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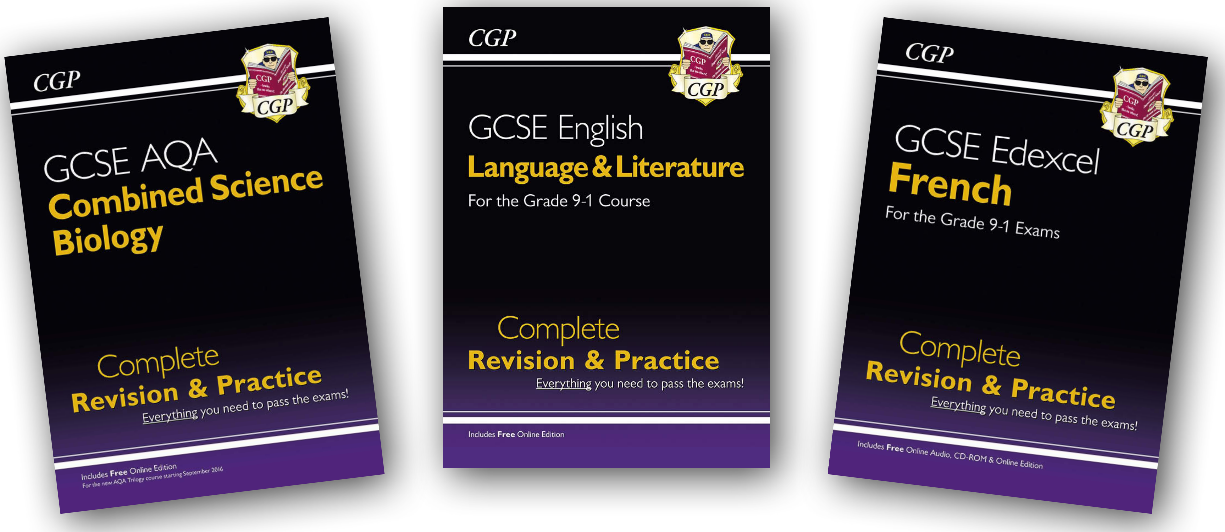

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GCSE
Mathematics
Foundation Level

There's a lot to get your head around in GCSE Maths, that's for sure. But don't worry — this life-saving CGP book is packed with brilliant notes and worked examples that make every topic easy to understand.

What's more, every section is rounded off with warm-up questions and exam-style practice — there's even a full set of mock exams at the end of the book!

Complete
Revision & Practice

Everything you need to pass the exams!

Contents

Section One — Number

Section Two — Algebra

Section Three — Graphs

Section Four — Ratio, Proportion and Rates of Change

Throughout this book you'll see grade stamps like these:
You can use these to focus your revision on easier or harder work.
But remember — to get a top grade you have to know **everything**, not just the hardest topics.

Contents

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Types of Number and BODMAS

Ah, the glorious world of GCSE Maths. Get stuck into this first section and you'll be an expert in no time. Here are some handy definitions of different types of number, and a bit about what order to do things in.

Integers

GRADE 2

An integer is another name for a whole number — either a positive or negative number, or zero.

Examples

Integers: –365, 0, 1, 17, 989, 1 234 567 890

Not integers: 0.5, $\frac{2}{3}$, $\sqrt{7}$, $13\frac{3}{4}$, –1000.1, 66.66, π

Square and Cube Numbers

GRADE 2

1) When you multiply a whole number by itself, you get a square number:

1^2	2^2	3^2	4^2	5^2	6^2	7^2	8^2	9^2	10^2	11^2	12^2	13^2	14^2	15^2
1	4	9	16	25	36	49	64	81	100	121	144	169	196	225
(1×1)	(2×2)	(3×3)	(4×4)	(5×5)	(6×6)	(7×7)	(8×8)	(9×9)	(10×10)	(11×11)	(12×12)	(13×13)	(14×14)	(15×15)

2) When you multiply a whole number by itself, then by itself again, you get a cube number:

1^3	2^3	3^3	4^3	5^3	10^3
1	8	27	64	125	1000
(1×1×1)	(2×2×2)	(3×3×3)	(4×4×4)	(5×5×5)	(10×10×10)

3) You should know these basic squares and cubes by heart — they could come up on a non-calculator paper, so it'll save you time if you already know what they are.

