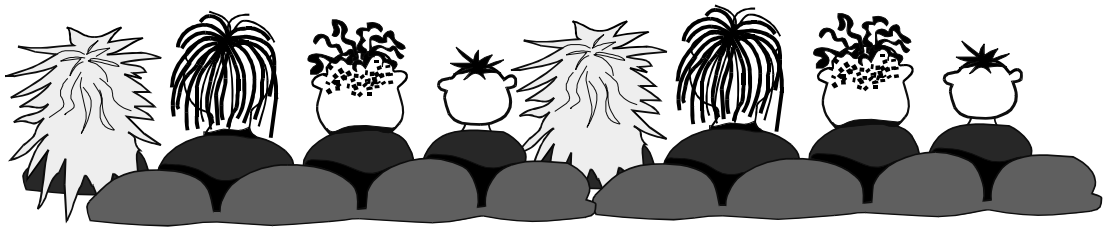
a) Calculate the probability that Fabrizio fails to score with his two penalties.

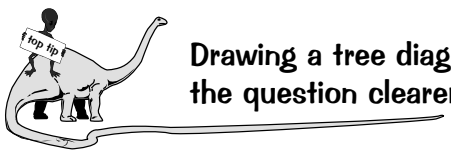
b) Calculate the probability that he scores only one goal.

c) Calculate the probability that Fabrizio scores on neither or both of his 2 attempts.

Q19 Trevor and his 2 brothers and 5 friends are seated at random in a row of 8 seats at the cinema. What is the probability that Trevor has one brother on his immediate left and one on his immediate right?

Careful here — you have to include the probability that Trevor sits in one of the six middle seats.



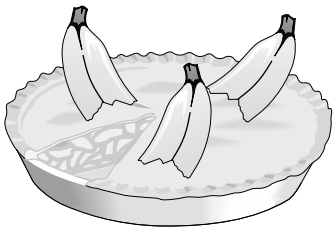


Drawing a tree diagram might be a bit of a faff, but it can really help to make the question clearer. So if you're stuck, give the old tree diagram a try.

Probability

Q20 A survey asked people if they like bananas and pies.

- 12 people said they only like bananas
- 11 people said they only like pies
- 7 people said they like both
- 3 people said they like neither



a) Draw a Venn diagram to show this data.
b) A person from the survey is chosen at random.
What is the probability that the chosen person:
i) likes bananas?
ii) doesn't like pies?

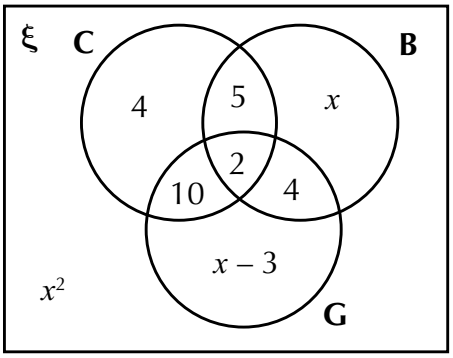
Q21 All 100 students in a year group sat a test in English and a test in Science.
Everyone passed at least one of the tests.
82 of the students passed the English test and 57 passed the Science test.

a) Show this information on a Venn diagram.
b) One student is chosen at random from the year group.
Given that this student had passed the Science test,
find the probability that she had also passed the English test.

Q22 One year, 121 films were shown at a film club.
The films were divided into three overlapping categories:

- comedies (C)
- films based on books (B)
- films that received good reviews when they were originally released (G)

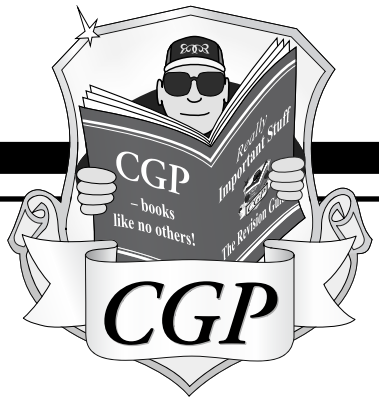
The number of films in each of the categories is shown in the Venn diagram.



a) Find the value of x.
b) A film is selected at random. Find the probability that it received good reviews.
c) Given that a randomly chosen film is a comedy and is based on a book, find the probability that it received good reviews when it was originally released.

MEW144

CGP



Edexcel International GCSE

Mathematics

For the Grade 9-1 Course

Answers

Contents

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Answers: P1 — P7

Section One — Numbers

Page 1 — Order of Operations

Q1 **a)** 8 **b)** 5 **c)** 6.56 **d)** 11.22 **e)** –0.90 **f)** 319.98 **g)** 5.5 **h)** 983 **i)** 9.17 **j)** 0

Q2 **a)** 8 **b)** 73 **c)** 113 **d)** 7 **e)** 22.57 **f)** 8.67 **g)** 1 **h)** 1.42 **i)** –488.76 **j)** –0.26

Q3 **a)** 3 **b)** 0.1 **c)** 16 **d)** 8.33 **e)** –0.01 **f)** 70.88 **g)** –176.95 **h)** 0.21 **i)** 0.58 **j)** 0.27 **k)** 0.01 **l)** –10.64

Page 2 — Types of Number

Q1 4

Q2 –3 °C

Q3 **a)** $6 \div 2 = 3$, rational
b) $\sqrt{16} = 4$, rational
c) $\sqrt{5} = 2.23606...$, irrational
d) $3 \div 8 = 0.375$, rational
e) $\sqrt[3]{25} = 2.92401...$, irrational
f) Rational

Q4 **a)** the third cube number (27)
b) the fourth square number (16)

Q5 **a)** 2
b) e.g. 29
c) 19
d) 19 and 2
e) e.g. 1 or 25

Q6 **a)**

1	(2)	(3)	4	(5)	6	(7)	8	9	10
(11)	12	(13)	14	15	16	(17)	18	(19)	20
21	22	(23)	24	25	26	27	28	(29)	30
(31)	32	33	34	35	36	(37)	38	39	40
(41)	42	(43)	44	45	46	(47)	48	49	50
51	52	(53)	54	55	56	57	58	(59)	60
(61)	62	63	64	65	66	(67)	68	69	70
(71)	72	(73)	74	75	76	77	78	(79)	80
81	82	(83)	84	85	86	87	88	(89)	90
91	92	93	94	95	96	(97)	98	99	100

b) 3 of: 11 (11), 13 (31), 17 (71), 37 (73), 79 (97)
c) e.g. 3 is a factor of 27

Q7 113

Q8 There's just one: 2 is the only even prime.

Page 3 — Square Roots and Cube Roots

Q1 **a)** 7.7 **b)** 4.4 **c)** 5.8 **d)** 14.1 **e)** 22.8 **f)** 8.7 **g)** 27.4 **h)** 0.9 **i)** 13.0 **j)** 85.0 **k)** 1000.0 **l)** 5.2

Q2 **a)** 2 and –2 **b)** 4 and –4 **c)** 3 and –3 **d)** 7 and –7 **e)** 5 and –5 **f)** 10 and –10 **g)** 12 and –12 **h)** 8 and –8 **i)** 9 and –9

Q3 **a)** 16 **b)** 12 **c)** 11 **d)** 100 **e)** 1 **f)** 0.5

Q4 **a)** 4 **b)** 8 **c)** 5 **d)** 10 **e)** 6 **f)** 20

Q5 7 cm

Q6 240 m

Q7 4

Pages 4-5 — Multiples, Factors and Prime Factors

Q1 **a)** 12
b) 3
c) 1, 9
d) 1, 3, 9
e) P = 12, Q = 6

Q2 Any 5 of:
2 groups of 24, 3 groups of 16,
4 groups of 12, 6 groups of 8,
8 groups of 6, 12 groups of 4,
16 groups of 3, 24 groups of 2.

Q3 The Conversational French and Woodturning classes both have a prime number of pupils and so cannot be divided into equal groups.

Q4 **a)** 1, 8, 27, 64, 125
b) 8, 64
c) 27
d) 8, 64
e) 125

Q5 **a)** 2×3^2
b) $2^2 \times 5 \times 7$
c) 47

Q6 **a)** 2, 3, 5, 7, 11
b) 28
c) $2^2 \times 7$

Q7 **a)** 1, 3, 5, 7, 9
b) 25
c) 5^2

Q8 **a)** 495
b) $3 \times 5 \times 11$

Q9 **a)** 1, 4, 9, 16, 25, 36, 49, 64, 81, 100
b) 4, 16, 36, 64, 100
c) 9, 36, 81
d) 1, 64
e) Total = $385 = 5 \times 7 \times 11$

Q10 **a)** $50 \times 25 \times 16 = 20{,}000\ cm^3$
b) $2^5 \times 5^4$
c) 200. It is not enough to divide the large volume by the smaller volume as the shapes of the blocks are important too. It is possible to fit $16 \div 4 = 4$ small blocks across the width, $50 \div 5 = 10$ small blocks along the length and $25 \div 5 = 5$ small blocks down the height of the large block. This enables Gordon to fit $4 \times 10 \times 5 = 200$ small blocks into the big block.

Q11 **a)** 680
b) $2^2 \times 5 \times 17$
c) $2 \times 5 \times 17$
d) 5×17

Q12 42

Page 6 — LCM and HCF

Q1 **a)** 6, 12, 18, 24, 30, 36, 42, 48, 54, 60
b) 5, 10, 15, 20, 25, 30, 35, 40, 45, 50
c) 30

Q2 **a)** 1, 2, 3, 5, 6, 10, 15, 30
b) 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
c) 6

Q3 **a)** 20 **b)** 10 **c)** 2 **d)** 15 **e)** 15 **f)** 5 **g)** 32 **h)** 16 **i)** 16

Q4 **a)** 120 **b)** 120 **c)** 120 **d)** 45 **e)** 90 **f)** 180 **g)** 64 **h)** 192 **i)** 192

Q5 **a)** $15 = 3 \times 5$
$18 = 2 \times 3^2$
b) $2 \times 3 \times 3 \times 5 = 90$

Q6 **a)** $90 = 2 \times 3^2 \times 5$
$120 = 2^3 \times 3 \times 5$
b) $2 \times 3 \times 5 = 30$

Q7 **a)** $2^4 \times 3^2 = 144$
b) $2^2 \times 3^2 \times 5 \times 7 = 1260$

Q8 **a)** $2 \times 3 \times 5 = 30$
b) $2^2 \times 5 = 20$

Q9 **a)** 7th June (i.e. 6 days later, since 6 is the LCM of 2 and 3)
b) 16th June (i.e. 15 days later, since 15 is the LCM of 3 and 5)
c) Sunday (30 days later, since 30 is the LCM of 2, 3 and 5 — i.e. 4 weeks and 2 days later)
d) Lars (it's 14 days after 1st June, and 14 is a multiple of 2 but not 3 and 5)

Q10 HCF of 36, 42 and 84 is 6.
$36 \div 6 = 6$, $42 \div 6 = 7$, $84 \div 6 = 14$
$6 + 7 + 14 = 27$ friends

Pages 7-9 — Fractions

Q1 **a)** $\frac{1}{64}$ **b)** $\frac{1}{9}$ **c)** $\frac{1}{18}$ **d)** $3\frac{29}{32}$ **e)** $5\frac{5}{32}$ **f)** $\frac{81}{100\,000}$

Q2 **a)** 1 **b)** 4 **c)** $\frac{1}{2}$ **d)** $\frac{2}{5}$ **e)** $\frac{10}{33}$ **f)** 1000

Q3 **a)** $\frac{1}{4}$ **b)** $\frac{5}{6}$ **c)** $\frac{1}{2}$ **d)** $4\frac{3}{8}$ **e)** $5\frac{3}{8}$ **f)** 1

Q4 $3\frac{7}{15}$, so the bowl will be big enough.

Q5 **a)** 0 **b)** $\frac{1}{2}$ **c)** $-\frac{1}{6}$ **d)** $1\frac{7}{8}$ **e)** $-3\frac{1}{8}$ **f)** $\frac{4}{5}$

Answers: P7 — P18

Q6 **a)** $\frac{3}{4}$ **b)** $\frac{5}{12}$ **c)** $\frac{7}{15}$
d) $4\frac{3}{4}$ **e)** 4 **f)** $1\frac{1}{5}$
g) $\frac{5}{8}$ **h)** $-\frac{1}{24}$ **i)** $4\frac{3}{5}$
j) $1\frac{1}{30}$ **k)** 1 **l)** $\frac{44}{75}$

Q7 **a)** 1/12 **b)** 1/4 **c)** 2/3

Q8 **a)** 3/4 of the programme
b) 5/8 of the programme
c) 1/8 of the programme

Q9 3/5 of the kitchen staff are girls.
2/5 of the employees are boys.

Q10 7/30 of those asked had no opinion.

Q11 **a)** 12/30 = 2/5
b) 6 days

Q12 **a)** Each box will hold 16 sandwiches. So 5 boxes will be needed for 80 sandwiches.
b) 25 inches tall

Q13 **a)** $\frac{1}{18}$ **b)** $\frac{1}{4}$

Q14 **a)** 48 km^2 **b)** $\frac{5}{8}$

Q15 **a)** 8 people **b)** $\frac{7}{20}$
c) $\frac{1}{4}$ **d)** 57 people
e) 65 people

Q16 After the 1st bounce the ball reaches 4 m, after the 2nd $2\frac{2}{3}$ m, after the 3rd $1\frac{7}{9}$ m.

Q17 **a)** 100 g flour **b)** 350 g
c) $\frac{2}{7}$ **d)** 300 g

Q18 £31.06

Pages 10-11 — Fractions, Decimals and Percentages

Q1 **a)** 25% **e)** 41.52%
b) 50% **f)** 84.06%
c) 75% **g)** 39.62%
d) 10% **h)** 28.28%

Q2 **a)** 0.5 **e)** 0.602
b) 0.12 **f)** 0.549
c) 0.4 **g)** 0.431
d) 0.34 **h)** 0.788

Q3 **a)** 50% **e)** 4%
b) 25% **f)** 66.7%
c) 12.5% **g)** 26.7%
d) 75% **h)** 28.6%

Q4 **a)** 1/4 **e)** 41/500
b) 3/5 **f)** 62/125
c) 9/20 **g)** 443/500
d) 3/10 **h)** 81/250

Q5 85%

Q6 Grade 6

Q7 **a)** 0.3 **e)** 1.75
b) 0.37 **f)** 0.125
c) 0.4 **g)** 0.6
d) 0.375 **h)** 0.05

Q8

0.5	0.2	0.125	1.6	0.25	3.5	0.15	0.45
1/2	1/5	1/8	8/5	1/4	7/2	3/20	9/20

Q9 **a)** $0.8\dot{3}$ **e)** $0.\dot{9}\dot{0}$
b) $0.\dot{7}$ **f)** $0.\dot{4}60317\dot{}$
c) $0.\dot{6}\dot{3}$ **g)** $0.\dot{4}7\dot{8}$
d) $0.\dot{4}\dot{7}$ **h)** $0.\dot{5}89\dot{1}$

Q10 **a)** $\frac{3}{5}$ **e)** $\frac{1}{3}$
b) $\frac{3}{4}$ **f)** $\frac{2}{3}$
c) $\frac{19}{20}$ **g)** $\frac{1}{9}$
d) $\frac{16}{125}$ **h)** $\frac{16}{99}$

Q11 **a)** $\frac{2}{9}$ **e)** $\frac{4}{33}$
b) $\frac{4}{9}$ **f)** $\frac{545}{999}$
c) $\frac{8}{9}$ **g)** $\frac{251}{333}$
d) $\frac{80}{99}$ **h)** $\frac{52}{333}$

Pages 12-14 — Percentages

Q1 **a)** £1.28 **b)** 629 kg
c) 16 mins

Q2 **a)** 0.2 **c)** 0.02
b) 0.35 **d)** 0.625

Q3 **a)** $\frac{1}{5}$ **c)** $\frac{7}{10}$
b) $\frac{3}{100}$ **d)** $\frac{421}{500}$

Q4 **a)** 12.5% **c)** 30%
b) 23% **d)** 34%

Q5 85%

Q6 72.5%

Q7 **a)** £4275 **b)** £6840

Q8 1.6%

Q9 500%

Q10 £358.80

Q11 £244.40

Q12 23 028

Q13 Car 1 costs £8495 – (0.15 × £8495) = £8495 – £1274.25 = £7220.75.
Car 2 costs £8195 – (0.12 × £8195) = £8195 – £983.40 = £7211.60.
So car 2 is the cheapest.

Q14 £5980

Q15 £152.75, So NO, he couldn't afford it.

Q16 31%

Q17 13%

Q18 **a)** 67.7% **b)** 93.5%
c) 38.1%

Q19 £80

Q20 **a)** 300 **b)** 4 whole years

Q21 £236.25

Q22 38%

Q23 Final cost of stereo
$= x \times (1 + 0.35) \times (1 - 0.2) = x \times 1.08$
So, the shop's overall profit is 8%.

Q24 House value now
$= y \times (1 + 0.1) \times (1 - 0.05)$
$= y \times 1.045$
So, if they sell now they will make a profit of 4.5%.

Page 15 — Interest and Depreciation

Q1 **a)** £473.47 **c)** £779.42
b) £612.52 **d)** £1065

Q2 Splitting the investment. £2.21 better.

Q3 **a)** £7877.94 **d)** £10 646.54
b) £27 116.06 **e)** £7184.25
c) £9980.90 **f)** £5843.70

Q4 **a)** 4% compound interest gives £1040
5% simple interest gives £1050
£5 a month gives £1060
£5 a month account pays more.
b) 4% compound interest gives £4440.73
5% simple interest gives £4500
£5 a month gives £3600
5% simple interest pays more.
c) 4% compound interest gives £5864.84
5% simple interest gives £4950
£5 a month gives £3700
4% compound interest pays more.

Q5 **a)** £270 **d)** £8012
b) £790 **e)** £5100
c) £1130

Pages 16-17 — Ratios

Q1 **a)** 3:4 **d)** 9:16
b) 1:4 **e)** 7:2
c) 1:2 **f)** 9:1

Q2 **a)** 6 cm **d)** 1.5 cm
b) 11 cm **e)** 2.75 cm
c) 30.4 m **f)** 7.6 m

Q3 **a)** £8, £12
b) 80 m, 70 m
c) 100 g, 200 g, 200 g.
d) 1hr 20 m, 2 hr 40 m, 4 hrs.

Q4 John 4, Peter 12

Q5 400 ml, 600 ml, 1000 ml

Q6 30

Q7 Jane £40, Holly £48, Rosemary £12

Q8 £12

Q9 **a)** 245 girls **b)** 210 boys

Q10 **a)** £39 **b)** £140

Q11 **a)** 1:300 **b)** 6 m
c) 3.3 cm

Q12 **a)** 15 kg **b)** 30 kg
c) 8 kg cement, 24 kg sand and 48 kg gravel.

Q13 **a)** 30 fine **b)** 15 not fine
c) 30/45 = 2/3

Q14 **a)** 45 Salt & Vinegar
b) 90 bags sold altogether

Page 18 — Proportion

Q1 85

Q2 £247.80

Q3 112 hrs

Q4 £96.10

Q5 96 sheep

Q6 **a)** 9.33 cm **b)** 30.45 km

Q7 **a)** 400 g
b) 300 g

Answers: P18 — P26

c) She will need 350 g of butter so she doesn't have enough.
d) 4

Q8 44 cows

Q9 **a)** 55.3 cm **c)** 20.4 °C
b) 51.5 cm **d)** 19.5 °C

Pages 19-20 — Rounding Numbers

Q1 **a)** 62.2 **b)** 62.19
c) 62.194 **d)** 19.62433
e) 6.300 **f)** 3.142

Q2 **a)** 1330 **b)** 1330
c) 1329.6 **d)** 100
e) 0.02 **f)** 0.02469

Q3 **a)** 457.0 **b)** 456.99
c) 456.987 **d)** 457
e) 460 **f)** 500

Q4 2.83

Q5 **a)** 0.704 (to 3 s.f. — the least number of significant figures used in the question).
b) 3.25 (to 3 s.f. — the least number of significant figures used in the question).