BODMAS

Brackets, Other, Division, Multiplication, Addition, Subtraction

GRADE 2

BODMAS tells you the ORDER in which these operations should be done:
Work out Brackets first, then Other things like squaring, then Divide / Multiply groups of numbers before Adding or Subtracting them.

EXAMPLE:

Find the reciprocal of $\sqrt{4 + 6 \times (12 - 2)}$.

$\sqrt{4+6\times(12-2)} = \sqrt{4+6\times10}$ — It's not obvious what to do inside the square root — so use BODMAS. Brackets first...

$= \sqrt{4+60}$ — ... then multiply...

$= \sqrt{64}$ — ... then add.

$= 8$ — Take the square root

1) The reciprocal of 8 is $\frac{1}{8}$. — Finally, take the reciprocal (the reciprocal of a number is just 1 ÷ the number).

Brackets, Other, Division, Multiplication, Addition, Subtraction

It's really important to check your working on BODMAS questions. You might be certain you did the calculation right, but it's surprisingly easy to make a slip.

Wordy Real-Life Problems

Wordy questions can be a bit off-putting — for these you don't just have to do the maths, you've got to work out what the question's asking you to do. It's really important that you begin by reading through the question really carefully...

Don't Be Scared of **Wordy Questions**

Relax and work through them step by step:

1) READ the question carefully. Work out what bit of maths you need to answer it.
2) Underline the INFORMATION YOU NEED to answer the question — you might not have to use all the numbers they give you.
3) Write out the question IN MATHS and answer it, showing all your working clearly.

EXAMPLES:

1. **A return car journey from Carlisle to Manchester uses $\frac{4}{7}$ of a tank of petrol. It costs £56 for a full tank of petrol. How much does the journey cost?**

You want to know $\frac{4}{7}$ of £56, so in maths: $\frac{4}{7} \times £56 = £32$

See p.15-17 for more on using fractions.

Don't forget the units in your final answer — this is a question about cost in pounds, so the units will be £.

2. **A water company has two different price plans for water usage:**

PLAN A	PLAN B
No fixed cost	Fixed cost of £12
23p per unit	Plus 11p per unit

A household uses 102 units of water in one month. Which price plan would be cheaper for this household in that particular month?

Cost with plan A: $102 \times £0.23 = £23.46$

Cost with plan B: $£12 + (102 \times £0.11) = £23.22$

So for this household in that particular month, plan B would be cheaper.

3. **Ben buys dog food in boxes of 20 packets. Each box costs £12.50. He has 3 dogs which each eat 2 packets per day. How much will it cost Ben to buy enough boxes of food for all of his dogs for 4 weeks?**

Number of packets for 3 dogs for 1 day = $3 \times 2 = 6$

Number of packets for 3 dogs for 4 weeks = $6 \times 7 \times 4 = 168$

Number of boxes needed = $168 \div 20 = 8.4$

Ben can't buy part of a box, so he needs to buy 9 boxes.

$9 \times £12.50 = £112.50$

Just because it's wordy doesn't mean it's difficult...

A lot of these wordy questions have a few stages to the working — if you take them one step at a time, they're really not bad at all. Read them carefully, pick the important bits and underline them — it helps.

Multiplying and Dividing by 10, 100, etc.

You need to know the stuff on this page — it's quite simple and they're likely to test you on it in the exam.

1) Multiplying and Dividing Any Number by 10

GRADE 1

Multiply: Move the decimal point ONE place BIGGER and if it's needed, ADD A ZERO on the end.

E.g. 23.6 × 10 = 236

Divide: Move the decimal point ONE place SMALLER and if it's needed, REMOVE ZEROS after the decimal point.

E.g. 23.6 ÷ 10 = 2.36

2) Multiplying and Dividing Any Number by 100

GRADE 1

Multiply: Move the decimal point TWO places BIGGER and ADD ZEROS if necessary.

E.g. 34 × 100 = 3400

Divide: Move the decimal point TWO places SMALLER and REMOVE ZEROS after the decimal point.

E.g. 340 ÷ 100 = 3.4

3) Multiplying and Dividing by 1000 or 10 000

GRADE 1

Multiply: Move the decimal point so many places BIGGER and ADD ZEROS if necessary.

Divide: Move the decimal point so many places SMALLER and REMOVE ZEROS after the decimal point.