Q6 **a)** £1100 **d)** £3
b) £88 **e)** £376
c) £300 **f)** £44

Q7 23 kg

Q8 £5.07

Q9 235 km

Q10 £19

Q11 £4.77

Q12 235 cm

Q13 470 cm

Q14 1810 g

Q15 13 s

Page 21 — Estimating

Q1 Mark's tank is approximately 4500 cm^3, so it won't be big enough.

Q2 **a)** $6500 \times 2 = 13\,000$
b) $8000 \times 1.5 = 12\,000$
c) $40 \times 1.5 \times 5 = 300$
d) $45 \div 9 = 5$
e) $35\,000 \div 7000 = 5$
f) $\frac{55 \times 20}{10} = 55 \times 2 = 110$
g) $7000 \times 2 = 14\,000$
h) $100 \times 2.5 \times 2 = 500$
i) $20 \times 20 \times 20 = 8000$
j) $8000 \div 80 = 100$
k) $62\,000 \div 1000 = 62$
l) $3 \div 3 = 1$

Q3 Approximately $15\,000 - (1500 + 2500 + 1500 + 1500 + 3000) = 5000$

Q4 **a)** $\frac{150 + 50}{150 - 50} = \frac{200}{100} = 2$
b) $\frac{20 \times 10}{\sqrt{400}} = \frac{200}{20} = 10$
c) $\frac{2000 \times 4}{20 \times 5} = \frac{8000}{100} = 80$
d) $\frac{10^2 \div 10}{4 \times 5} = \frac{10}{20} = 0.5$

Q5 **a)** $2 \times (3 \times 3) + 2 \times (2 \times 3.5) = 36\ m^2$
b) 3 tins

Q6 **a)** 6.9 (accept 6.8)
b) 10.9 (accept 10.8)
c) 9.2 (accept 9.1)
d) 4.1 (accept 4.2)
e) 9.9 (accept 9.8)
f) 5.8 (accept 5.9)

Pages 22-23 — Bounds

Q1 **a)** 64.785 kg **b)** 64.775 kg

Q2 **a)** 1.75 m
b) $1.85 \times 0.75 = 1.3875\ m^2$

Q3 **a)** 2.525 l **b)** 2.475 l

Q4 **a)** 95 g
b) Upper bound = 97.5 g, lower bound = 92.5 g
c) No, since the lower bound for the electronic scales is 97.5 g, which is greater than the upper bound for the scales in part **a)**.

Q5 **a)** Upper bound = 13.5, lower bound = 12.5
b) Upper bound = 12.55, lower bound = 12.45
c) To calculate the upper bound for C multiply the upper bound for A by the upper bound for B; $13.5 \times 12.55 = 169.425$
To calculate the lower bound for C multiply the lower bound for A by the lower bound for B; $12.5 \times 12.45 = 155.625$

Q6 **a)** Upper bound = 5 minutes 32.5 seconds, lower bound = 5 minutes 27.5 seconds.
b) The lower bound for Jimmy's time is 5 minutes 25 seconds, which is lower than the lower bound for Douglas' time (5 minutes 25.5 seconds).

Q7 **a)** Upper bound = 945, lower bound = 935.
b) Upper bound = 5.565, lower bound = 5.555.
c) To find the upper bound for R, divide the upper bound for S by the lower bound for T; $945 \div 5.555 = 170.117...$
To find the lower bound for R, divide the lower bound for S by the upper bound for T; $935 \div 5.565 = 168.014...$
d) $940 \div 5.56 = 170$ (to 2 s.f. — the upper and lower bounds both round to 170 to 2 s.f., but give different answers to 3 s.f.).

Q8 At least 18.2 m^2

Q9 The upper bound for the distance is 127.5 km. The lower bound for the time is 1 hour and 45 minutes = 1.75 hours. The maximum value of the average speed is $127.5 \div 1.75 = 72.857...$ km/hour.

Q10 **a)** Perimeter $= 2(12 + 4) = 32$ cm. Maximum possible error $= 4 \times 0.1$ cm $= 0.4$ cm.
b) Maximum possible error in P is $2(x + y)$.

Pages 24-25 — Standard Form

Q1 **a)** 35.6 **b)** 3560
c) 0.356 **d)** 35600
e) 8.2 **f)** 0.00082
g) 0.82 **h)** 0.0082
i) 1570 **j)** 0.157
k) 157000 **l)** 15.7

Q2 **a)** 2.56×10^0 **b)** 2.56×10
c) 2.56×10^{-1} **d)** 2.56×10^4
e) 9.52×10 **f)** 9.52×10^{-2}
g) 9.52×10^4 **h)** 9.52×10^{-4}
i) 4.2×10^3 **j)** 4.2×10^{-3}
k) 4.2×10 **l)** 4.2×10^2

Q3 **a)** 3.47×10^2 **b)** 7.3004×10
c) 5×10^0 **d)** 9.183×10^5
e) 1.5×10^7 **f)** 9.371×10^6
g) 7.5×10^{-5} **h)** 5×10^{-4}
i) 5.34×10^0 **j)** 6.2103×10^2
k) 1.49×10^4 **l)** 3×10^{-7}

Q4 6×10^{-3}

Q5 1×10^9, 1×10^{12}

Q6 9.46×10^{12}

Q7 6.9138×10^4

Q8 1.2×10^{-2} (mm)

Q9 **a)** Mercury
b) Jupiter
c) Mercury
d) Neptune
e) Venus and Mercury
f) Jupiter, Neptune and Saturn

Q10 **a)** 6×10^9 **e)** 5.6×10^{16}
b) 1.89×10^7 **f)** 3.99×10^4
c) 4×10^4 **g)** 4.3473×10^6
d) 2×10^2 **h)** 1.748×10^4

Q11 **a)** 2.4×10^{10}
b) 1.6×10^6
c) 1.8×10^5

Q12 1.04×10^{13} is greater by 5.78×10^{12}

Q13 1.3×10^{-9} is smaller by 3.07×10^{-8}

Q14 **a)** 4.2×10^7 **b)** 3.8×10^{-4}
c) 1.0×10^7 **d)** 1.12×10^{-4}
e) 8.43×10^5 **f)** 4.232×10^{-3}
g) 1.7×10^{18} **h)** 2.83×10^{-4}
i) 1×10^{-2}

Q15 7×10^6

Q16 6.38×10^8 cm

Q17 3.322×10^{-27} kg

Q18 **a)** 1.8922×10^{16} m
b) 4.7305×10^{15} m

Q19 **a)** 510000000 km^2
b) 3.62×10^8 km^2
c) 148000000 km^2

Pages 26-27 — Sets and Venn Diagrams

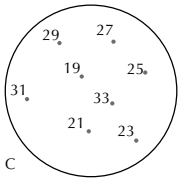
Q1 **a)** E = {prime numbers less than 12}
b) E = {2, 3, 5, 7, 11}

Answers: P26 — P32

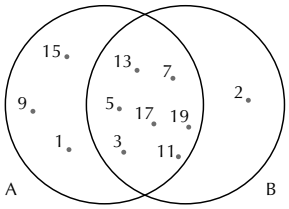
Q2 **a)** L **b)** E.g. $1 \in K$ and $1.1 \notin K$

Q3 **a)** B = {–3, 1, 7, 8, 9, 12, 21}

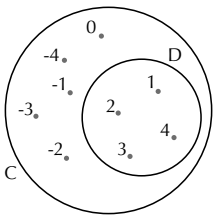
b)



Q4 **a)**



b)



Q5 **a)** 5
b) 2, 4, 7, 8
c) 8
d) 3, 5, 9

Q6 **a)** People who answered the survey
b) 9
c) $n(D \cup P) = 58$ and $n(D \cap P) = 3$

Q7 **a)** All the cows he counted
b) 19
c) $n(P) = 83$
d) $n(C \cup G) = 59$
e) $n(C') = 80$
f) $n(C \cap P) = 25$
g) $n(C \cap G \cap P) = 11$
h) $n(G \cap P \cap C') = 23$

Q8 **a)** false **g)** true
b) false **h)** false
c) true **i)** true
d) false **j)** false
e) false **k)** true
f) false **l)** false

Section Two — Algebra

Pages 28-30 — Powers and Roots

Q1 **a)** 16
b) 1000
c) $3 \times 3 \times 3 \times 3 \times 3 = 243$
d) $4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4096$
e) $1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 = 1$
f) $5 \times 5 \times 5 \times 5 \times 5 \times 5 = 15\,625$

Q2 **a)** 2^8 (or 256)
b) 12^5 (or 248 832)
c) x^5 **d)** m^3
e) y^4 **f)** z^6

Q3 **b)** 10^7 **c)** 10^6
d) 10^8
e) Simply add the powers.

Q4 **b)** 2^3 **c)** 4^2
d) 8^3
e) Simply subtract the powers.

Q5 **a)** true **b)** true
c) false **d)** false
e) true **f)** false
g) false **h)** true
i) false **j)** true
k) true **l)** false

Q6 **a)** 3^{-3} **d)** 3^{-12}
b) 4^{25} **e)** 4^6
c) 10^{-13} **f)** 5^3

Q7 **a)** 275 **b)** 0.123
c) 53 400 **d)** 6.40×10^{-5}
e) 2.37 **f)** 2.31
g) 10.4 **h)** 0.843
i) 2.25 **j)** 2.18
k) 0.244 **l)** 0.965

Q8 **a)** 8.76 **b)** 4.17
c) 19.4 **d)** 219
e) 108 **f)** 91.9
g) 13.6 **h)** 17.8
i) 5.06

Q9 **a)** 0.008 **b)** 0.25
c) 1.53×10^{-5} **d)** 0.667
e) 2.24 **f)** 1.82
g) 1.55 **h)** 2.60
i) 0.512 **j)** 1.21
k) 0.0352 **l)** 7.28

Q10 **a)** 1.49 **b)** 20.1
c) 2.50 **d)** 6.55
e) 1.08 **f)** 8.78
g) 0.707 **h)** –0.380

Q11 **a)** 9.14 **b)** 1.50
c) 0.406 **d)** 476
e) 0.0146 **f)** 1.22
g) 84.5 **h)** 0.496
i) 165 **j)** 8.47

Q12 **a)** $k \times k$
b) $p \times p \times p \times q \times q$
c) $g \times t \times t$
d) $g \times g \times t \times t$
e) $-t \times -t$
f) $-(t \times t)$

Q13 **a)** a^3 **d)** c^3d^2
b) c^5 **e)** $4x^3$
c) a^2b^3 **f)** $6xy^3$

Q14 **a)** x^5 **m)** u^3
b) x **n)** 1
c) $-y^{13}$ **o)** gt
d) q^3 **p)** 1
e) b^9 **q)** k^{10}
f) $x^3 + x^2$ **r)** p^6
g) $-f^7$ **s)** v^2
h) x^7y^7 **t)** i
i) p^9q^6 **u)** g^7t^{14}
j) $-x$ **v)** x^{11}
k) 1 **w)** r^3
l) 1

Q15 **a)** $\frac{1}{k^2}$ **d)** y^2
b) $\frac{q^2}{p^3}$ **e)** a^4v^2
c) $\frac{g}{t^2}$ **f)** $\frac{a^4}{bv^2}$

Q16 **a)** $\frac{1}{h^6}$ **d)** $\frac{1}{v^2}$
b) $\frac{1}{g^3}$ **e)** $\frac{1}{w}$
c) $\frac{1}{t^4}$ **f)** $\frac{1}{a^4}$

Q17 **a)** $3a^3$ **f)** $\frac{1}{6j^2}$
b) p^4q^5 **g)** $\frac{b^{\frac{7}{10}}}{8}$
c) x^3y^4 **h)** $\frac{3}{u^{\frac{1}{2}}}$
d) x^2 **i)** $10d^{\frac{1}{8}}$
e) $2x^{\frac{1}{2}}$

Page 31 — Algebra Basics

Q1 **a)** –27°C **d)** +18°C
b) –22°C **e)** +15°C
c) +12°C **f)** –12°C

Q2 Expression **b)** is larger by 1.

Q3 **a)** $-4x$ **b)** $18y$

Q4 **a)** –1000, –10 **c)** 144, 16
b) –96, –6 **d)** 0, 0

Q5 –4

Q6 **a)** $-6xy$ **g)** $\frac{-5x}{y}$
b) $-16ab$ **h)** 3
c) $8x^2$ **i)** –4
d) $-16p^2$ **j)** –10
e) $\frac{10x}{y}$ **k)** $4x$
f) $\frac{-10x}{y}$ **l)** $-8y$

Q7 **a)** $15x^2 - x$
b) $13x^2 - 5x$
c) $-7x^2 + 12x + 12$
d) $30abc + 12ab + 4b$
e) $18pq + 8p$
f) $17ab - 17a + b$
g) $4pq - 5p - 9q$
h) $16x^2 - 4y^2$
i) $abc + 10ab - 11cd$
j) $-2x^2 + y^2 - z^2 + 6xy$

Q8 **a)** $x^2 + 4x + 3x + 12 = x^2 + 7x + 12$
b) $4x^2 + 6x + 6x + 9 = 4x^2 + 12x + 9$
c) $15x^2 + 3x + 10x + 2$
$= 15x^2 + 13x + 2$

Page 32 — Formulas from Words

Q1 **a)** $y = x + 5$ **d)** $y = x + 6^2$
b) $y = 7x + 4$ **e)** $y = x^2 \div 8$
c) $y = (x - 7) \div 3$ **f)** $y = x^2 \div 12$

Q2 **a)** $c = 25n$
b) $c = (25 + 1.25)n = 26.25n$

Q3 **a)** $N = n + 23$ **d)** $N = xn$
b) $N = n - 14$ **e)** $N = nx^2$
c) $N = 2n$

Q4 **a) i)** $4d$ cm **ii)** d^2 cm^2
b) i) $a + b + c$ cm **ii)** $\frac{1}{2}cz$ cm^2

Q5 $C = 10 + 5h$

Q6 $T = (73 + 27)p + 15l = 100p + 15l$

Q7 $S = (3 + \frac{1}{3}w)d$

Answers: P33 — P37

Page 33 — Multiplying Out Brackets

Q1 **a)** $4x + 4y - 4z$
b) $x^2 + 5x$
c) $-3x + 6$
d) $9a + 9b$
e) $-a + 4b$
f) $2x - 6$
g) $4e^2 - 2f^2 + 10ef$
h) $16m - 8n$
i) $6x^2 + 2x$
j) $-2ab + 11$
k) $-2x^2 - xz - 2yz$
l) $3x - 6y - 5$
m) $-3a - 4b$
n) $14pqr + 8pq + 35qr$
o) $x^3 + x^2$
p) $4x^3 + 8x^2 + 4x$
q) $8a^2b + 24ab + 8ab^2$
r) $7p^2q + 7pq^2 - 7q$
s) $16x - 8y$

Q2 **a)** $x^2 - 2x - 3$
b) $x^2 + 2x - 15$
c) $x^2 + 13x + 30$
d) $x^2 - 7x + 10$
e) $x^2 - 5x - 14$
f) $28 - 11x + x^2$
g) $6x - 2 + 9x^2 - 3x = 9x^2 + 3x - 2$
h) $6x^2 - 12x + 4x - 8 = 6x^2 - 8x - 8$
i) $4x^2 + x - 12x - 3 = 4x^2 - 11x - 3$
j) $4x^2 - 8xy + 2xy - 4y^2$
$= 4x^2 - 4y^2 - 6xy$
k) $12x^2 - 8xy + 24xy - 16y^2$
$= 12x^2 - 16y^2 + 16xy$
l) $9x^2 + 4y^2 + 12xy$

Q3 $15x^2 + 10x - 6x - 4 = 15x^2 + 4x - 4$

Q4 $4x^2 - 4x + 1$

Q5 **a)** $x^3 + x^2 - 4x - 4$
b) $x^3 + 10x^2 + 31x + 30$
c) $2x^3 - 15x^2 + 27x - 10$
d) $-x^3 - 4x^2 + 11x + 30$
e) $-2x^3 - 7x^2 - 2x + 3$
f) $6x^3 + 74x^2 + 144x + 40$
g) $-x^3 - 3x^2 + 9x + 27$
h) $4x^3 - 3x - 1$
i) $x^3 + 9x^2 + 27x + 27$

Q6 **a)** $(4x + 6)$ m
b) $(-3x^2 + 17x - 10)$ m^2

Q7 **a)** $(8x + 20)$ cm
b) $40x$ cm^2
c) $40x - 12x = 28x$ cm^2

Q8 **a)** Perimeter — $3x + 29$ cm
Area — $\frac{7x + 126}{2}$ cm^2
b) Perimeter — $(8x + 4)$ cm
Area — $(3x^2 + 14x - 24)$ cm^2
c) Perimeter — $(16x - 4)$ cm
Area — $(16x^2 - 8x + 1)$ cm^2
d) Perimeter — $(10x + 4)$ cm
Area — $(6x^2 - 5x - 6)$ cm^2

Page 34 — Factorising

Q1 **a)** $a^2(b + c)$
b) $a^2(5 + 13b)$
c) $a^2(2b + 3c)$
d) $a^2(a + y)$
e) $a^2(2x + 3y + 4z)$
f) $a^2(b^2 + ac^2)$

Q2 **a)** $x(x - 5)$
b) $2(x + 3)$
c) $3x(x + 4)$
d) $2x(2x - 3)$
e) $3xy(1 + 4x)$
f) $3(3x + 5)$
g) $5x(3xy - 5)$
h) $4pq(q - 5 + 2p)$
i) $2x(5x^3 + 3)$
j) $5x^2(3x - 4)$
k) $7x(3x + 2)$
l) $5xy(z + 4u)$

Q3 **a)** $4xyz(1 + 2) = 12xyz$
b) $4xyz\ (2 + 3) = 20xyz$
c) $8xyz(1 + 2x)$
d) $4xyz^2(5xy + 4)$

Q4 **a)** $(x + 3)(x - 3)$
b) $(y + 4)(y - 4)$
c) $(5 + z)(5 - z)$
d) $(6 + a)(6 - a)$
e) $(2x + 3)(2x - 3)$
f) $(3y + 2)(3y - 2)$
g) $(5 + 4z)(5 - 4z)$
h) $(1 + 6a)(1 - 6a)$
i) $(x^2 + 6)(x^2 - 6)$
j) $(x^2 + y^2)(x^2 - y^2)$
k) $(1 + ab)(1 - ab)$
l) $(10x + 12y)(10x - 12y)$

Q5 **a)** $(x + 2)(x - 2)$
b) $(12 + y^2)(12 - y^2)$
c) $(1 + 3xy)(1 - 3xy)$
d) $(7x^2y^2 + 1)(7x^2y^2 - 1)$

Q6 **a)** $16a^2b^2(4b - a)$
b) $q(p + r - pqr)$
c) $3(m^2 - 8)$
d) $b^2(b^2 - ab + c)$
e) $(a^2 - 13)(a^2 + 13)$
f) $3ab(3b - c)$
g) $(9 - z)(9 + z)$
h) $(6m - 5n)(6m + 5n)$
i) $mn(m + 3 - 2n^2)$
j) $(11p - 3q)(11p + 3q)$
k) $12(12x^2 - 9y^2 - 5z^2)$
l) $(8ab - 7cd)(8ab + 7cd)$

Page 35 — Manipulating Surds

Q1 **a)** $\sqrt{15}$ **d)** x
b) 2 **e)** 8
c) x **f)** $\sqrt{5}$

Q2 3π cm^2

Q3 **a)** 1 **e)** $3\sqrt{5}$
b) $5\sqrt{3}$ **f)** $5\sqrt{2}$
c) $2\sqrt{2}$ **g)** $\sqrt{2}$
d) $7 + 4\sqrt{3}$ **h)** $3(\sqrt{2} - 1)$

Q4 **a)** $(1 + \sqrt{5})(1 - \sqrt{5}) = -4$, rational
b) $\frac{1 + \sqrt{5}}{1 - \sqrt{5}} = -\frac{1}{2}(3 + \sqrt{5})$, irrational

Q5 **a)** $(x + y)(x - y) = -1$, rational
b) $\frac{x + y}{x - y} = -3 - 2\sqrt{2}$, irrational

Q6 **a)** $\frac{\sqrt{2}}{2}$ **e)** $\sqrt{2} - 1$
b) $\frac{\sqrt{2}}{2}$ **f)** $3 - \sqrt{3}$
c) $\frac{\sqrt{10}a}{10}$ **g)** $\frac{2[\sqrt{6} - 1]}{5}$
d) $\frac{\sqrt{xy}}{y}$ **h)** $\frac{3 + \sqrt{5}}{2}$

Q7 $3\sqrt{3}$

Q8 $\sqrt{16} \times \sqrt{2} + 3\sqrt{2} = 7\sqrt{2}$

Q9 $19 + 6\sqrt{2}$

Pages 36-37 — Solving Equations

Q1 1

Q2 **a)** $x = \pm 3$ **d)** $x = \pm 3$
b) $x = \pm 6$ **e)** $x = \pm 1$
c) $x = \pm 3$

Q3 **a)** $x = 5$ **d)** $x = -6$
b) $x = 4$ **e)** $x = 5$
c) $x = 10$ **f)** $x = 9$

Q4 **a)** $x = 5$ **e)** $x = 6$
b) $x = 2$ **f)** $x = 5$
c) $x = 8$ **g)** $x = \pm 2$
d) $x = 17$

Q5 **a)** 15.5 cm **b)** 37.2 cm

Q6 £15.50

Q7 **a)** $x = 9$ **g)** $x = 15$
b) $x = 2$ **h)** $x = 110$
c) $x = 3$ **i)** $x = \pm 6$
d) $x = 3$ **j)** $x = 66$
e) $x = 4$ **k)** $x = 700$
f) $x = -1$ **l)** $x = 7\frac{1}{2}$

Q8 **a)** Joan — £x
Kate — £$2x$
Linda — £$(x - 232)$
b) $4x = 2632$
$x = 658$
c) Kate — £1316
Linda — £426

Q9 **a)** $2x + 32$ cm
b) $12x$ cm^2
c) $x = 3.2$

Q10 **a)** $x = 0.75$ **d)** $x = -1$
b) $x = -1$ **e)** $x = 4$
c) $x = -6$ **f)** $x = 13$

Q11 $x = 8$

Q12 $x = 1$

Q13 8 yrs

Q14 39, 35, 8

Q15 **a)** $y = 22$ **f)** $x = 7$
b) $x = 8$ **g)** $x = \pm 3$
c) $z = -5$ **h)** $x = \pm 4$
d) $x = 19$ **i)** $x = \pm 7$
e) $x = 23$

Q16 $x = 1\frac{1}{2}$

Q17 **a)** $x = 5$ **b)** $x = 9$

Answers: P37 — P43

Q18 $x = 1\frac{1}{2}$ AB = 5 cm
AC = 5½ cm
BC = 7½ cm

Pages 38-39 — Rearranging Formulas

Q1 **a)** $h = \frac{10 - g}{4}$ **b)** $c = 2d - 4$
c) $k = 3 + \frac{j}{2}$ **d)** $b = \frac{3a}{2}$
e) $g = \frac{8f}{3}$ **f)** $x = 2(y + 3)$
g) $t = 6(s - 10)$ **h)** $q = \pm\frac{\sqrt{p}}{2}$

Q2 **a)** $c = \frac{w - 500m}{50}$
b) 132

Q3 **a) i)** £38.00 **ii)** £48.00
b) $c = 28 + 0.25n$
c) $n = 4(c - 28)$
d) i) 24 miles **ii)** 88 miles
iii) 114 miles

Q4 **a)** $x = \pm\sqrt{y + 2}$
b) $x = y^2 - 3$
c) $s = \pm 2\sqrt{r}$
d) $g = 3f - 10$
e) $z = 5 - 2w$
f) $x = \pm\sqrt{\frac{3v}{h}}$
g) $a = \frac{v^2 - u^2}{2s}$
h) $u = \pm\sqrt{v^2 - 2as}$
i) $g = \frac{4\pi^2 l}{t^2}$

Q5 **a)** £Jx **b)** $P = T - Jx$
c) $J = \frac{T - P}{x}$ **d)** £16

Q6 **a) i)** £2.04 **ii)** £3.48
b) C = (12x + 60) pence
c) $x = \frac{C - 60}{12}$
d) i) 36 **ii)** 48 **iii)** 96

Q7 **a)** $x = \frac{z}{y + 2}$
b) $x = \frac{b}{a - 3}$
c) $x = \frac{y}{4 - z}$
d) $x = \frac{3z + y}{y + 5}$
e) $x = \frac{-2}{y - z}$ or $\frac{2}{z - y}$
f) $x = \frac{2y + 3z}{2 - z}$
g) $x = \frac{-y - wz}{yz - 1}$ or $\frac{y + wz}{1 - yz}$
h) $x = \frac{-z}{4}$

Q8 **a)** $p = \frac{4r - 2q}{q - 3}$
b) $g = \frac{5 - 2e}{f + 2}$
c) $b = \frac{3c + 2a}{a - c}$
d) $q = \pm\sqrt{\frac{4}{p - r}} = \pm\frac{2}{\sqrt{p - r}}$
e) $a = \frac{2c + 4b}{4 + c - d}$
f) $x = \pm\sqrt{\frac{-3y}{2}}$
g) $k = \pm\sqrt{\frac{14}{h - 1}}$
h) $x = \left(\frac{4 - y}{2 - z}\right)^2$
i) $a = \frac{b^2}{3 + b}$
j) $m = -7n$
k) $e = \frac{d}{50}$
l) $y = \frac{x}{3x + 2}$

Q9 **a)** $y = \frac{x}{x - 1}$
b) $y = \frac{-3 - 2x}{x - 1}$ or $\frac{2x + 3}{1 - x}$
c) $y = \pm\sqrt{\frac{x + 1}{2x - 1}}$
d) $y = \pm\sqrt{\frac{1 + 2x}{3x - 2}}$

Page 40 — Factorising Quadratics

Q1 **a)** $(x + 5)(x - 2)$
$x = -5$, $x = 2$
b) $(x - 3)(x - 2)$
$x = 3$, $x = 2$
c) $(x - 1)^2$
$x = 1$
d) $(x - 3)(x - 1)$
$x = 3$, $x = 1$
e) $(x - 5)(x + 4)$
$x = 5$, $x = -4$
f) $(x + 1)(2x - 5)$
$x = -1$, $x = \frac{5}{2}$
g) $(3x + 7)(x - 1)$
$x = -\frac{7}{3}$, $x = 1$
h) $(x + 7)^2$
$x = -7$
i) $(x - 5)(2x + 3)$
$x = 5$, $x = -\frac{3}{2}$

Q2 **a)** $(x + 9)(x - 4)$
$x = -9$, $x = 4$
b) $x(x - 5)$
$x = 0$, $x = 5$
c) $(x - 7)(x + 3)$
$x = 7$, $x = -3$
d) $(x - 24)(x - 2)$
$x = 24$, $x = 2$
e) $(x + 7)(x - 2)$
$x = -7$, $x = 2$
f) $(x - 6)(x + 3)$
$x = 6$, $x = -3$

Q3 $x = ½$, $x = -½$

Q4 **a)** $(x^2 - x)$ m^2
b) $x = 3$

Q5 **a)** $x(x + 1)$ cm^2
b) $x = 3$

Q6 **a)** x^2 m^2
b) $12x$ m^2
c) $x^2 + 12x - 64 = 0$
$x = 4$

Q7 **a)** area = $l(l - 0.75)$ cm^2
b) i) area = $(16l^2 - 12l)$ cm^2
ii) $16l^2 - 12l - 340 = 0 \Rightarrow l = 5$

Q8 **a)** $\frac{4}{x + 5}$ **b)** $\frac{x + 2}{x - 3}$
c) $\frac{2x + 3}{3x + 1}$

Pages 41-42 — The Quadratic Formula

Q1 **a)** 1.87, 0.13
b) 2.39, 0.28
c) 1.60, –3.60
d) 1.16, –3.16
e) 0.53, –4.53
f) –11.92, –15.08
g) –2.05, –4.62
h) 0.84, 0.03

Q2 **a)** –2, –6 **b)** 0.67, –0.5
c) 3, –2 **d)** 2, 1
e) 3, 0.75 **f)** 3, 0
g) 0.67 **h)** 0, –2.67
i) 4, –0.5 **j)** 4, –5
k) 1, –3 **l)** 5, –1.33
m)1.5, –1 **n)** –2.5, 1
o) 0.5, 0.33 **p)** 1, –3
q) 2, –6 **r)** 2, –4

Q3 **a)** 0.30, –3.30 **b)** 3.65, –1.65
c) 0.62, –1.62 **d)** –0.55, –5.45
e) –0.44, –4.56 **f)** 1.62, –0.62
g) 0.67, –4.00 **h)** –0.59, –3.41
i) 7.12, –1.12 **j)** 13.16, 0.84
k) 1.19, –4.19 **l)** 1.61, 0.53
m)0.44, –3.44 **n)** 2.78, 0.72

Q4 **a)** 1.7, –4.7 **b)** –0.27, –3.73
c) 1.88, –0.88 **d)** 0.12, –4.12
e) 4.83, –0.83 **f)** 1.62, –0.62
g) 1.12, –1.79 **h)** –0.21, –4.79
i) 2.69, –0.19 **j)** 2.78, 0.72
k) 1, 0 **l)** 1.5, 0.50

Q5 $x^2 - 3.6x + 3.24 = 0$
$x = 1.8$

Q6 **a)** $x^2 + 2.5x - 144.29 = 0$
$x = 10.83$
b) 48.3 cm

Page 43 — Completing the Square

Q1 **a)** $(x - 2)^2 - 9$
b) $(x - 1)^2$
c) $(x + \frac{1}{2})^2 + \frac{3}{4}$
d) $(x - 3)^2$
e) $(x - 3)^2 - 2$
f) $(x - 2)^2 - 4$
g) $(x + 1\frac{1}{2})^2 - 6\frac{1}{4}$
h) $(x - \frac{1}{2})^2 - 3\frac{1}{4}$
i) $(x - 5)^2$
j) $(x - 5)^2 - 25$
k) $(x + 4)^2 + 1$
l) $(x - 6)^2 - 1$

Q2 **a)** $x = 0.30$, $x = -3.30$
b) $x = 2.30$, $x = -1.30$
c) $x = 0.65$, $x = -4.65$
d) $x = 0.62$, $x = -1.62$
e) $x = 4.19$, $x = -1.19$

Answers: P43 — P47

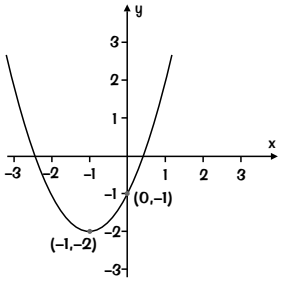
f) $x = 2.82$, $x = 0.18$
g) $x = 1.46$, $x = -0.46$
h) $x = 2.15$, $x = -0.15$

Q3 **a)** $4 \pm \sqrt{29}$
b) $1 \pm \sqrt{6}$
c) $-3 \pm 2\sqrt{5}$
d) $-4 \pm 2\sqrt{3}$
e) $\frac{-3 \pm \sqrt{17}}{2}$
f) $\frac{-7 \pm \sqrt{37}}{2}$
g) $\frac{5 \pm 5\sqrt{5}}{2}$
h) $\frac{9 \pm 3\sqrt{5}}{2}$
i) $-1 \pm \frac{3\sqrt{2}}{2}$
j) $1 \pm \frac{2\sqrt{3}}{3}$
k) $\frac{7 \pm \sqrt{73}}{4}$
l) $\frac{5 \pm \sqrt{13}}{6}$

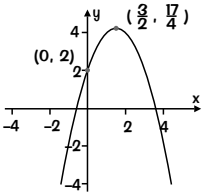
Q4 **a)** $(5, -28)$
b) $\left(\frac{7}{2}, -\frac{109}{4}\right)$
c) $(-2, -10)$
d) $\left(\frac{5}{8}, -\frac{41}{16}\right)$

Q5 **a)** $(-2, -5)$, minimum
b) $\left(\frac{11}{2}, \frac{97}{4}\right)$, maximum
c) $\left(-\frac{3}{2}, \frac{41}{4}\right)$, maximum
d) $(-2, -19)$, minimum
e) $\left(-\frac{5}{4}, -\frac{33}{8}\right)$, minimum
f) $(-1, 5)$, maximum

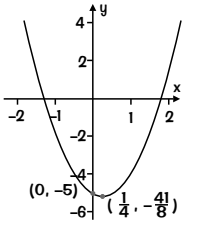
Q6 **a)**



b)



c)



Q7 $y = \left(x + \frac{m}{2}\right)^2 - \frac{m^2}{4} + n$
This has its turning point when $x = -\frac{m}{2} = 1$. So $m = -2$.
When $x = 1$, $y = -\frac{m^2}{4} + n = -5$.
So $n = -4$.

Pages 44-45 — Algebraic Fractions

Q1 **a)** $\frac{3xy}{z}$
b) $\frac{12b^2}{c}$
c) $\frac{1}{3xy^2z^3}$
d) $\frac{q^3}{2r^3}$

Q2 **a)** $\frac{2}{xy}$
b) $\frac{3a^2b}{2}$
c) $\frac{y}{2x^2}$
d) $\frac{2qr^2}{3}$
e) $\frac{8x^2z^2}{y}$
f) $\frac{90ac^4}{b}$
g) $\frac{x^3}{5}$
h) $\frac{12a^3b^2}{5}$
i) $\frac{3a^4c^3}{2bd}$
j) 1
k) $\frac{3rt^2}{2}$
l) $\frac{d^6}{e^3f}$

Q3 **a)** $2x^2y$
b) a
c) $\frac{3x^2}{y}$
d) $\frac{pq}{2}$
e) $2ef$
f) $5x^3$
g) $\frac{12yz}{x}$
h) $\frac{4a^3}{b}$
i) $\frac{5a^3}{b}$
j) $\frac{2x}{y^2z}$
k) $\frac{6}{n}$
l) $\frac{7g}{f}$

Q4 **a)** $\frac{3a-4}{2}$
b) $\frac{2x-y}{4}$
c) $\frac{5x+6}{3}$

Q5 **a)** $x = 5$
b) $x = 2$

Q6 **a)** $\frac{3+y}{2x}$
b) $\frac{1+y}{x}$
c) $\frac{2xy}{z}$
d) $\frac{6x+1}{3}$
e) $\frac{7x+6}{x}$
f) $\frac{14x+y}{6}$
g) $\frac{3x+2+y}{24}$
h) $\frac{x+2y-2}{10}$
i) $\frac{7x}{6}$
j) $\frac{37x}{42}$
k) $\frac{x(y+3)}{3y}$
l) $\frac{xyz+4x+4z}{4y}$

Q7 **a)** $\frac{4x-5y}{3}$
b) $\frac{4x-1}{y}$
c) $\frac{4x+3y-2}{2x}$
d) $\frac{2-2x}{x}$
e) $\frac{-1}{4x}$
f) $\frac{4x-y}{6}$
g) $\frac{z}{15}$
h) $\frac{m(12-n)}{3n}$
i) $\frac{b(14-a)}{7a}$
j) $\frac{-p+5q}{10}$
k) $\frac{-3p-4q}{4}$
l) $\frac{9x-4y+xy}{3y}$

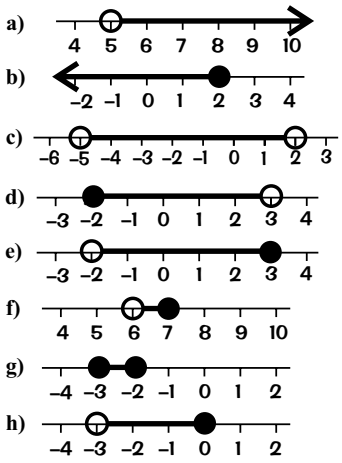
Q8 **a)** $\frac{a^2}{b^2}$
b) 1
c) $\frac{3}{2r}$
d) $\frac{mn(pm+1)}{p^2}$
e) $\frac{2x}{x^2-y^2}$
f) $\frac{11}{6x}$
g) $\frac{2(a^2+b^2)}{a^2-b^2}$
h) $\frac{3}{4}$
i) $\frac{3x-6y}{8}$

Q9 **a)** $\frac{2(a^2+b^2)}{a^2-b^2}$
b) $\frac{8x^2+10x+11}{(2x-3)(2x+5)}$
c) $\frac{2x}{y}$
d) $\frac{5}{3(2x+1)}$
e) $\frac{1}{(x+4)(x-3)}$
f) $\frac{8(x+6)}{3}$

Pages 46-47 — Inequalities

Q1 **a)** $9 \le x < 13$
b) $-4 \le x < 1$
c) $x \ge -4$
d) $x < 5$
e) $x > 25$
f) $-1 < x \le 3$
g) $0 < x \le 5$
h) $x < -2$

Q2



Q3 **a)** $x > 3$
b) $x < 4$
c) $x \le 5$
d) $x \le 6$
e) $x \ge 7.5$
f) $x < 4$
g) $x < 7$
h) $x < 4$
i) $x \ge 3$
j) $x > 11$
k) $x < 3$
l) $x \ge -\frac{1}{2}$
m) $x \le -2$
n) $x > 5$
o) $x < 15$
p) $x \ge -2$

Q4 Largest integer for x is 2.

Q5 $\frac{11-x}{2} < 5$, $x > 1$

Q6 **a)** $1 < x < 8$
b) $0 \le x \le 8$
c) $2 < x < 4$
d) $-4 \le x < -1$
e) $5 \le x \le 10$
f) $-1 < x < 23$
g) $-2 \le x < 4$
h) $-51 < x \le -11$
i) $-16 < x \le 5$

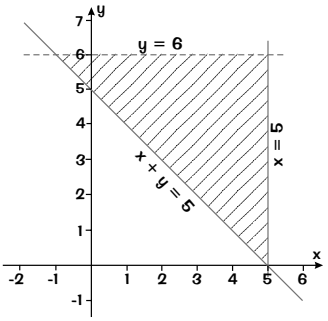
Q7 $1130 \le 32x$
36 classrooms are needed.