You always move the decimal point this much:
1 place for 10, 2 places for 100,
3 places for 1000, 4 places for 10 000, etc.

4) Multiply and Divide by Numbers like 20, 300, etc.

GRADE 2

MULTIPLY by 2 or 3 or 8 etc. FIRST, then move the decimal point so many places BIGGER ( ) according to how many zeros there are.

DIVIDE by 4 or 3 or 7 etc. FIRST, then move the decimal point so many places SMALLER (i.e. to the left).

EXAMPLES:

1. **Calculate 234 × 200.**

234 × 2 = 468
468 × 100 = 46800

2. **Calculate 960 ÷ 300.**

960 ÷ 3 = 320
320 ÷ 100 = 3.2

Multiplying — move the decimal point to the right

When dividing move the decimal point to the left. This stuff might seem easy, but you still need to practise it. Look at how many zeros there are in the multiplier and move the decimal point that many places.

Multiplying and Dividing Whole Numbers

You need to be really happy doing multiplications and divisions without a calculator — they're likely to come up in your non-calculator exam.

Multiplying Whole Numbers

The Traditional Method:

1) Split it into separate multiplications.
2) Add up the results in columns (right to left).

There are lots of other multiplication methods — make sure you're comfortable using whichever method you prefer.

EXAMPLES:

1. **Work out 46 × 27**

```
     4 6
  ×  2 7
  ------
   3 2₄2  —— This is 7 × 46
   9₁2 0  —— This is 20 × 46
  ------
 1 2 4 2  —— This is 322 + 920
```

2. **Work out 243 × 18**

```
     2 4 3
   ×   1 8
  --------
   1 9₃4₂4  —— This is 8 × 243
   2 4 3 0  —— This is 10 × 243
  --------
   4₁3 7 4  —— This is 1944 + 2430
```

Dividing Whole Numbers

EXAMPLE: **What is 748 ÷ 22?**

For questions like this, it's useful to write out the first few multiples of the number you're dividing by.

E.g
1 × 22 = 22
2 × 22 = 44
3 × 22 = 66
4 × 22 = 88...

Short Division Method

1) Set out the division as shown.

number you're dividing by — 22 ⟌ 748 — number you're dividing

2) Look at the first digit under the line. 7 doesn't divide by 22, so put a zero above and look at the next digit.

```
     0
22 ⟌ 7 4 8
```

3) 22 × 3 = 66, so 22 into 74 goes 3 times, with a remainder of 74 – 66 = 8.

```
     0 3
22 ⟌ 7 4 ⁸8
```

carry the remainder

4) 22 into 88 goes 4 times exactly.

```
     0 3 4
22 ⟌ 7 4 ⁸8
```

the top line has the final answer

So 748 ÷ 22 = 34

The other common method for dividing is long division — if you prefer this method, make sure you know it really well, so you'll have no problems with any division in your exam.

Make sure you know how to multiply and divide without a calculator

Practice makes perfect when it comes to multiplication and division. Go through the worked examples on this page and make sure you can follow the methods, then practise, practise, practise...

Multiplying and Dividing with Decimals

You might get a scary non-calculator question on multiplying or dividing using decimals. Luckily, these aren't really any harder than the whole-number versions. You just need to know what to do in each case.

Multiplying Decimals

GRADE 3

1) Start by <u>ignoring</u> the decimal points. Do the multiplication using <u>whole numbers</u>.
2) Count the <u>total</u> number of digits after the <u>decimal points</u> in the original numbers.
3) Make the answer have the <u>same number</u> of decimal places.

EXAMPLE: **Work out 4.6 × 2.7**

46 × 27 = 1242 — We worked this out on page 4.

4.<u>6</u> × 2.<u>7</u> has <u>2 digits</u> after the decimal points.

4.6 × 2.7 = 12.42

Dividing a Decimal by a Whole Number

GRADE 3

For these, you just set the question out like a whole-number division <u>but</u> put the <u>decimal point</u> in the answer <u>right above</u> the one in the question.

EXAMPLE: **What is 52.8 ÷ 3?**

Put the decimal point in the answer above the one in the question.

$$\begin{array}{r} 1\ . \\ 3\overline{)5\,{}^{2}2.\,8} \end{array}$$

3 into 5 goes once, carry the remainder of 2.

$$\begin{array}{r} 1\ 7\ . \\ 3\overline{)5\,{}^{2}2.\,{}^{1}8} \end{array}$$

3 into 22 goes 7 times, carry the remainder of 1.

$$\begin{array}{r} 1\ 7\ .\ 6 \\ 3\overline{)5\,{}^{2}2.\,{}^{1}8} \end{array}$$

3 into 18 goes 6 times exactly.

So 52.8 ÷ 3 = 17.6

Dividing a Number by a Decimal

GRADE 3

Two-for-one here — this works if you're dividing a whole number by a decimal, or a decimal by a decimal.

EXAMPLE: **What is 36.6 ÷ 0.12?**

1) The trick here is to write it as a fraction: $36.6 \div 0.12 = \frac{36.6}{0.12}$
2) Get rid of the decimals by multiplying top and bottom by 100 (see p.3): $= \frac{3660}{12}$
3) It's now a decimal-free division that you know how to solve:

$$\begin{array}{r} 0 \\ 12\overline{)3\,{}^{3}6\ 6\ 0} \end{array}$$

12 into 3 won't go so carry the 3

$$\begin{array}{r} 0\ 3 \\ 12\overline{)3\,{}^{3}6\ 6\ 0} \end{array}$$

12 into 36 goes 3 times exactly

$$\begin{array}{r} 0\ 3\ 0 \\ 12\overline{)3\,{}^{3}6\ 6\,{}^{6}0} \end{array}$$

12 into 6 won't go so carry the 6

$$\begin{array}{r} 0\ 3\ 0\ 5 \\ 12\overline{)3\,{}^{3}6\ 6\,{}^{6}0} \end{array}$$

12 into 60 goes 5 times exactly

So 36.6 ÷ 0.12 = 305

To divide decimals by decimals, first turn them into whole numbers

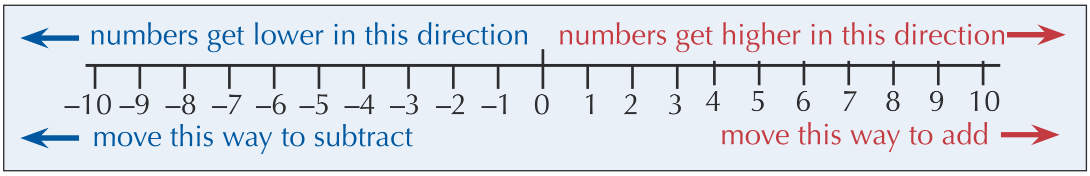
Multiply your decimals by 10, 100, etc. to get rid of the decimal points. This will give you an equivalent fraction. But be careful to multiply both the top and bottom by the same amount.

Negative Numbers

Numbers less than zero are negative. You should be able to add, subtract, multiply and divide with them.

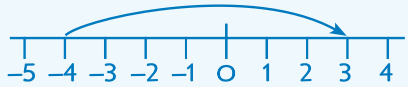
Adding and Subtracting with Negative Numbers

Use the number line for addition and subtraction involving negative numbers:



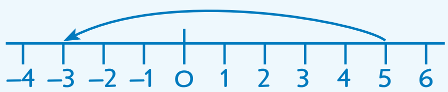
EXAMPLES:

What is –4 + 7? Start at –4 and move 7 places in the positive direction:



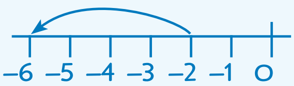
So –4 + 7 = 3

Work out 5 – 8 Start at 5 and move 8 places in the negative direction:



So 5 – 8 = –3

Find –2 – 4 Start at –2 and move 4 places in the negative direction:



So –2 – 4 = –6

Use These Rules for Combining Signs

+	+	makes	+
+	–	makes	–
–	+	makes	–
–	–	makes	+

These rules are ONLY TO BE USED WHEN:

1) Multiplying or dividing

EXAMPLES:

(invisible + sign)

Find: a) –2 × 3 – + makes – so –2 × 3 = –6

b) –8 ÷ –2 – – makes + so –8 ÷ –2 = 4

Be careful when squaring or cubing. Squaring a negative number gives a positive number (e.g. –2 × –2 = 4) but cubing a negative number gives a negative number (e.g. –3 × –3 × –3 = –27).