Q8 50 guests (including bride and groom),
$900 \ge 18x$

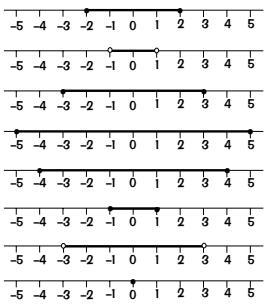
Q9 $x \ge 2$, $y > 1$, $x + y \le 5$

Answers: P47 — P51

Q10



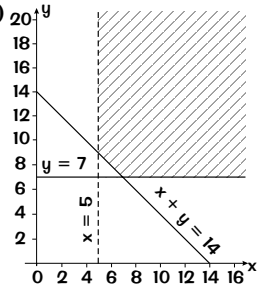
Q11 **a)**



b) **c)** **d)** **e)** **f)** **g)** **h)**

Q12 **a)** $x > 5, \quad y \geq 7, \quad x + y \geq 14$

b)



Q13 **a)** $-2 < x < 3$

b) $-5 \leq x \leq -1$

c) $x < -2$ or $x > -1$

d) $x \leq 2$ or $x \geq 8$

e) $x < -2$ or $x > 9$

f) $-7 < x < -5$

g) $x \leq 4$ or $x \geq 5$

h) $-3 < x < 1$

i) $-2 \leq x \leq -\frac{3}{2}$

j) $x < \frac{1}{2}$ or $x > 4$

k) $-\frac{2}{3} < x < 1$

l) $x \leq -4$ or $x \geq \frac{5}{4}$

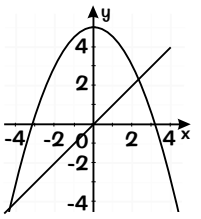
Page 48 — Simultaneous Equations and Graphs

Q1 **a)** $x = 3, y = 3$ **b)** $x = 2, y = 5$
c) $x = 1, y = 2$ **d)** $x = 1, y = 2$
e) $x = 1, y = 4$ **f)** $x = 1, y = 2$
g) $x = 2, y = 3$ **h)** $x = 2, y = 3$
i) $x = 5, y = 2$ **j)** $x = 3, y = 4$

Q2 **a)** $x = 0, x = 1$
b) $x = 2.7, x = -0.7$
c) $x = 3.4, x = -2.4$
d) $x = 1.6, x = -2.6$
e) $x = 0.7$
f) $x = 3.4, x = -2.4$
g) $x = 1.6, x = -2.6$

Q3

x	-4	-3	-2	-1	0	1	2	3	4
$-\frac{1}{2}x^2$	-8	-4.5	-2	-0.5	0	-0.5	-2	-4.5	-8
+5	5	5	5	5	5	5	5	5	5
y	-3	0.5	3	4.5	5	4.5	3	0.5	-3



a) $x = 3.2, x = -3.2$
b) $x = 4, x = -4$
c) $x = 2.3, x = -4.3$

Page 49 — Simultaneous Equations

Q1 **a)** $x = 4, y = 18$ OR $x = -3, y = 11$
b) $x = 6, y = 28$ OR $x = -3, y = 1$
c) $x = 1.5, y = 4.5$ OR $x = -1, y = 2$
d) $x = -3, y = 33/5$ OR $x = 2, y = \frac{28}{5}$
e) $x = -\frac{1}{4}, y = \frac{17}{4}$ OR $x = -3, y = 40$
f) $x = -\frac{2}{3}, y = \frac{31}{3}$ OR $x = -4, y = 57$

Q2 **a)** $x = 1, y = 2$
b) $x = 0, y = 3$
c) $x = -1\frac{1}{2}, y = 4$
d) $x = 5, y = 23$ OR $x = -2, y = 2$
e) $x = \frac{1}{3}, y = -\frac{29}{3}$ OR $x = 4, y = 38$
f) $x = \frac{1}{2}, y = -\frac{3}{2}$ OR $x = -2, y = 6$
g) $x = 1, y = 9$
h) $x = 8, y = -\frac{1}{2}$
i) $x = -1, y = 3$

Q3 **a)** $6x + 5y = 430$
$4x + 10y = 500$
b) $x = 45, y = 32$

Q4 7 chickens
4 cats

Q5 5 g (jellies are 4 g)

Q6 $3y + 2x = 18$
$y + 3x = 6$ $\quad x = 0, y = 6$
$4y + 5x = 7$
$2x - 3y = 12$ $\quad x = 3, y = -2$
$4x - 6y = 13$
$x + y = 2$ $\quad x = 2\frac{1}{2}, y = -\frac{1}{2}$

Q7 $5m + 2c = 344$
$4m + 3c = 397$
$m = 34$p, $c = 87$p

Q8 $x = 12, y = 2$

Page 50 — Sequences

Q1 **a)** 10, 12, 14; even numbers
b) 9, 11, 13; odd numbers
c) 25, 36, 49; square numbers
d) 125, 216, 343; cube numbers

Q2 **a)** 31, 36, 41 **b)** 5
c) $5n + 1$ **d)** 101

Q3 **a)** 10, 12, 14, $2n$
b) 9, 11, 13, $2n - 1$
c) 25, 30, 35, $5n$
d) 17, 20, 23, $3n + 2$
e) 19, 22, 25, $3n + 4$
f) 32, 37, 42, $5n + 7$
g) 46, 56, 66, $10n - 4$
h) 82, 89, 96, $7n + 47$

Q4 **a)** $4n - 3$
b) 75 is not in the sequence because when the expression is set to equal 75, n is not a whole number.

Q5 24, 35, 48

Q6 **a)** $16\frac{7}{8}, 16\frac{9}{16}, 16\frac{23}{32}, 16\frac{41}{64}$
b) The 10th term will be the mean of the 8th and 9th terms.

Q7 592

Q8 **a)** $d = 2, a = -1$
b) 9800

Page 51 — Proof

Q1 $(2n + 1)^2 - (2n - 1)^2 - 10$
$= (4n^2 + 4n + 1) - (4n^2 - 4n + 1) - 10$
$= 8n - 10$
Dividing this by 8 gives $n - \frac{5}{4}$
(not a whole number), so the expression is not a multiple of 8.

Q2 $n + (n + 1) + (n + 2)$
$= 3n + 3 = 3(n + 1)$
Dividing this by 3 gives n + 1 (a whole number), so the sum is divisible by 3.

Q3 $2a \times 2b = 4ab = 2 \times 2ab$. This is divisible by 2 and so is even.

Q4 $2n + (2n + 2) + (2n + 4)$
$= 6n + 6 = 6(n + 1)$
Dividing this by 6 gives n + 1 (a whole number), so the sum is a multiple of 6.

Q5 **a)** E.g. 3 and 1 are both odd numbers but if you add them together you get 4, which is even so the statement is wrong.
b) E.g. If $n = 6$, $n^2 = 36$, which is divisible by 4, but 6 is not divisible by 4, so the statement is wrong.

Q6 E.g. If $a = 1$ and $b = -1$, then $a^2 = 1$ and $b^2 = 1$. So $a^2 = b^2$ but a does not equal b, so the statement is wrong.

Q7 $5^{20} - 5^{19}$
$= 5 \times 5^{19} - 5^{19}$
$= 5^{19}(5 - 1) = 4 \times 5^{19}$
$= 2 \times 2 \times 5^{19}$
$= 2n$ where $n = 2 \times 5^{19}$
So $5^{20} - 5^{19}$ is even.

Q8 $3^8 - 1$ can be factorised to $(3^4 - 1)(3^4 + 1)$, so it has factors that are not equal to itself or 1. Therefore, it is not a prime number.

Q9 $\frac{y^2 + 1}{y^2} - \frac{x^2 + 1}{x^2} = \frac{x^2 - y^2}{(xy)^2}$
$x^2 - y^2 < 0$ since $x > 0$, $y > 0$ and $x < y$.
Also, $(xy)^2$ is always positive.
Therefore, $\frac{x^2 - y^2}{(xy)^2} < 0$
which means $\frac{y^2 + 1}{y^2} - \frac{x^2 + 1}{x^2} < 0$

Answers: P51 — P56

Q10 $n^2 - 2n + 2 + (n + 1)^2 - 2(n + 1) + 2$
$= n^2 - 2n + 2 + n^2 + 2n + 1 - 2n - 2 + 2$
$= 2n^2 - 2n + 3$
$= 2n^2 - 2n + 2 + 1$
$= 2(n^2 - n + 1) + 1$
$= 2x + 1$ where $x = (n^2 - n + 1)$
So the sum of two consecutive terms is an odd number.

Q11 If $x^2 + 3 > 2x + 1$,
then $x^2 - 2x + 2 > 0$
and $(x - 1)^2 + 1 > 0$
This is always true as $(x - 1)^2$ cannot be negative. So the inequality always holds and Fay is correct.

Page 52 — Direct and Inverse Proportion

Q1 $y = 20$

Q2 $y = 184.8$

Q3 $y = 2$

Q4 $x = 2$

Q5

x	1	2	3	4	5	6
y	48	24	16	12	9.6	8

Q6

x	1	2	5	10
y	100	25	4	1

x	2	4	6	8
y	24	6	$2^2/_3$	1.5

Q7 4 kg

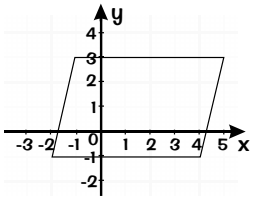
Q8 **a)** $r = 96$ **b)** $s = 4$
c) $r = 600$ **d)** $s = -8$

Q9 9.5 N kg^{-1}

Section Three — Graphs, Functions and Calculus

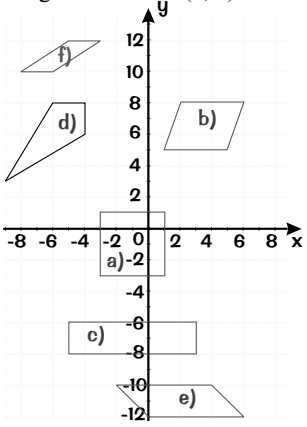
Pages 53-54 — Coordinates

Q1



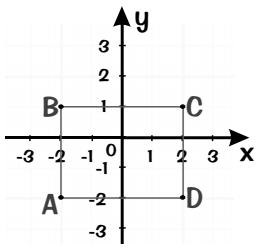
missing coordinate = (5, 3)

Q2



a) B is (1, –3) **b)** C is (5, 5)
c) A is (–5, –8) **d)** D is (–4, 6)
e) D is (0, –12) **f)** C is (–3, 12)

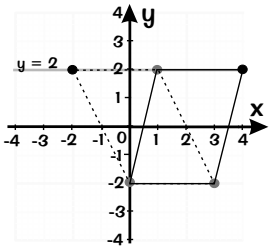
Q3



C = (2, 1), D = (2, –2)

Q4 (-2, 7)

Q5



Possible coordinates = (–2, 2) and (4, 2).

Q6 **a)** (3,4) **b)** (5.5,5)
c) (5.5,11) **d)** (8.5,9)
e) (3,3.5) **f)** (9.5,9.5)
g) (20,41.5) **h)** (30.5,20.5)

Q7 (110, 135)

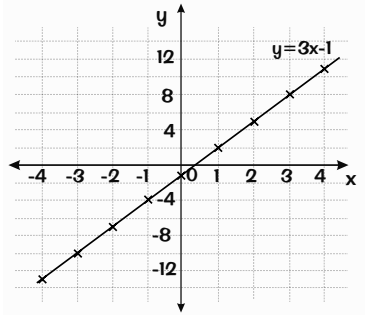
Q8 **a)** (2,5.5) **b)** (0.5,1.5)
c) (2,–2.5) **d)** (1,–1)
e) (2,3) **f)** (4,–0.5)
g) (–13,–12.5) **h)** (–5,–7)

Pages 55-56 — Straight-Line Graphs

Q1 **a)** B **f)** F
b) A **g)** C
c) F **h)** B
d) G **i)** D
e) E **j)** H

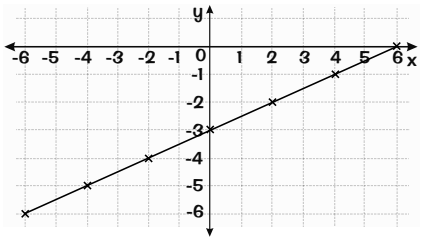
Q2

x	-4	-3	-2	-1	0	1	2	3	4
3x	-12	-9	-6	-3	0	3	6	9	12
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
y	-13	-10	-7	-4	-1	2	5	8	11



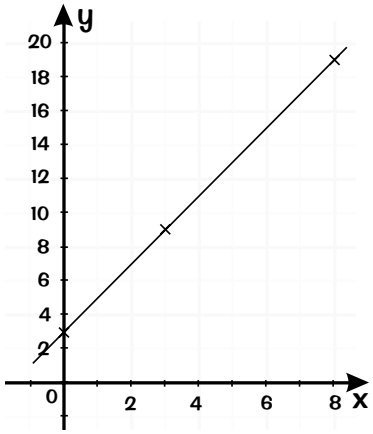
Q3

x	-6	-4	-2	0	2	4	6
1/2 x	-3	-2	-1	0	1	2	3
-3	-3	-3	-3	-3	-3	-3	-3
y	-6	-5	-4	-3	-2	-1	0



Q4

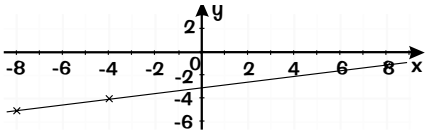
x	0	3	8
y	3	9	19



a) 13 **c)** 4
b) 7 **d)** 7

Q5

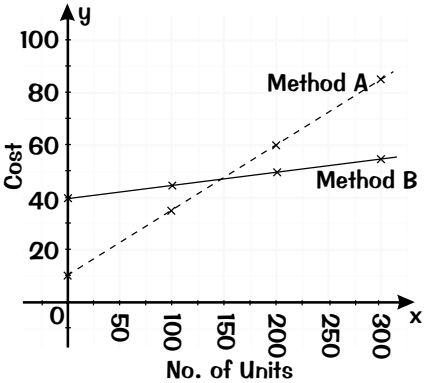
x	-8	-4	8
y	-5	-4	-1



a) –2.5 **c)** 4
b) –3 **d)** 6

Q6

Number of Units used	0	100	200	300
Cost using method A	10	35	60	85
Cost using method B	40	45	50	55



a) i) £27.50 **ii)** £43.50
b) Method A
c) 150 units

Answers: P57 — P62

Page 57 — Finding the Gradient

Q1 **a)** $-\frac{1}{2}$ **g)** 4
b) 3 **h)** 1
c) $-\frac{1}{4}$ **i)** –1
d) –2 **j)** $\frac{1}{3}$
e) $-\frac{2}{3}$ **k)** $-\frac{1}{2}$
f) $-\frac{8}{3}$ **l)** 3

Q2 **a)** 2 **d)** –2
b) $\frac{1}{2}$ **e)** $\frac{1}{2}$
c) –1 **f)** $-\frac{3}{4}$

Q3 **a)** A and C
b) (1, 2)

Q4 The gradient is –0.23 so it's a red run.

Page 58 — "y = mx + c"

Q1 **a)** $m = 4$, (0, 3)
b) $m = 3$, (0, –2)
c) $m = 2$, (0, 1)
d) $m = –3$, (0, 3)
e) $m = 5$, (0, 0)
f) $m = –2$, (0, 3)
g) $m = –6$, (0, –4)
h) $m = 1$, (0, 0)
i) $m = -\frac{1}{2}$, (0, 3)
j) $m = \frac{1}{4}$, (0, 2)
k) $m = \frac{4}{3}$, (0, 2)

Q2 **a)** $y = \frac{7}{2}x - 1$ **d)** $y = \frac{1}{4}x - 3$
b) $y = \frac{1}{2}x + 4$ **e)** $y = -\frac{1}{2}x$
c) $y = -\frac{1}{5}x + 7$ **f)** $y = -2x - 6$

Q3 **a)** $y = x + 4$ **c)** $y = –x$
b) $y = 3x + 2$ **d)** $y = –3x + 4$

Q4 **a)** $y = x$ **c)** $y = –3x + 3$
b) $y = 3x$ **d)** $y = –2x – 4$

Q5 **a)** $x = 4$ **c)** $y = 7$
b) $x = 8$ **d)** $y = 9$

Q6 (7, 20) and (5, 14)

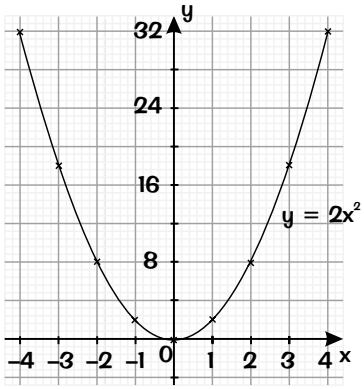
Q7 **a)** $m = 3$
b) $y = 3x + 1$

Q8 **a)** $m = –2$
b) $y = –2x – 3$

Page 59 — Quadratic Graphs

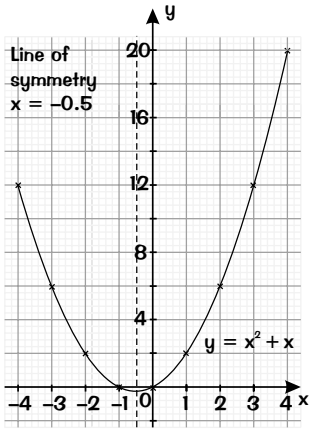
Q1

x	–4	–3	–2	–1	0	1	2	3	4
$y=2x^2$	32	18	8	2	0	2	8	18	32



Q2

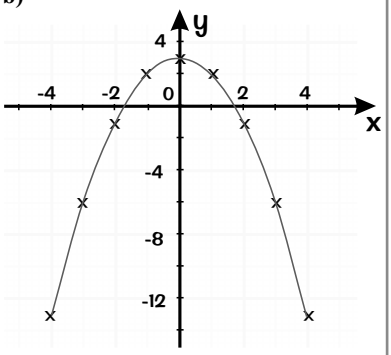
x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$y=x^2+x$	12	6	2	0	0	2	6	12	20



Q3 **a)**

x	–4	–3	–2	–1	0	1	2	3	4
3	3	3	3	3	3	3	3	3	3
$–x^2$	–16	–9	–4	–1	0	–1	–4	–9	–16
$y=3–x^2$	–13	–6	–1	2	3	2	–1	–6	–13

b)



c) 3

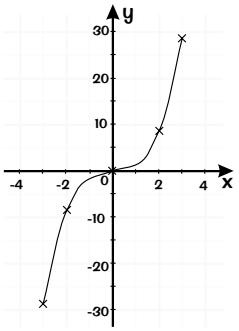
Pages 60-64 — Harder Graphs

Q1 **a)** Reciprocal **f)** Reciprocal
b) Reciprocal **g)** Straight line
c) Reciprocal **h)** Cubic
d) Quadratic **i)** Cubic
e) Quadratic **j)** Cubic

Q2 **a)** ix **g)** viii
b) iv **h)** vi
c) iii **i)** x
d) vii **j)** v
e) xi **k)** ii
f) xii **l)** i

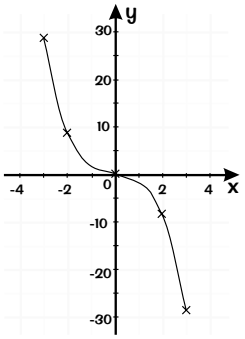
Q3

x	-3	-2	-1	0	1	2	3
$y=x^3$	-27	-8	-1	0	1	8	27



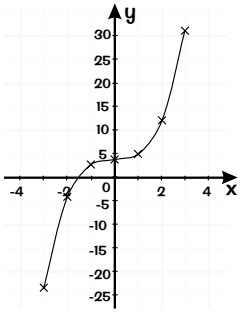
Q4

x	-3	-2	-1	0	1	2	3
$y=-x^3$	27	8	1	0	-1	-8	-27



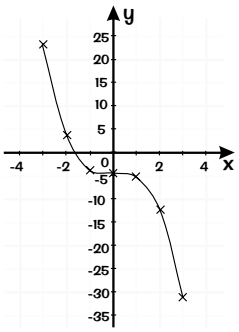
Q5

x	-3	-2	-1	0	1	2	3
x^3	-27	-8	-1	0	1	8	27
$y=x^3+4$	-23	-4	3	4	5	12	31



Q6

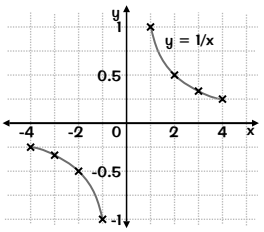
x	-3	-2	-1	0	1	2	3
$-x^3$	27	8	1	0	-1	-8	-27
$y=-x^3-4$	23	4	-3	-4	-5	-12	-31



Answers: P63 — P66

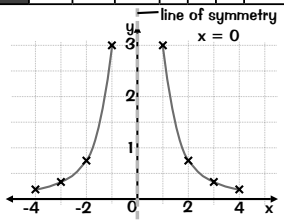
Q7

x	-4	-3	-2	-1	0	1	2	3	4
y=1/x	-0.25	-0.33	-0.5	-1	n/a	1	0.5	0.33	0.25



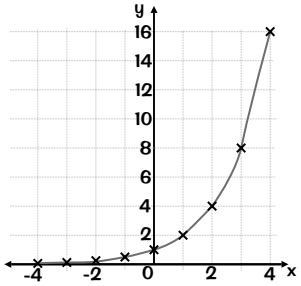
Q8

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$y=3/x^2$	0.2	0.3	0.8	3	0	3	0.8	0.3	0.2



Q9

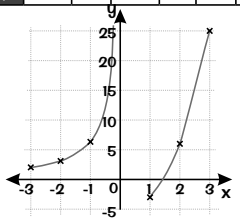
x	-4	-3	-2	-1	0	1	2	3	4
$y=2^x$	0.06	0.1	0.3	0.5	1	2	4	8	16



c) Anything to the power of 0 is 1.