2) Two signs appear next to each other

EXAMPLES:

Work out: a) 5 – –4 – – makes + so 5 – –4 = 5 + 4 = 9

b) 4 + –6 – –7 + – makes – and – – makes +
so 4 + –6 – –7 = 4 – 6 + 7 = 5

Number lines are handy for adding or subtracting negative numbers

To multiply or divide negative numbers, you need to use the rules in the burgundy box.
Don't just learn them — make sure you know when you can use them too.

Warm-Up and Worked Exam Questions

Doing maths without a calculator becomes easier the more you practise. These warm-up questions will help to get your brain in gear. Work through them without using your calculator.

Warm-up Questions

1) Without using a calculator, find the value of $3 + 22 \times 3 - 14$.
2) Hank owns a fruit stall. He sells 11 apples and 5 oranges for £2.71. One apple costs 16p. How much does one orange cost?
3) Work out: a) 12.3×100 b) 2.4×20
4) Work out: a) $2.45 \div 10$ b) $4000 \div 800$
5) Work out the following:
   a) 28×12 b) 104×8 c) 3.2×56 d) 0.6×10.2
6) Work out the following:
   a) $96 \div 8$ b) $242 \div 2$ c) $33.6 \div 0.6$ d) $45 \div 1.5$
7) Work out: a) -4×-3 b) $-4 + -5 + 3$ c) $(3 + -2 - 4) \times (2 + -5)$ d) $120 \div -40$

Worked Exam Questions

These questions already have the answers filled in. Have a careful read through the working and handy hints before you have a go at the exam questions on the next page.

1 Theo has a 500 ml bottle of a fizzy drink. Poppy has 216 ml of the same fizzy drink in a glass. Theo gives Poppy some of his drink so that they each have the same amount. (Grade 3)

How much drink does Theo give to Poppy?

Total amount of drink = 500 ml + 216 ml = 716 ml

Find how much fizzy drink there is in total.

$$2\overline{)7^11^16} \quad 358$$

= 358 ml each

Divide this by 2 to find the amount they should each have.

500 ml – 358 ml = 142 ml,
so Theo gives 142 ml of his drink to Poppy.

..........142.......... ml
[2 marks]

2 Work out how many of each individual item below were sold if: (Grade 3)

a) Sophie spent £14 on rulers that cost £0.70 each.

$$14 \div 0.7 = \frac{14}{0.7} = \frac{140}{7}$$

$$7\overline{)1^14\,0} \quad 020 = 20$$

Multiply top and bottom by 10, then do the division as normal.

..........20..........
[2 marks]

b) Michael spent £2.76 on pencils that cost £0.12 each.

$$2.76 \div 0.12 = \frac{2.76}{0.12} = \frac{276}{12}$$

$$12\overline{)2^27^36} \quad 023 = 23$$

Don't worry if you have to keep carrying numbers — just continue like normal.

..........23..........
[2 marks]

Exam Questions

3 Write down the value of each of the following:

a) 9^2 (Grade 1)

....
[1 mark]

b) 4^3 (Grade 2)

....
[1 mark]

4 Three numbers multiply to give 288. Two of the numbers are –3 and 12. (Grade 2)

What is the third number?

....
[3 marks]

5 Alanna buys 15 tickets for a concert for herself and some friends.
Each ticket is the same price. She pays with £200 and gets £5 change. (Grade 3)

How much does each ticket cost?

£ ....
[3 marks]

6 Georgie is a sales representative. She drives to different companies to sell air conditioning units. (Grade 4)

When she has to travel, her employer pays fuel expenses of 30p per mile.
She drives to a job in the morning and drives home again later that day.
She is also given £8 to cover any food expenses for each day that she is not in the office.

The distances to her jobs for this week are shown on the right.

Find Georgie's total expenses for this week.

Monday: Buckshaw, 30 miles
Tuesday: in office
Wednesday: Wortham, 28 miles
Thursday: Harborough, 39 miles
Friday: Scotby, 40 miles

Hint: think carefully about the total distance travelled.

£ ....
[4 marks]

Prime Numbers

There's no getting around it — prime numbers are as important as they sound.
Luckily, they're not too difficult.

PRIME Numbers Don't Divide by Anything

Prime numbers are all the numbers that DON'T come up in times tables:

2 3 5 7 11 13 17 19 23 29 31 37 ...