Q10

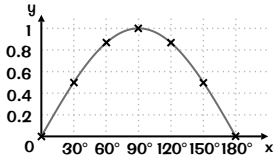
x	-3	-2	-1	0	1	2	3
3^x	0.04	0.1	0.3	1	3	9	27
6/x	-2	-3	-6	n/a	6	3	2
$y=3^x - 6/x$	2.04	3.1	6.03		-3	6	25



Q11 **a)** B **b)** E **c)** D

Q12

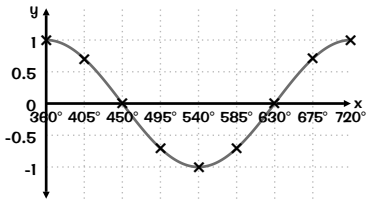
x	0°	30°	60°	90°	120°	150°	180°
sin x	0	0.5	0.87	1	0.87	0.5	0
cos x	1	0.87	0.5	0	-0.5	-0.87	-1
tan x	0	0.58	1.73	—	-1.73	-0.58	0



Q13

x	360°	405°	450°	495°	540°
cos x	1	0.71	0	-0.71	-1

x	585°	630°	675°	720°
cos x	-0.71	0	0.71	1



Page 65 — Functions

Q1 **a)** $f(x) = 3 + x$
b) $f(x) = 790 - 41x$
c) $f(x) = 3(9x^2 + 2)$

Q2 **a)** $x = 4.5$ **d)** $x = \pm 2$
b) $x = \pm 1$ **e)** $x = 0.27$ or -7.27
c) $x = \pm 0.31$ **f)** $x = 5$

Q3 **a)** $x > 4$ **d)** $x < -3.5$
b) $x \leq 0$ **e)** $x = -0.75$
c) $x = 0$

Q4 **a)** $f(8) = 44$
b) $g(3) = -6$
c) $f(-4) = -4$
d) $gf(x) = 3 - (4x + 12)^2$
e) $fg(x) = 4(3 - x^2) + 12$
f) $gf(2) = -397$

Q5 **a)** $h^{-1}(x) = x - 6$
b) $f^{-1}(x) = \frac{11}{x} - 1$
c) $g^{-1}(h(x)) = \frac{4}{3}(6 + x)$
d) $f^{-1}(g(x)) = \frac{44}{3x} - 1$
e) $h^{-1}(f(5)) = -4\frac{1}{6}$
f) $h^{-1}(g(-1)) = -6.75$

Q6 **a)** $f(-1) = -\frac{2}{3}$ **d)** $n^{-1}(2) = -14.5$
b) $g(9) = 554$ **e)** 12
c) $kj(-3) = 210$ **f)** 0.75

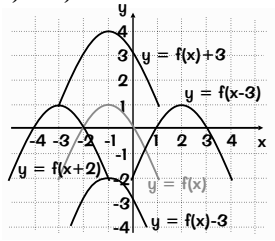
Q7 **a)** $hi(x) = (11 \div (-x + \frac{x^2}{2})) - 8$

b) $m^{-1}(x) = \frac{10(x - 3) + 4}{18} = \frac{5x - 9}{9}$

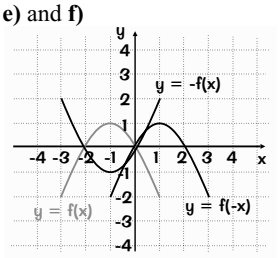
c) $p^{-1}(q(x)) = \frac{(\frac{13}{x-2}) - 5}{8} = \frac{23 - 5x}{8x - 16}$

Pages 66-67 — Graph Transformations

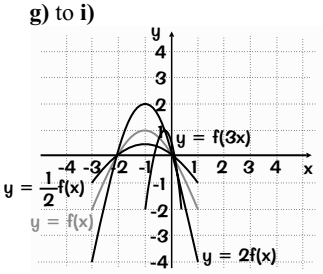
Q1 **a)** to **d)**



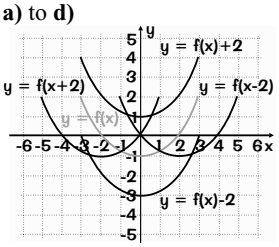
e) and **f)**



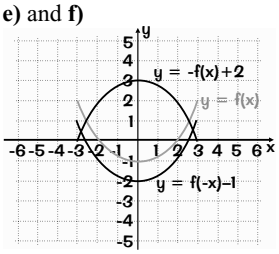
g) to **i)**



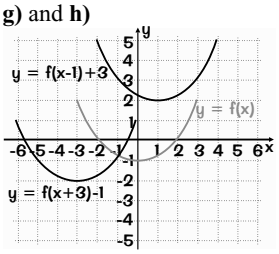
Q2 **a)** to **d)**



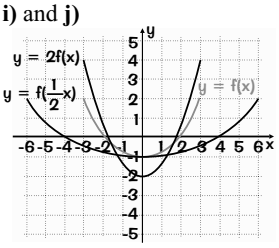
e) and **f)**



g) and **h)**



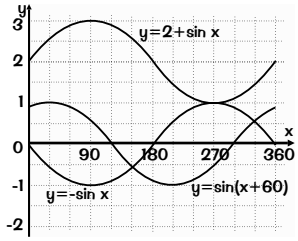
i) and **j)**



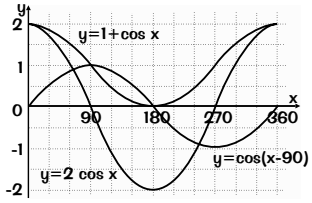
Q3 Graph is reflected in the x-axis and the y-axis.

Answers: P67 — P74

Q4



Q5 **a)** to **c)**



Q6 **a)** $(4, \frac{3}{2})$ **e)** (0, 0)
b) $(1, \frac{7}{2})$ **f)** $(-1, \frac{3}{2})$
c) $(1, -\frac{3}{2})$ **g)** $(\frac{4}{3}, \frac{11}{2})$
d) (1, 3)

Q7 $y = -x^3 - 2x + 4$

Pages 68–71 — Differentiation

Q1 **a)** $4x^3$ **g)** $2x^3$
b) $2x$ **h)** 33
c) $13x^{12}$ **i)** 1
d) $4x$ **j)** 0
e) $15x^2$ **k)** $-9x^2$
f) $28x^3$ **l)** $-4x^{15}$

Q2 **a)** $7a^6$ **c)** 1
b) $50t^4$ **d)** $-4w^5$

Q3 **a)** $5x^4$ **f)** $21x^2 + 12x$
b) $14x^6$ **g)** $32x^7 + 2x$
c) 1 **h)** $15x^4 + 3x^2 + 1$
d) $4x + 1$ **i)** $3x^8 + 5x^4 + 2x$
e) $9x^8 + 3$

Q4 $36x^3 + 3x^2 + 8x + 6$

Q5 $15x^4 + 28x^3 + 24x^2 + 4x + 10$

Q6 $40x^7 + 24x^5 + 48x^3 + 14x$

Q7 $44d^3 + 36d^2 + 18d + 14$

Q8 **a)** $-12x^2 - 2x$ **d)** $6x^2 + 2x - 8$
b) $15x^2 + 6x + 1$ **e)** $-1/x^2$
c) $-18x^2 - 4x$ **f)** $-2/x^3$

Q9 **i)** 1 **ii)** 1.75

Q10 **a)** –2, –2 **g)** –1.5, –6
b) 5, 44 **h)** 3, 12
c) –2, 4 **i)** 3, 0.75
d) 2, –4 **j)** –2, –0.5
e) –2, 4 **k)** 2, –0.25
f) 6, 24 **l)** –2, 0.25

Q11 $dy/dx = 3x^2 + 6x + 1$. Gradient at $x = 2$ is $3(2^2) + (6\times2) + 1 = 12 + 12 + 1 = 25$. This is positive, so must be Graph B as Graphs A and C have negative gradients at $x = 2$.

Q12 (2, 14)

Q13 (0.5, 8)

Q14 **a)** Between 23 minutes and 2 hours 37 minutes = 2 hours 14 minutes.
b) $v = -2t + 3$
c) after 30 mins, $v = 2$ km/h, after 1 hour, $v = 1$ km/h

Q15 **a)** $d = 2t^2(t + 1) = 2(6)^2(6 + 1) = 2 \times 36 \times 7 = 504$ metres
b) Velocity $= d(d)/dt = 6t^2 + 4t$
So after 6 seconds: Velocity $= 6(6)^2 + 4(6) = 216 + 24 = 240$ m/s
c) Acceleration = d(velocity)/dt $= 12t + 4$. So after 6 seconds acceleration $= 12(6) + 4 = 76$ m/s^2

Q16 **a)** $v = 4(40)^2 + 2(40) + 3$
$v = 6483$ m/s
b) Acceleration $= dv/dt = 8t + 2$, so acceleration after 40 s is: $8(40) + 2 = 322$ m/s^2

Q17 **a)** $dy/dx = 4x$, so turning point is when $4x = 0$, so $x = 0$.
Turning point is (0, 0).
Graph is $y = ax^2$ graph, a > 0, so turning point is a minimum.
b) $dy/dx = 10x + 1$, so turning point is when $10x + 1 = 0$, so $x = -0.1$.
Turning point is (–0.1, –0.05).
Graph is $y = ax^2$ graph, a > 0, so turning point is a minimum.
c) $dy/dx = 6x + 2$, so turning point is when $6x + 2 = 0$, so $x = -1/3$.
Turning point is $(-1/3, -5\frac{1}{3})$.
Graph is $y = ax^2$ graph, a > 0, so turning point is a minimum.
d) $dy/dx = -2x + 4$, so turning point is when $-2x + 4 = 0$, so $x = 2$.
Turning point is (2, –4).
Graph is $y = ax^2$ graph, a < 0, so turning point is a maximum.
e) $y = x^2 - 4x - 32$, so $dy/dx = 2x - 4$
Turning point is when $2x - 4 = 0$, so $x = 2$.
So turning point is (2, –36).
Graph is $y = ax^2$ graph, a > 0, so turning point is a minimum.

Q18 **a)** $4x^3 - 2x - 3$
b) $20x^4 + 36x^3$
c) $2x^2 + \frac{2}{x^3} + 10$
d) $3x^2 + 15x - \frac{12}{x^5}$

Q19 **a)** $dy/dx = x^2 - 2x - 3$
So stationary points are when $x^2 - 2x - 3 = (x + 1)(x - 3) = 0$.
So $x = -1$ and $x = 3$.
So stationary points are $(-1, -6\frac{1}{3})$ and (3, –17).
b) $dy/dx = 4x^2 - 32x + 48$
So stationary points are when $x^2 - 8x + 12 = (x - 2)(x - 6) = 0$.
So $x = 2$ and $x = 6$.
So stationary points are $(2, 42\frac{2}{3})$ and (6, 0).
c) $dy/dx = 2x^2 - 9x - 5$
So stationary points are when $2x^2 - 9x - 5 = (2x + 1)(x - 5) = 0$.
So $x = -0.5$ and $x = 5$.
So stationary points are $(-0.5, -\frac{17}{24})$ and $(5, -56\frac{1}{6})$.
d) $dy/dx = 3x^2 + 12x + 12$
So stationary points are when $x^2 + 4x + 4 = (x + 2)(x + 2) = 0$.
So $x = -2$.
So stationary point is (–2, –7).

Q20 **a)** $C = -20(0)^3 + 40(0)^2 - 10 = -10$ °C.
b) $dC/dt = -60t^2 + 80t$
So $dC/dt = -60(0.5)^2 + 80(0.5) = 25$ °C.
c) $dC/dt = 0$, so $-60t^2 + 80t = 0$,
Using the quadratic formula:
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$t = \frac{-80 \pm \sqrt{80^2 - 4(-60)(0)}}{2(-60)}$$
so $t = 0$ and $1\frac{1}{3}$
The heater was switched off after 1 hour 20 minutes.

Section Four — Geometry and Measure

Page 72 — Scale Drawings

Q1 10 cm long and 7.5 cm wide
Q2 65 cm long and 17 cm wide
Q3 13 mm wide gap, 78 cm wide oven.
Q4 **a)** Room 4 cm long and 3 cm wide
b) Window 2 cm, door 0.75 cm
Q5 **a)** 3.3 cm **c)** 48.8 km
b) 13.2 km
Q6 **a)** 12.25 m **b)** 4.02 m^2

Pages 73-74 — Geometry

Q1 **a)** $x = 47°$ **b)** $y = 154°$
c) $z = 22°$ **d)** $p = 35°$, $q = 45°$

Q2 **a)** $a = 146°$
b) $m = 131°$, $z = 48°$
c) $x = 68°$, $p = 112°$
d) $s = 20°$, $t = 90°$

Q3 **a)** $a = 130°$
b) $b = 56°$
c) $c = 48°$

Q4 **a)** $x = 96°$, $p = 38°$
b) $a = 108°$, $b = 23°$, $c = 95°$
c) $d = 120°$, $e = 60°$, $f = 60°$, $g = 120°$
d) $h = 155°$, $i = 77.5°$, $j = 102.5°$, $k = 77.5°$

Q5 **a)** $b = 70°$ $c = 30°$
$d = 50°$ $e = 60°$
$f = 150°$
b) $g = 21°$ $h = 71°$
$i = 80°$ $j = 38°$
$k = 92°$
c) $l = 35°$ $m = 145°$
$n = 55°$ $p = 125°$

Answers: P74 — P79

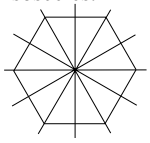
Q6 **a)** $x = 162°$ $y = 18°$
b) $x = 87°$ $y = 93°$
$z = 93°$
c) $a = 30°$ $2a = 60°$
$5a = 150°$ $4a = 120°$

Q7 **a)** $a = 141°$, $b = 141°$, $c = 39°$,
$d = 141°$, $e = 39°$
b) $a = 47°$, $b = 47°$, $c = 133°$,
$d = 43°$ $e = 43°$
c) $m = 140°$, $n = 140°$, $p = 134°$,
$q = 46°$, $r = 40°$

Pages 75-76 — Polygons

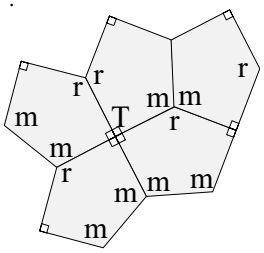
Q1 Isosceles.

Q2

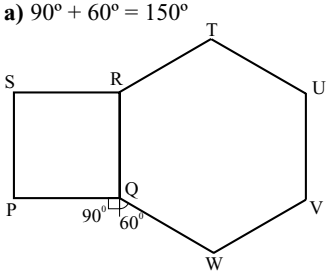


order of rotational symmetry = 6.

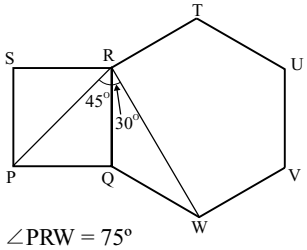
Q3 **a)** Angles at a point sum to 360°, hence m + m + r = 360°.
Angles in a pentagon sum to 540°. We know two angles are 90°, so we are left with 360°. The only angles left are m, m and r so m + m + r must equal 360°.
b) r°.
c)



Q4 **a)** 90° + 60° = 150°



b)



∠PRW = 75°

c) 180 – (360/n) = 150
180n – 360 = 150n
30n = 360 ⇒ n = 12

Q5 540° – (100° + 104° + 120°)
= 216° for two equal angles
∴1 angle = 108°

Q6 **a)** Interior angle = 165°
b) Exterior angle = 180° – 165° = 15°
Sum of exterior angles = 15 × 24
= 360°

Q7 **a)** $\frac{360}{5} = 72°$

b) $\frac{180 - 72}{2} = 54°$

c) **i)** 90° **ii)** 36°

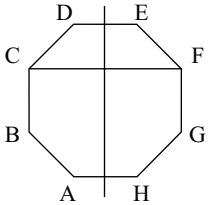
d) Lines ST and BE are parallel, so angle ABE = angle BAS = 36° (alternate angles).
Triangle ABE is isosceles, so angle BEA = angle ABE = 36°.

Q8 (2n – 4)90 = 2520, n = 16

Q9 **a)** $(\frac{360}{5}) \div 2 = 36°$

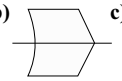
b) OX = 5 cos 36° = 4.045 cm.
Hence MX = 5 – 4.045 = 0.95 cm.

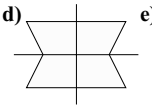
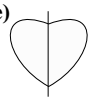
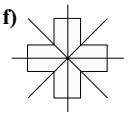
Q10 a)



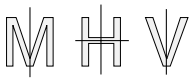
b) Angle CDE = angle DEF
$= \frac{(2 \times 8 - 4)90}{8} = 135$
so angle EFC = $\frac{360 - 2(135)}{2} = 45°$
OR exterior angle = 45° = angle EFC (alternate angles).

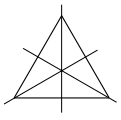
Page 77 — Symmetry

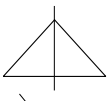
Q1 **a)**  **b)**  **c)** 

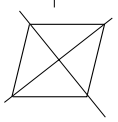
d)  **e)**  **f)** 

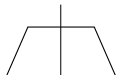
Q2 **a)** 6 **b)** 8 **c)** 5 **d)** 3

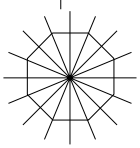
Q3 
1 2 1
Order of Rotation
1 1 2 2


Q4 **a)**  Order of Rotation = 3

b)  Order of Rotation = 1

c)  Order of Rotation = 2

d)  Order of Rotation = 1

e)  Order of Rotation = 8

f)  Order of Rotation = 2

Pages 78-80 — Circle Geometry

Q1 BD bisects AC and meets it at an angle of 90°, so BD must be a diameter of the circle.
So BD = 2 × 9 = 18 m

Q2 **a)** BD = 5 cm (as the tangents BD and CD are equal).
b) Angle COD = 70° (= 180° – (20° + 90°)), since the tangent CD meets the radius OC at an angle of 90°.
c) Angle COB = 140° (since angle BOD equals angle COD).

Q3 Both 90°

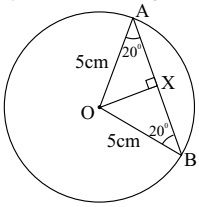
Q4 **a)** BAD = 80° (opposite angle C in cyclic quadrilateral)
b) EAB = 180 – 80 – 30 = 70°

Q5 **a)** BOE = 106° (angle at centre)
b) ACE = 32° (angle in opposite segment)

Q6 **a)** ACD = 70° (angle in opposite segment)
b) BAD = 180 – (30 + 70) = 80° (opposite angles of a cyclic quadrilateral total 180°)

Q7 **a)** Angles in the same segment.
b) 3x + 40 = 6x – 50
90 = 3x
30 = x
angle ABD = 3(30) + 40 = 130°

Q8 There are 2 ways of answering this question.



A diameter through O bisects the chord at X so cos 20° = $\frac{AX}{5} \Rightarrow$
AX = 4.698 and
AB = 9.40 cm.
or by the sine rule $\frac{AB}{\sin 140°} = \frac{5}{\sin 20°}$
AB = $\frac{5 \sin 140°}{\sin 20°}$ = 9.40 cm

Q9 **a)** Angle ACB is an angle in a semicircle, so it is a right angle.
So area of ABC = ½ × AC × BC
= ½AC² = 64 cm²
By Pythagoras,
AB² = AC² + BC² = 2AC²
= 4(½AC²)
= 4 × 64 = 256
So AB = $\sqrt{256}$ = 16 cm
b) BX = 3AX and
BX + AX = 16 cm, so BX = 12 cm and AX = 4 cm
AB and DE are intersecting chords, so AX × BX = DX × EX
12 × 4 = 6 × EX
EX = 48 ÷ 6 = 8 cm
So DE = 8 + 6 = 14 cm

Answers: P79 — P85

Q10 **a)** Angle ABD = 70° (angle at centre = 2 × angle at circumference)
b) Angle ABC = 90° (angle in semicircle)
c) Angle DBC = 20° (90° – 70°)

Q11 **a)** 90° (angle in a semicircle)
b) The angle at A = 90° (tangent and radius are perpendicular).
The third angle in the triangle is 180 – 90 – 23 = 67° and so $x = 90 - 67 = 23°$.
Or, by opposite segment theorem: x = angle ABC = 23°.

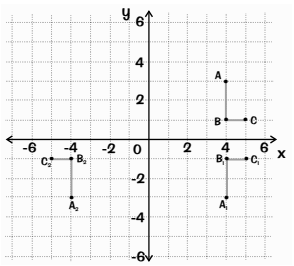
Q12 **a)** With AD as a chord, angle ABD = ACD = 30° (same segment); angle AXB = 85° (vertically opposite angles).
The third angles must be the same in both triangles so the triangles must be similar.
b) Ratio of lengths = $\frac{4}{8} = \frac{1}{2}$
so XB = 7.25 cm
c) angle BDC = 180 – 85 – 30 = 65°

Q13 **a)** 90° (angle in a semicircle)
b) Pythagoras is needed here:
$AC^2 + 3^2 = 10^2$
$AC^2 = 100 - 9 = 91$
AC = 9.54 cm
c) AD = 5 cm so DC = 9.54 – 5 = 4.54 cm then Pythagoras gives
$(4.54)^2 + 3^2 = (DOB)^2$
$20.606 + 9 = (DOB)^2$
So DOB = 5.44 cm

Q14 **a)** Both angles are 90° (angle in a semicircle)
b) 3 × 6.5 = 5 × BX
So BX = 3.9 cm
c) 2.5 × (2.5 + 3 + 6.5) = 6 × YF
So YF = 5 cm

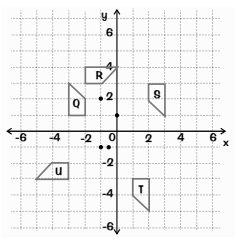
Pages 81-82 — The Four Transformations

Q1 **a)**, **b)**, **c)** — see diagram.



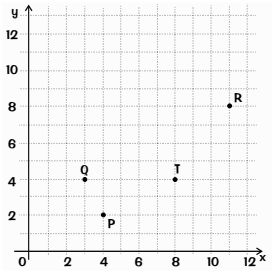
d) Rotation of +180° (or –180) about (0, 0)

Q2 **a)**, **b)**, **d)**, **e)** — see diagram



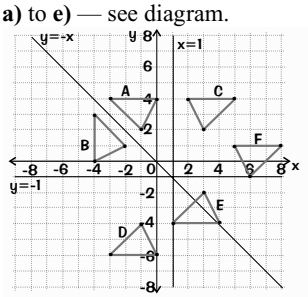
c) Rotation of +180° (or –180) about (0, 2).
f) 90° rotation anticlockwise about $\left(-\frac{1}{2}, -\frac{1}{2}\right)$.

Q3 **a)**



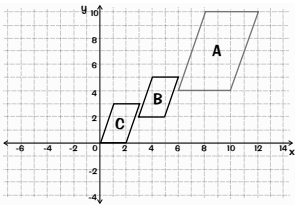
b) $\overrightarrow{QO} = \begin{pmatrix}-3\\-4\end{pmatrix}$
$T = \begin{pmatrix}11\\8\end{pmatrix} + \begin{pmatrix}-3\\-4\end{pmatrix} = \begin{pmatrix}8\\4\end{pmatrix}$
see diagram
c) $\begin{pmatrix}-1\\2\end{pmatrix} + \begin{pmatrix}8\\4\end{pmatrix} + \begin{pmatrix}-3\\-4\end{pmatrix} + \begin{pmatrix}-4\\-2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$

Q4 **a)** to **e)** — see diagram.



f) Rotation of +180° (or –180), centre (3, 0)

Q5 **a)**, **b)** — see diagram.

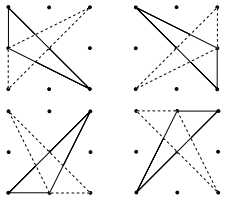


c) Ratio of areas C:A = 1:4

Pages 83-84 — Congruence, Similarity and Enlargement

Q1 **a)** Angle A shared. Parallel lines make corresponding angles equal so the triangles are similar.
b) Ratio of lengths given by
$\frac{AB}{AD} = \frac{12}{20} = \frac{3}{5}$
So $x = 25 \times \frac{3}{5} = 15$ cm
Also $\frac{y+10}{y} = \frac{5}{3}$
$\Rightarrow 2y = 30$, $y = 15$ cm

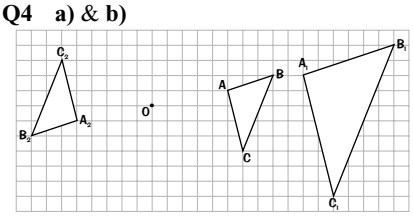
Q2



Hence 7 ways to draw another.

Q3 **a)** Triangles APQ and STC (both isosceles and share either angle A or C)
b) Ratio AC:AQ = 24:7.5 = 3.2:1 so
$AP = 15 \times \frac{1}{3.2} = 4.6875$ cm
PT = 24 – 2 (4.6875) = 14.625 cm
c) Using $\frac{1}{2}$(base)(height)
$= \frac{1}{2}(24)(9) = 108$ cm²
d) Scale factor = $\frac{1}{3.2}$
Area scale factor = $\frac{1}{10.24}$
Area of triangle APQ
$= 108 \times \frac{1}{10.24} = 10.5$ cm²
e) 108 – 2 (10.5) = 87 cm²

Q4 **a)** & **b)**



c) triangle $A_2B_2C_2$

Q5 Widths in ratio 2:3, so volumes in ratio 8:27.
Volume = $30 \times \frac{27}{8} = 101$ litres

Q6 **a)** All lengths must be enlarged in the same ratio for them to be similar.
b) 4 litres

Q7 **a)** 2 end faces 2 × (2 × 3) = 12 cm²
2 side faces 2 × (5 × 3) = 30 cm²
Top & bottom 2 × (5 × 2) = 20 cm²
Total = 62 cm²
b) SF for length = 1:4
SF for area = 1:16
new area = 62 × 16 = 992 cm²

Q8 **a)** volume = $\frac{1}{3}(\pi \times 100^2)(100)$
$= 1047198$ cm³ $= 1.05$ m³
b) 50 cm
c) ratio = $1:2^3 = 1:8$
d) Volume of small cone =
$1.05 \times \frac{1}{8} = 0.131$ m³
e) volume of portion left =
1.05 – 0.131 = 0.919
so ratio = 0.919:0.131 = $\frac{0.919}{0.131}$:1
= 7:1

Pages 85-87 — Perimeter and Area

Q1 Area 24 cm², perimeter 20 cm
Q2 Area 25 cm², perimeter 20 cm

Q3 **a)** Area = (4 × 4) – (1 × 2 + ½ × π × 1²)
+ ½ × π × 2²
= 16 – 3.5708 + 6.2832
= 18.7 m² (1 d.p.)
b) Three 1 litre tins of paint are needed for two coats.
c) Perimeter = 1 + 1 + (½ × π × 2)
+ 1 + 1 + 4 + (½ × π × 4) + 4
= 12 + 3π = 21.4 m (1 d.p.)

Q4 **a)** l = 24, w = 12, area = 288 m²

Answers: P85 — P91

b) 1 Carpet tile = 0.50 × 0.50 = 0.25 m²
So 288 m² ÷ 0.25 = 1152 tiles are required.
c) £4.99 per m² => £4.99 for 4 tiles
Total cost = (1152 ÷ 4) × 4.99 = £1437.12

Q5 Area = 120 cm²

Q6 Each square = 0.6 m × 0.6 m = 0.36 m².
Total area of material = 6 × 0.36 = 2.16 m²

Q7 Perimeter = $4 \times \sqrt{9000}$ = 379.47 m (2 d.p.)
Natasha ran: 11 × 379.47 = 4200 m (to nearest 100 m)

Q8 48 ÷ 5 = 9.6 m length. Area of 1 roll = 11 m × 0.5 m = 5.5 m².
48 m² ÷ 5.5 m² = $8\frac{8}{11}$ rolls of turf required. Of course 9 should be ordered.

Q9 Base length = 4773 ÷ 43 = 111 mm.

Q10 Area of metal blade = ½ × 35 × (70 + 155) = 3937.5 mm²

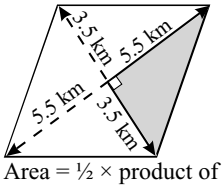
Q11 Area of larger triangle = ½ × 14.4 × 10 = 72 cm².
Area of inner triangle = ½ × 5.76 × 4 = 11.52 cm².
Area of metal used for a bracket = 72 – 11.52 = 60.48 cm² so NO, bracket is too heavy for the fixing.

Q12 T_1: ½ × 8 × 16 = 64 m²
Tr_1: ½ × 8 × (8 + 16) = 96 m²
Tr_2: ½ × 4 × (8 + 12) = 40 m²
T_2: ½ × 8 × 12 = 48 m²
Total area of glass sculpture = 248 m²

Q13 Area = ½ × 8.2 × 4.1 = 16.81 m²
Perimeter = 10.8 + 4.5 + 8.2 = 23.5 m.

Q14 **a)** Area of each isosceles triangle = ½ × 2.3 × 3.2 = 3.68 m²
b) Area of each side = $(\sqrt{3.2^2 + 1.15^2}) \times 4$ = 13.6 m²
Groundsheet = 2.3 × 4 = 9.2 m²
c) Total material = 2 × 3.68 + 9.2 + 2 × 13.6 = 43.8 m²

Q15



Area = ½ × product of diagonals = ½ × 7 × 11 = 38.5 km².

Q16 B = major sector
C = chord
D = tangent

Q17 **a)** 117.607 m² to 3 d.p.
b) 45.216 = 45 m to 2 s.f.
c) 46.5 m to 1 d.p.
d) 14.152 cm² to 3 d.p.
e) 12.0 cm² to 3 s.f.
f) 25.98 cm² to 2 d.p.

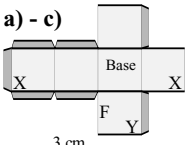
Q18 **a)** Area = area of a full circle radius 10 cm. A = πr^2 = 3.14 × 10² = 314 cm².
Circumference = π × D = 3.14 × 20 = 62.8 cm.
Perimeter = 62.8 + 20 = 82.8 cm
b) Area = (area of a full circle radius 15 cm) + (area of a rectangle 15 × 30 cm) = (π × 15²) + (15 × 30) = 1156.5 cm².
Perimeter = (Circumference of a full circle radius 15 cm) + 15 +15 (two shorter sides of rectangle) = (π × 30) + 30 = 124.2 cm.
c) Area = Outer semi circle – Inner semi circle = 510.25 m².
Perimeter = ½ Circumference of larger + ½ Circumference of inner + 5 + 5 = ½ × π × 70 + ½ × π × 60 + 10 = 214.1 m.

Q19 **a)** ABDC = $\frac{60}{360} \times \pi(30)^2 - \frac{60}{360} \times \pi(20)^2$
= 261.8 mm²
b) 2(½π5²) = 78.5 mm².
Hence 261.8 + 78.5 = 340.3 mm².

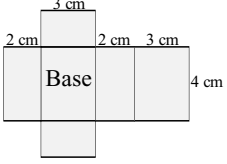
Q20 **a)** 80/360 × π5² = 17.45 cm²
b) Area of triangle AOB = $\frac{1}{2} \times 5 \times 5 \times \sin 80°$ = 12.31 cm²
Shaded Area = 17.45 – 12.31 = 5.14 cm²

Pages 88-89 — Surface Area

Q1 **a) - c)**



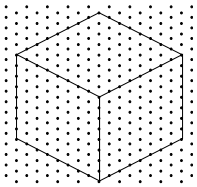
Q2



Other arrangements are possible.

Q3 **a)** H, F and D
b) Line symmetry through lines AF, DH, BG and CE. Rotational symmetry of order 4.
c) 5 faces and vertices, 8 edges.

Q4 **a)** I
b) 64 cm²
c) 64 × 6 = 384 cm²
d)



Q5 Net B

Q6 No, Hannah would need more than 603 cm².

Q7 Surface area = 4 × π × 3² = 113.10 cm² (to 2 d.p.)

Q8 Surface area of cone = $\pi rl + \pi r^2$
= (π × 1.5 × 8) + (π × 1.5²)
= 44.77 cm² (to 2 d.p)
Height of triangular prism = $\sqrt{3^2 - 1.5^2}$ = $\sqrt{6.75}$ = 2.598... cm
Surface area of triangular prism = $2(\frac{1}{2} \times 3 \times 2.598...) + 3(3 \times 8)$
= 79.79 cm² (to 2 d.p.)
Therefore the triangular prism has the largest surface area.

Q9 Surface area of hemisphere = $\pi r^2 + \frac{1}{2}(4\pi r^2)$
$75\pi = 3\pi r^2$
$r^2 = 25$, radius = 5 cm

Q10 AB² = 2² + 1.5² AB = 2.5 m
1 panel on roof = ½AB × $\frac{5}{2}$
= 1.25 × 2.5 = 3.125 m²
Front of greenhouse = (2.5 × 4) + (½ × 4 × 1.5) = 13 m²
Total = 3.125 + 13 = 16.125 m²

Pages 90-92 — Volume

Q1 **a)** $\frac{1}{2}\pi(0.35)^2$ = 0.192 m²
b) 0.1924 × 3 = 0.577 m³

Q2 **a)** π(2.5² – 2²) = 7.07 m²
£16 × 7.07 = £113.12 = £110 to nearest £10.
b) Volume = π(2)² × 0.50 = 6.28 m³
so use 6.28 × 15 = 94 ml treatment to the nearest ml.

Q3 **a)** Volume Cube = Volume Cylinder
$10^3 = \pi r^2 \times 10$ so $r^2 = \frac{10^2}{\pi}$,
r = 5.64 cm
b) S.A. of cylinder = $2\pi rh + 2\pi r^2$ = $2\pi \times 5.64... \times 10 + 2\pi \times (5.64...)^2$
= 554.49 cm²

Q4 **a)** π(5)²(16) = 1257 cm³
b) π(5)²h = 600
$h = \frac{600}{25\pi}$ = 7.64 cm

Q5 (3)(3)(0.5) – π(0.7)²(0.5) = 3.73 cm³

Q6 Volume = $\frac{1}{3} \times (230 \times 230) \times 139$
= 2 451 033 m³

Q7 (π × (2)² × 110) + (½(14 + 20) × 6 × 20) = 3422.30 cm³
2 × 3422.30 = 6844.60 cm³ = 6.84 l

Q8 **a)** (60)(30) + (30)(120) = 5400 cm²
b) 5400 × 100 = 540000 cm³ = 0.54 m³

Q9 Volume = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times 15^3$
= 14137 cm³

Q10 $$\frac{\text{volume of cylinder}}{\text{volume of hemisphere}} = \frac{\pi r^2 h}{\frac{1}{2} \times \frac{4}{3}\pi r^3}$$
$$= \frac{6\pi r^2 h}{4\pi r^3} = \frac{3h}{2r}$$
$$\frac{3h}{2r} = 3 \quad \Rightarrow \quad h = 2r$$

Answers: P92 — P98

Q11 a) $\frac{1}{2}(\frac{4}{3}\pi(1.3)^3) + \pi(1.3)^2 \times 1.8$
$+ \frac{1}{3}\pi(1.3)^2 \times 1.2 = 16.28$ cm^3

b) Volume of sand in hemisphere and cone parts remain the same so change is in cylindrical part.
Therefore $h + 0.3 = 1.8$,
$h = 1.5$ cm.

c) Volume of sand transferred =
$\frac{1}{2}(\frac{4}{3}\pi(1.3)^3) + \pi(1.3)^2 \times 1.5$
$= 12.57$ cm^3
Time Taken = $\frac{12.57}{0.05} \approx 251$ secs.
= 4 minutes 11 secs

Q12 a) Volume of ice cream
$= \frac{1}{3}\pi(R^2H - r^2h) + \frac{1}{2}(\frac{4}{3}\pi R^3)$
$= \frac{1}{3}\pi(2.5^2 \times 10 - 1^2 \times 4)$
$+ \frac{1}{2}(\frac{4}{3}\pi \times 2.5^3)$
$= 93.99$ cm^3 of ice cream.

b) Outer surface area of cone
$= \pi Rl$
Using Pythagoras,
$l^2 = 10^2 + 2.5^2 = 106.25$,
$l = 10.3$ cm. So S.A. =
$\pi \times 2.5 \times 10.3 = 81.0$ cm^2

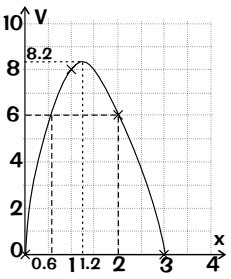
Q13 Vol. increase is a cylinder of height 4.5 cm. So vol. increase =
$\pi(5)^2 \times 4.5 = 353.4$ cm^3.
Volume of each marble = $\frac{353.4}{200}$
$= 1.767$ cm^3
$\frac{4}{3}\pi r^3 = 1.767 \Rightarrow r = 0.75$ cm

Q14 a) $x(3 - x)(5 - x)$ m^3 or $x^3 - 8x^2 + 15x$

b)

x	0	1	2	3
V	0	8	6	0

c)



d) about 8.2 m^3

e)

ends	2(1.2)(1.8) =	4.32 +
side faces	2(1.2)(3.8) =	9.12 +
tops	2(3.8)(1.8) =	13.68
So area is about		27.12 m^2

f) $x = 2$ or $x = 0.6$
If $x = 0.6$:

ends	2(0.6)(2.4) =	2.88 +
side faces	2(0.6)(4.4) =	5.28 +
tops	2(2.4)(4.4) =	21.12
		29.28 m^2

If $x = 2$:

ends	2(2)(1) =	4 +
side faces	2(2)(3) =	12 +
tops	2(1)(3) =	6
		22 m^2

Maximum Total S.A. ≈ 29.28 m^2

Page 93 — Time

Q1 **a)** 5 am **b)** 2.48 pm **c)** 3.16 am **d)** 3.58 pm **e)** 10.30 pm **f)** 12.01 am

Q2 **a)** 2330 **b)** 1022 **c)** 0015 **d)** 1215 **e)** 0830 **f)** 1645

Q3 145 mins

Q4 **a)** 8 hours **b)** 10 hours **c)** 11 hrs 56 mins **d)** 47 hrs 48 mins

Q5 **a)** 3 hrs 15 mins **b)** 24 mins **c)** 7 hrs 18 mins **d)** 1 hr 12 mins

Q6 **a)** 2.33 hrs **b)** 3.1 hrs **c)** 0.33 hrs

Q7 **a)** Train 3 **b)** Train 1 **c)** 1208

Pages 94-96 — Speed, Density and Pressure

Q1 **a)** 100/11= 9.09 m/s (to 2 d.p)
b) 32.73 km/h

Q2 540 km/h

Q3 Journey takes 3 hrs 39 mins.
07.05 to 10.30 is 3 hrs 25 mins.
So Pete will not be in London on time.

Q4 **a)** 98.9 km/h **b)** 72.56 s **c)** 99.2 km/h

Q5 **a)** 5 hrs 31 mins 30 s **b)** 405 km **c)** 73.3 km/h

Q6 2.15pm

Q7 **a)** 2.23 hrs (2 hrs 14 mins)
b) 1 hr 49 mins + 10 mins
= 1 hr 59 mins
c) 1346 and 1401

Q8 The first athlete ran at 16000 ÷ (60 × 60) = 4.44 m/s, so was faster than the second athlete (at 4 m/s). The first athlete would take 37.5 mins to run 10 km; the second would take 41.7 mins.

Q9 **a)** 487.5 km
b) 920.8 km
c) 497.1 km/h

Q10 **a)** 8.13 m/s
b) 7.30 m/s

Q11 **a)** 220 km
b) 5 mins

Q12 1 hr 27 mins.

Q13 **a)** 4.9 m/s
b) 24.5 m/s
c) 14.7 m/s
d) 17.64 km/h, 88.2 km/h, 52.92 km/h.

Q14 **a)** 0.75 g/cm^3
b) 0.6 g/cm^3
c) 0.8 g/cm^3
d) 700 kg/m^3 = 0.7 g/cm^3

Q15 **a)** 62.4 g
b) 96 g
c) 3744 g (3.744 kg)
d) 75 g

Q16 **a)** 1176 cm^3
b) 278 cm^3 (to 3 s.f.)
c) 2500 cm^3
d) 45 500 cm^3 (to 3 s.f.)

Q17 34.71 g

Q18 20968 cm^3

Q19 Vol. = 5000 cm^3 = 5 litres

Q20 1.05 g/cm^3

Q21 **a)** SR flour 1.16 g/cm^3;
granary flour 1.19 g/cm^3
b) 378 ml

Q22 165.23 g

Q23 150 N/m^2

Q24 0.8 m^2

Q25 448 N

Q26 30.59 cm

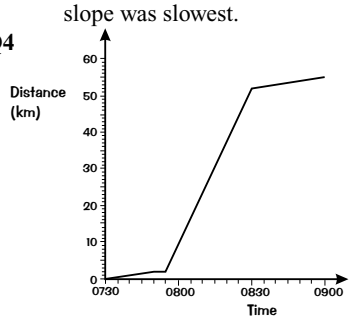
Pages 97-98 — D/T and S/T Graphs

Q1 **a)** 4 km
b) 15 mins and 45 mins
c) 2.4 km/h
d) 1100
e) 10 km/h
f) 1030

Q2 **a)** 1 hr 25 mins
b) 1 hr 15 mins
c) 25.4 km/h
d) 86.4 km/h
e) No. Can't get to Ingleton and back.

Q3 **a)** A 80.0 km/h, fastest.
B 57.1 km/h
C 66.7 km/h
D 44.4 km/h
E 50.0 km/h
b) steepest slope was fastest, least steep slope was slowest.

Q4

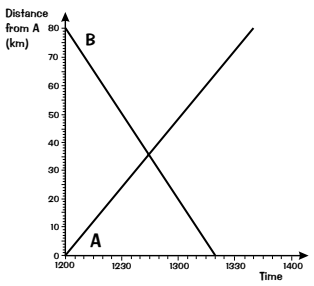


He waited for 5 mins.

Q5 **a)** 3 hours
b) 4 to 6 hours into the journey
c) Travelling at a constant speed of 15 km/h

Q6 **a)** acceleration
= change in speed ÷ time
= (100 – 60)/1 = 40 km/h^2
b) The first hour.

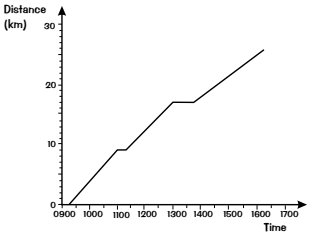
Q7 **a)**



b) accept 1243-1245
c) accept 35-36 km

Answers: P98 — P105

Q8 a)



b) 25.75 km **c)** 3.68 km/h
d) Her fastest speed was in the first section (steepest graph) — her speed was 5.14 km/h.

Page 99 — Unit Conversions

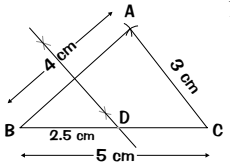
Q1 **a)** 200 cm **i)** 6000 mm
b) 33 mm **j)** 2000 kg
c) 4000 g **k)** 3 kg
d) 0.6 kg **l)** 86 mm
e) 0.65 km **m)** 0.55 tonnes
f) 9000 g **n)** 354 cm
g) 0.007 kg **o)** 7 mm
h) 0.95 kg **p)** 4.2 l
Q2 **a)** 0.47 m **b)** 470 mm
Q3 **a)** 300 cm **c)** 0.003 km
b) 3000 mm
Q4 **a)** 0.2 km **c)** 7 km
b) 2 km **d)** 0.02 km^2
Q5 **a)** 167 cm **c)** 0.11 cm^2
b) 33.3 cm **d)** 0.056 cm^2
Q6 **a)** £4.69 **b)** £51.07

Page 100 — Conversion Graphs

Q1 **a) i)** £5 **ii)** £9.50 **iii)** £17
b) No (each 4.5 km journey costs more than £8)
Q2 **a)** \$4.50 **c)** £2
b) \$2.25 **d)** £3.65 (+/– £0.05)
Q3 **a) i)** 12-13 miles
ii) 43-44 miles
iii) 56-57 miles
b) i) 63-65 km
ii) 15-17 km
iii) 47-49 km

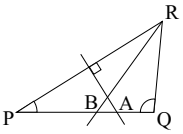
Page 101 — Constructions

Q1



Not to scale

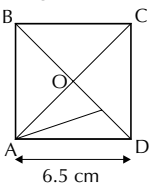
Q2



Not to scale

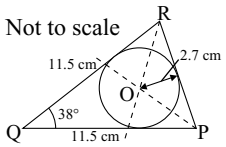
Length BA = 0.87 cm

Q3



Not to scale

Q4

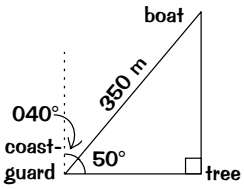


Radius of the circle = 2.7 cm

Page 102 — Bearings

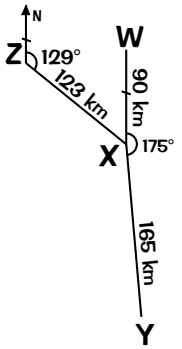
Q1 **a)** 245° **b)** 310°
c) 035° **d)** 131°
e) 297°, 028°, 208°

Q2 **a)**



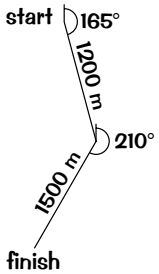
i) 268 m
ii) 225 m
b) $350^2 = 122\,500$.
$225^2 + 268^2 = 122\,449$

Q3



a) 96 km
b) 255 km
c) 266 km
d) 156°
e) 082°
f) 177°

Q4



2500 m, 010°

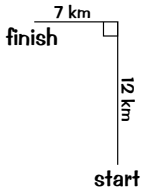
Section Five — Pythagoras and Trigonometry

Pages 103-105 — Pythagoras' Theorem

Q1 **a)** 10.8 cm **f)** 7.89 m
b) 6.10 m **g)** 9.60 cm
c) 5 cm **h)** 4.97 cm
d) 27.0 mm **i)** 6.80 cm
e) 8.49 m **j)** 8.5 cm
Q2 a = 3.32 cm f = 8.62 m
b = 6 cm g = 6.42 m
c = 6.26 m h = 19.2 mm
d = 5.6 mm i = 9.65 m
e = 7.08 mm j = 48.7 mm
Q3 k = 6.55 cm q = 7.07 cm
l = 4.87 m r = 7.50 m
m = 6.01 m s = 9.45 mm
n = 12.4 cm t = 4.33 cm
p = 5.22 cm u = 7.14 m
Q4 9.7 m
Q5 **a)** 12 cm, 7.94 cm
b) 40.9 cm
c) 89.7 cm^2
Q6 314 m
Q7 91.9 cm
Q8 5.0 m
Q9 4.58 m
Q10 AB: 5 (don't need Pythagoras)
CD: $\sqrt{10} = 3.16$
EF: $\sqrt{13} = 3.61$
GH: $\sqrt{8} = 2.83$
JK: $\sqrt{5} = 2.24$
LM: $\sqrt{26} = 5.10$
PQ: $\sqrt{20} = 4.47$
RS: $\sqrt{45} = 6.71$
TU: $\sqrt{13} = 3.61$
Q11 **a) i)** 5
ii) $\sqrt{17} = 4.12$
iii) 5
iv) $\sqrt{58} = 7.62$
v) $\sqrt{26} = 5.10$
b) parallelogram
Q12 **a)** $\sqrt{41} = 6.40$
b) $\sqrt{98} = 9.90$
c) $\sqrt{53} = 7.28$
d) $\sqrt{34} = 5.83$
e) 4 (don't need Pythagoras here)
f) $\sqrt{37} = 6.08$
Q13 **a)** $\sqrt{10} = 3.16$
b) $\sqrt{130} = 11.40$
c) $\sqrt{8} = 2.83$
d) $\sqrt{233} = 15.26$
e) $\sqrt{353} = 18.79$
f) $\sqrt{100} = 10$
Q14 192 km

Answers: P105 — P110

Q15



13.9 km from the starting point.
150° to return to base.

Pages 106-108 — Trigonometry — Sin, Cos, Tan

		(tan)	(sin)	(cos)
Q1	**a)**	0.306	0.292	0.956
	b)	8.14	0.993	0.122
	c)	0.0875	0.0872	0.996
	d)	0.532	0.469	0.883
	e)	1	0.707	0.707

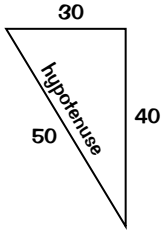
Q2 a = 1.40 cm c = 5.31 cm
b = 6 cm d = 10.8 cm
θ = 28.1°

Q3 e = 12.6 cm g = 6.71 m
f = 11.3 cm h = 30.1 cm
θ = 49.5°

Q4 i = 4.89 cm k = 5.32 cm
j = 3.79 cm l = 41.6 cm
θ = 52.4°

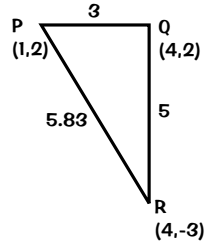
Q5 m = 11.3 cm t = 59.8 cm
n = 18.8 cm u = 14.5 cm
p = 8.62 cm v = 11.7 cm
q = 21.3 cm w = 11.7 cm
r = 54.6°

Q6 **a)**



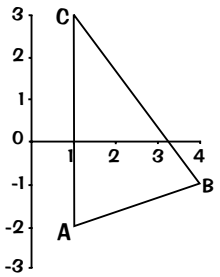
b) 36.9°

Q7 **a)**



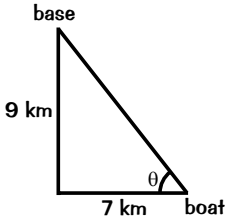
b) 59.0°
c) 31.0°

Q8 **a)**



b) 71.6°
c) 36.9°
d) 71.5°

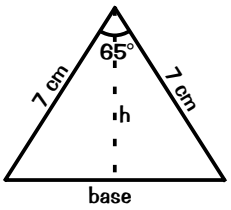
Q9 2.1 m
Q10 62°
Q11 20.5°
Q12



θ = 52.1°, bearing = 322°

Q13 **a)** both 30.8 cm
b) 27.5 cm **c)** 385 cm²

Q14

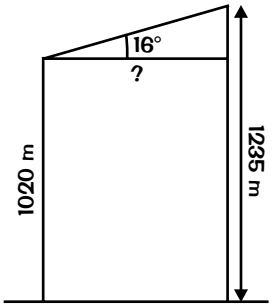


height = 5.90, base = 7.52,
so area = 22.2 cm².

Q15 **a)** 8.23 cm
b) 4.75 cm **c)** 39.1 cm²

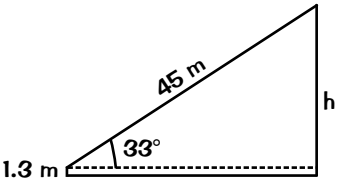
Q16 **a)** 10.8 cm
b) 150.8 cm² **c)** 21.0°

Q17



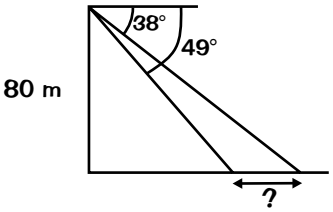
750 m

Q18



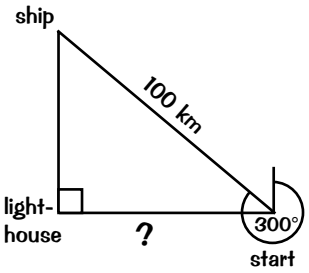
25.8 m

Q19



a) 102.4 m, 69.5 m
b) 32.9 m

Q20



86.6 km

Pages 109-110 — The Sine and Cosine Rules

Q1 a = 4.80 cm f = 5.26 cm
b = 25.8 mm g = 9.96 cm
c = 13.0 cm h = 20.2 mm
d = 8.89 m i = 3.72 m
e = 18.4 cm j = 8.29 cm

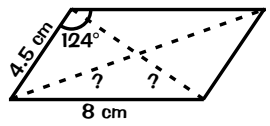
Q2 k = 51° r = 64°
l = 46° s = 18°
m = 43° t = 49°
p = 45° u = 88°
q = 36°

Q3 **a)** 46°
b) 52° **c)** 82°

Q4 **a)** 18.1 cm² **d)** 29.5 m²
b) 8.5 m² **e)** 25.5 cm²
c) 198.6 cm² **f)** 17.4 mm²

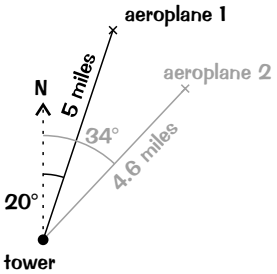
Q5 **a)** 28.8 km **b)** 295.5°

Q6



Diagonals 11.2 cm and 6.6 cm.

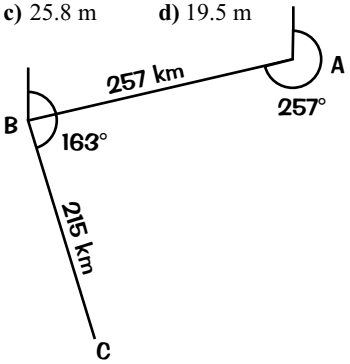
Q7



Distance = 1.2 miles.
The alarm should be ringing because the planes are less than 3 miles apart, so the software seems reliable.

Q8 **a)** 16.9 m **b)** 12.4 m
c) 25.8 m **d)** 19.5 m

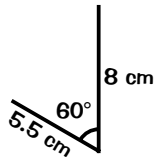
Q9



Answers: P110 — P116

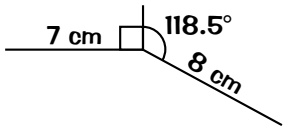
a) 86°
b) 323 km
c) 215°

Q10 a)



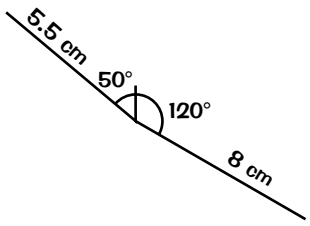
7.1 cm

b)



14.5 cm
(118.5° comes from the fact that the minute hand is at 19.75 mins. 19.75 ÷ 60 × 360 = 118.5.)

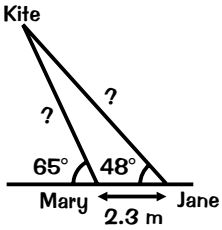
c)



13.5 cm

Q11 Height of building = 35 m

Q12



Mary's string = 5.85 m
Jane's string = 7.13 m

Page 111 — 3D Pythagoras and Trigonometry

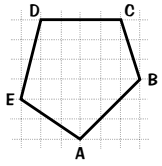
Q1 **a)** 59.0° **c)** 25 cm
b) 23.3 cm **d)** 21.1°
Q2 **a)** 42.5 cm **b)** 50.9 cm
Q3 **a)** 36.1 cm, 21.5 cm, 31.0 cm
b) 36.9 cm
Q4 **a)** 15.4 cm **b)** 20.4 cm
Q5 The 85p box
Q6 **a)** 3.82 cm
b) 45.8 cm^2
c) 137.5 cm^3

Page 112 — Sin, Cos and Tan for Larger Angles

Q1 $a = 5.7$ cm $e = 13.0$ cm
$B = 38.9°$ $F = 62.6°$
$c = 8.2$ cm $G = 115.4°$
$D = 140°$
Q2 **a)** 122.9° **c)** 135°
b) 170.0° **d)** 94.0°
Q3 32.1 m
Q4 **a)** 109°
b) Front = 28.5 m, roof = 107.6 m
c) 24.2 m
Q5 24.6 km
Q6 42.5° and 137.5°
Q7 153.5°

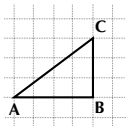
Pages 113-114 — Vectors

Q1 **a)**



b) i) $\begin{pmatrix}-1\\-4\end{pmatrix}$ **ii)** $\begin{pmatrix}4\\0\end{pmatrix}$ **iii)** $\begin{pmatrix}5\\4\end{pmatrix}$
c) Isosceles

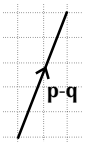
Q2 **a)**



b) Using Pythagoras,
$|\overrightarrow{AC}| = \sqrt{4^2 + 3^2} = 5$

Q3 **a)** $\begin{pmatrix}2\\1\end{pmatrix}$ **p+q**

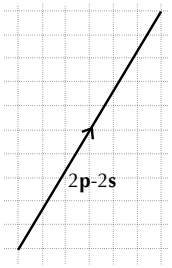
b) $\begin{pmatrix}2\\5\end{pmatrix}$



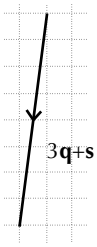
c) $\begin{pmatrix}6\\-2\end{pmatrix}$

d) $\begin{pmatrix}1\\1\end{pmatrix}$ **s+p**

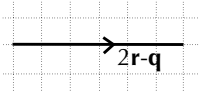
e) $\begin{pmatrix}6\\10\end{pmatrix}$



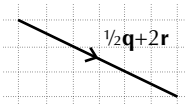
f) $\begin{pmatrix}-1\\-8\end{pmatrix}$



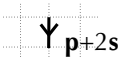
g) $\begin{pmatrix}6\\0\end{pmatrix}$



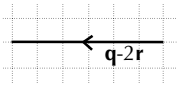
h) $\begin{pmatrix}6\\-3\end{pmatrix}$



i) $\begin{pmatrix}0\\-1\end{pmatrix}$



j) $\begin{pmatrix}-6\\0\end{pmatrix}$



Q4 **a)** 1 **f)** 5
b) 3.61 **g)** 8.60
c) 1 **h)** 8.49
d) 3.61 **i)** 9.43
e) 6.08 **j)** 11.18
Q5 **a)** $\begin{pmatrix}3\\3\end{pmatrix}$ **d)** 5.39
b) 4.24 **e)** $\begin{pmatrix}2\\-2\end{pmatrix}$
c) $\begin{pmatrix}5\\-2\end{pmatrix}$ **f)** 2.83
Q6 **a)** $2y$ **d)** $2y + 2x$
b) $y + x$ **e)** $4y + 2x$
c) $-y - x$ **f)** $2x$
Q7 **a) i)** $\overrightarrow{ED}$ or $\overrightarrow{AF}$ **v)** $\overrightarrow{BE}$
ii) $\overrightarrow{EF}$ or $\overrightarrow{DC}$ **vi)** $\overrightarrow{AC}$
iii) $\overrightarrow{AE}$ **vii)** $\overrightarrow{EC}$ or $\overrightarrow{AB}$
iv) $\overrightarrow{BA}$ **viii)** $\overrightarrow{EB}$
b) i) 48 cm^2 **ii)** 60 cm^2

Q8 $\overrightarrow{EG} = -\frac{3}{5}\underset{\sim}{a} + \frac{6}{5}\underset{\sim}{b}$

Q9 **a) i)** $2\mathbf{a}$ **ii)** $\mathbf{b} - 2\mathbf{a}$ **iii)** $\mathbf{a} - \mathbf{b}$
b) $\overrightarrow{AC} = -2\mathbf{b} + 2\mathbf{a} = 2(\mathbf{a} - \mathbf{b})$.
Since $\overrightarrow{AC}$ is a multiple of $\overrightarrow{PQ}$, they must both be in the same direction and therefore parallel.

Section Six — Statistics and Probability

Pages 115-116 — Mean, Median, Mode and Range

Q1 3 tries
Q2 mean = 1.333 (to 3 dp)
median = 1.5
mode = 2
range = 11
Q3 **a)** mean = £12,944, or £13,000 to the nearest £500
median = £12,000
mode = £7,500
b) mode
c) E.g. mean — they should use the highest value to attract people to the job.
Q4 **a)** 0 minutes **b)** 0 minutes
c) 0 minutes
d) No, according to the raw data.
Q5 73.5 kg
Q6 20 kg
Q7 97%
Q8 **a)** 22 **b)** 74
Q9 **a)** 3.5 **b)** 3.5 **c)** 5

Answers: P116 — P122

Q10 **a)** Both spend a mean of 2 hours.
b) The range for Jim is 3 hours and for Bob is 2 hours.
c) The amount of TV that Jim watches each night is more variable than the amount that Bob watches.

Q11 **a)** 1 day
b) 2 days
c) The statement is true according to the data.

Q12 **a)** mode
b) median **c)** mean

Page 117 — Quartiles and Comparing Distributions

Q1 **a)** 65 g **b)** The 2nd quartile (or Q_2)

Q2 **a)** 1020 – 80 = 940
b) 510 **c)** 700 **d)** 840

Q3 200

Q4 **a)** 325 **b)** 50

Q5 Mean (before)
= 3·61 fillings per child
Mean (after)
= 2·08 fillings per child
Mode (before)
= 4 fillings per child
Mode (after)
= 2 fillings per child
(all other things being equal, I'd say that the dental hygienist has decreased the number of fillings received by each child.)

Pages 118-119 — Frequency Tables — Finding Averages

Q1 **a)** 12 **b)** 12 **c)** 2

Q2 **a)**

Subject	M	E	F	A	S
Frequency	5	7	3	4	6

b) 36 French lessons **c)** English

Q3

Length (m)	4 and under	6	8	10	12	14 and over
Frequency	3	5	6	4	1	1

a) 8 m **b)** 8 m **c)** 14 m

Q4

Weight (kg)	Frequency	Weight × Frequency
51	40	2040
52	30	1560
53	45	2385
54	10	540
55	5	275

a) 52 kg **b)** 2 kg
c) 53 kg **d)** 52 kg (to nearest kg)

Q5 mean = 3.75
mode = 3
median = 4

Q6 **a)** 4 **b)** 3 **c)** 3.2 (to 1 dp)

Q7 **a) i)** False, mode is 8.
ii) False, they are equal.
iii) True
b) iv)

Page 120 — Grouped Frequency Tables

Q1 **a)**

Speed (km/h)	40≤s<45	45≤s<50	50≤s<55	55≤s<60	60≤s<65
Frequency	4	8	10	7	3
Mid-Interval	42.5	47.5	52.5	57.5	62.5
Frequency × Mid-Interval	170	380	525	402.5	187.5

Estimated mean = 52 km/h
(to nearest km/h)

b) 22 skiers **c)** 20 skiers

Q2 **a)**

Weight (kg)	Tally	Frequency	Mid-Interval	Frequency × Mid-Interval
200≤w<250	IIII	4	225	900
250≤w<300	~~IIII~~	5	275	1375
300≤w<350	~~IIII~~ II	7	325	2275
350≤w<400	II	2	375	750

b) 294 kg (to nearest kg)
c) 300 ⩽ w < 350 kg

Q3 **a)**

Number	0≤n<0.2	0.2≤n<0.4	0.4≤n<0.6	0.6≤n<0.8	0.8≤n<1
Tally	~~IIII~~ ~~IIII~~ II	~~IIII~~ I	~~IIII~~ ~~IIII~~ II	~~IIII~~ ~~IIII~~	~~IIII~~ III
Frequency	12	6	12	10	8
Mid-Interval	0.1	0.3	0.5	0.7	0.9
Frequency× Mid-Interval	1.2	1.8	6	7	7.2

b) 0 ⩽ n < 0.2 and 0.4 ⩽ n < 0.6
c) 0.4 ⩽ n < 0.6
d) 0.483 (3 dp)

Pages 121-122 — Cumulative Frequency

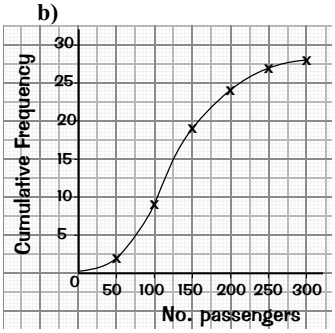
Q1 accept:
a) 133-134 **c)** 136-137
b) 127-128 **d)** 8-10

Q2 **a)**

Number of passengers	0≤n<50	50≤n<100	100≤n<150	150≤n<200	200≤n<250	250≤n<300
Frequency	2	7	10	5	3	1
Cumulative Frequency	2	9	19	24	27	28
Mid-Interval	25	75	125	175	225	275
Frequency × Mid-Interval	50	525	1250	875	675	275

Estimated mean = 130 passengers
(to nearest whole number)

b)

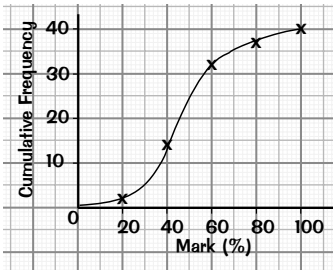


accept median of 118-122 passengers

c) 100 ⩽ n < 150

Q3 **a)**

Mark (%)	0≤m<20	20≤m<40	40≤m<60	60≤m<80	80≤m<100
Frequency	2	12	18	5	3
Cumulative Frequency	2	14	32	37	40

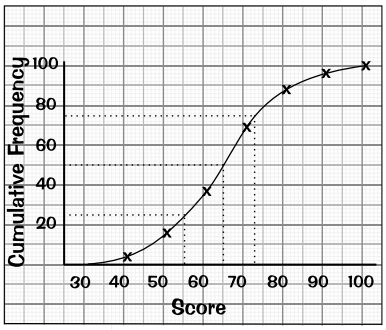


b) 36%-38%
c) 19%-21%
d) 45%-47%

Q4

Score	31≤s<41	41≤s<51	51≤s<61	61≤s<71	71≤s<81	81≤s<91	91≤s<101
Frequency	4	12	21	32	19	8	4
Cumulative frequency	4	16	37	69	88	96	100

a) 61 ⩽ s < 71
b) 61 ⩽ s < 71
c)



median = 65 (accept 64-66)
d) 73 – 55 = 18 (accept 17-19)

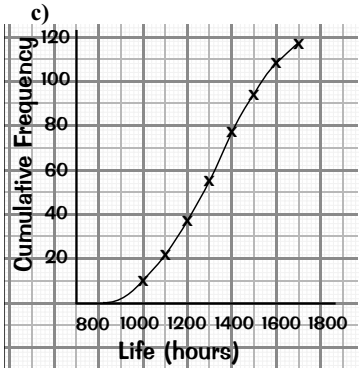
Q5 **a)**

Life (hours)	Frequency	Cumulative Frequency
900 ⩽ L < 1000	10	10
1000 ⩽ L < 1100	12	22
1100 ⩽ L < 1200	15	37
1200 ⩽ L < 1300	18	55
1300 ⩽ L < 1400	22	77
1400 ⩽ L < 1500	17	94
1500 ⩽ L < 1600	14	108
1600 ⩽ L < 1700	9	117

b) 1300 ⩽ L < 1400

Answers: P122 — P127

c)



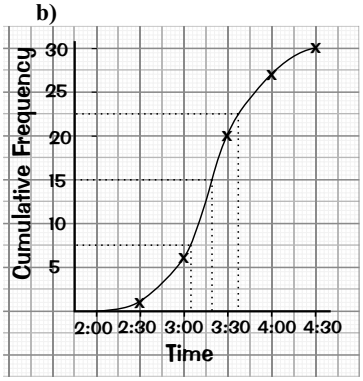
median = 1320 hours (±20)

d) lower quartile = 1150 (±20)
upper quartile = 1460 (±20)

Q6 a)

Time	2:00≤t<2:30	2:30≤t<3:00	3:00≤t<3:30	3:30≤t<4:00	4:00≤t<4:30
Tally	1	𝍸	𝍸 |||	𝍸 ||	|||
Frequency	1	5	14	7	3
Cumulative frequency	1	6	20	27	30

b)



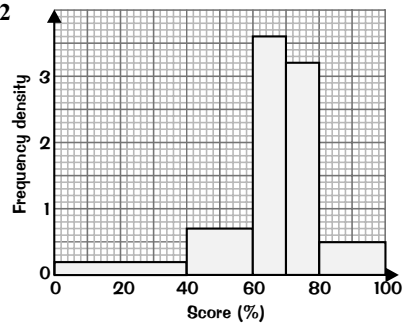
c) median = 3:19 (±3)
upper quartile = 3:37 (±3)
lower quartile = 3:05 (±3)

d) 0:32 (±5)

Pages 123-124 — Histograms and Frequency Density

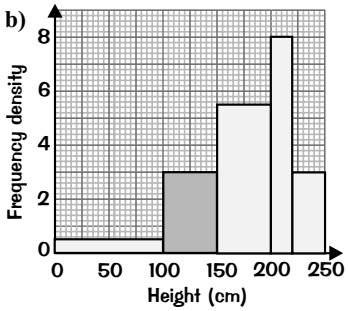
Q1 4 × 10 = 40 people

Q2



Q3 a) Frequency for $150 < x \leqslant 200$ = 275

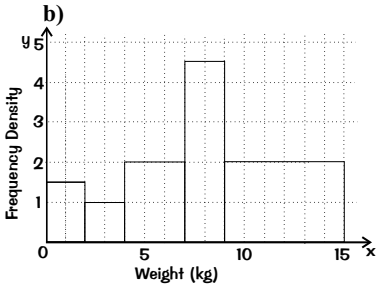
b)



Q4 a)

Weight (kg)	0≤w<2	2≤w<4	4≤w<7	7≤w<9	9≤w<15
Frequency	3	2	6	9	12
Frequency density	1.5	1	2	4.5	2

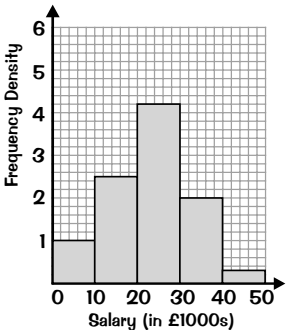
b)



c) 23 hives

Q5 a)

Salary (£1000s)	0 ≤ s < 10	10 ≤ s < 20	20 ≤ s < 30	30 ≤ s < 40	40 ≤ s < 50
Frequency	10	25	42	20	3
Frequency Density	1	2.5	4.2	2	0.3



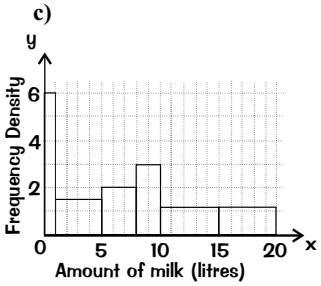
b) E.g. there are more people with higher salaries now than 10 years ago.

Q6 a)

Milk (litres)	Frequency	Frequency density	Mid-interval	Frequency × mid-interval
0≤C<1	6	6	0.5	3
1≤C<5	6	1.5	3	18
5≤C<8	6	2	6.5	39
8≤C<10	6	3	9	54
10≤C<15	6	1.2	12.5	75
15≤C<20	6	1.2	17.5	105

b) 8.2 litres (to 1 d.p.)

c)



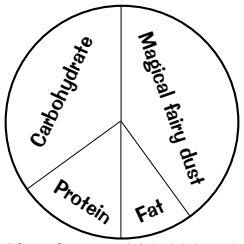
d) 18 days

Pages 125-127 — Other Graphs and Charts

Q1 $\frac{360°}{100}$ = 3.6° per gram

Carbohydrate	3.6 × 35 = 126°
Protein	3.6 × 15 = 54°
Fat	3.6 × 10 = 36°
Magical Fairy Dust	3.6 × 40 = 144°

360°



Q2 Sherrington 380,000 = 148º (approx)
2600 visitors = 1º
So, to the nearest 10,000:
Brompton = 2600 × 118º ≈ 310,000
Barny = 2600 × 44º ≈ 110,000
Livsea = 2600 × 50º ≈ 130,000

Q3 **c)**

Q4 It's not possible to tell whether more people voted for the Green Party in 2009, because you can't tell how many people voted in either election.

Q5 **a)** 60 **b)** 8 **c)** 4
d) e.g. Cola was the most popular and milk the least popular / cola was much more popular than milk.

Q6 **a)** Monday, Wednesday and Thursday.
b) Monday

Q7 **a)** 60% **b)** About 10%

Q8

	Van	Motor-bike	Car	Total
Travelling North	15	12	21	48
Travelling South	20	9	23	52
Total	35	21	44	100

a) 35 **c)** 9
b) 52 **d)** 21

Q9 **a)**

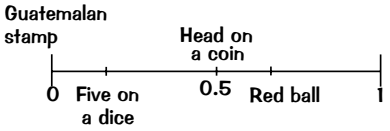
	Wings	No wings	Total
Antennae	6	4	10
No antennae	6	9	15
Total	12	13	25

b) 9
c) 25

Answers: P128 — P132

Pages 128-132 — Probability

Q1 **a)** 1/2 **c)** 1/6
b) 2/3 **d)** 0
And so should be arranged approximately like this on the number line.



Q2 Debbie's chance of winning would be 1/9. This is greater than 0.1, so she would choose to play.

Q3 The probability of a head is still 1/2

Q4 1 – 0.27 = 0.73 or 73/100

Q5 **a)** 5/12 **c)** 3/12 = 1/4
b) 4/12 = 1/3 **d)** 9/12 = 3/4

Q6 **a)** 40/132 = 10/33
b) P(car being blue or green) = 45/132
P(not blue or green) = 87/132
= 29/44

Q7

	1	2	3	4	5
1	1,1	1,2	1,3	1,4	1,5
2	2,1	2,2	2,3	2,4	2,5
3	3,1	3,2	3,3	3,4	3,5
4	4,1	4,2	4,3	4,4	4,5
5	5,1	5,2	5,3	5,4	5,5
6	6,1	6,2	6,3	6,4	6,5

Q8 **a)**

Outcome	Frequency
W	8
D	5
L	7

b) The 3 outcomes are not equally likely.
c) 1/4
d) They are most likely to win.
e) 14

Q9 **a)** $\frac{1}{13}$ **b)** $\frac{2}{39}$ **c)** $\frac{1}{36}$

Q10 **a)** $\frac{7}{12}$ **b)** $\frac{7}{12}$
c) The two events can both happen at the same time, since 3 is a white.

Q11 **a)** $\frac{2}{5}$ **b)** $\frac{4}{15}$ **c)** $\frac{2}{3}$

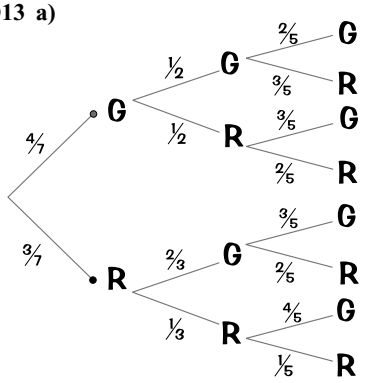
Q12 **a)** (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (2,7), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (3,7)

b)

	1	2	3	4	5	6	7
1	2	3	4	5	6	7	8
2	3	4	5	6	7	8	9
3	4	5	6	7	8	9	10

c) $\frac{1}{7}$ **d)** $\frac{11}{21}$
e) $\frac{2}{7}$ **f)** $\frac{5}{7}$
g) Subtract the answer to part **e)** from 1.

Q13 **a)**

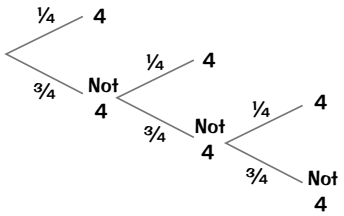


b) $\frac{18}{35}$
c) $\frac{3}{7}$

Q14 **a)** 14/40 or 0.35
b) 24/60 = 0.4
c) 38/100 = 0.38

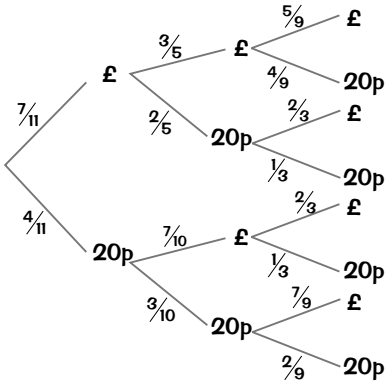
Q15 4 times

Q16



a) $\frac{3}{16}$ **b)** $\frac{37}{64}$

Q17 **a)**

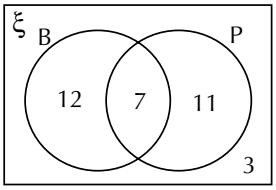


b) $\frac{28}{55}$ **c)** $\frac{46}{165}$

Q18 **a)** $\frac{1}{4}$
b) $\frac{1}{2}$ **c)** $\frac{1}{2}$

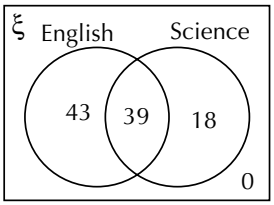
Q19 $\frac{1}{28}$

Q20 **a)**



b) i) $\frac{19}{33}$
ii) $\frac{15}{33} = \frac{5}{11}$

Q21 **a)**



b) $\frac{39}{57} = \frac{13}{19}$

Q22 **a)** $x^2 + 2x + 22 = 121$
So $x^2 + 2x - 99 = 0$
i.e. $(x + 11)(x - 9) = 0$
This means $x = 9$.
b) $\frac{22}{121} = \frac{2}{11}$
c) $\frac{2}{7}$

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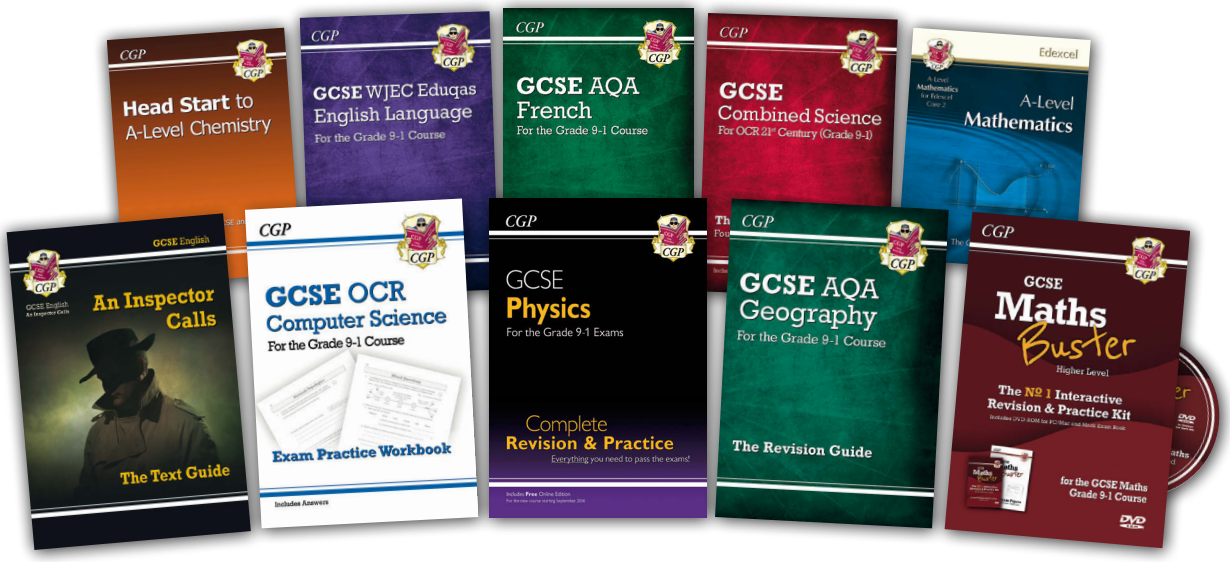

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