The only way to get ANY PRIME NUMBER is: $1 \times$ ITSELF

E.g. The only numbers that multiply to give 7 are 1×7
The only numbers that multiply to give 31 are 1×31

EXAMPLE: **Show that 24 is not a prime number.**

Just find another way to make 24 other than 1×24: $2 \times 12 = 24$

24 divides by other numbers apart from 1 and 24, so it isn't a prime number.

Five Important Facts

1) 1 is NOT a prime number.
2) 2 is the ONLY even prime number.
3) The first four prime numbers are 2, 3, 5 and 7.
4) Prime numbers end in 1, 3, 7 or 9 (2 and 5 are the only exceptions to this rule).
5) But NOT ALL numbers ending in 1, 3, 7 or 9 are primes, as shown here:
(Only the circled ones are primes.)

2	3	5	7
11	13	17	19
21	23	27	29
31	33	37	39
41	43	47	49
51	53	57	59
61	63	67	69

How to FIND Prime Numbers — a very simple method

1) All primes (above 5) end in 1, 3, 7 or 9. So ignore any numbers that don't end in one of those.
2) Now, to find which of them ACTUALLY ARE primes you only need to divide each one by 3 and by 7. If it doesn't divide exactly by 3 or by 7 then it's a prime.

This simple rule using just 3 and 7 is true for checking primes up to 120.

EXAMPLE: **Find all the prime numbers in this list:** **71, 72, 73, 74, 75, 76, 77, 78**

1 First, get rid of anything that doesn't end in 1, 3, 7 or 9: **71, ~~72~~, 73, ~~74~~, ~~75~~, ~~76~~, 77, ~~78~~**

2 Now try dividing 71, 73 and 77 by 3 and 7:

$71 \div 3 = 23.667$ $71 \div 7 = 10.143$ so 71 is a prime number
$73 \div 3 = 24.333$ $73 \div 7 = 10.429$ so 73 is a prime number
$77 \div 3 = 25.667$ BUT: $77 \div 7 = 11$ — 11 is a whole number,
so 77 is NOT a prime, because it divides by 7.

So the prime numbers in the list are 71 and 73.

Remember — prime numbers don't come up in times tables

You'll save time if you can answer prime number questions straight off, without having to test for primes.
Memorise as many primes as you can, then use the method in the box above for bigger numbers.

Multiples, Factors and Prime Factors

You might get asked to list multiples or find factors in the exam. This page will teach you how.

Multiples and Factors

The MULTIPLES of a number are just its times table.

EXAMPLE: **Find the first 8 multiples of 13.**

You just need to find the first 8 numbers in the 13 times table:
13 26 39 52 65 78 91 104

The FACTORS of a number are all the numbers that divide into it.

There's a method that guarantees you'll find them all:

1) Start off with 1 × the number itself, then try 2 ×, then 3 × and so on, listing the pairs in rows.
2) Try each one in turn. Cross out the row if it doesn't divide exactly.
3) Eventually, when you get a number repeated, stop.
4) The numbers in the rows you haven't crossed out make up the list of factors.

EXAMPLE: **Find all the factors of 24.**

Increasing by 1 each time

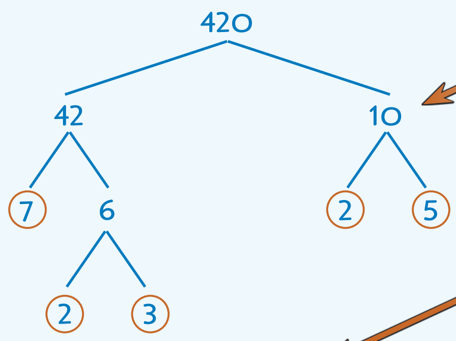
1 × 24
2 × 12
3 × 8
4 × 6
~~5 × —~~
6 × 4

So the factors of 24 are:
1, 2, 3, 4, 6, 8, 12, 24

Finding Prime Factors — The Factor Tree

Any number can be broken down into a string of prime numbers all multiplied together — this is called 'expressing it as a product of prime factors', or its 'prime factorisation'.

EXAMPLE: **Express 420 as a product of prime factors.**



So 420 = 2 × 2 × 3 × 5 × 7
= 2^2 × 3 × 5 × 7

To write a number as a product of its prime factors, use the Factor Tree method:

1) Start with the number at the top, and split it into factors as shown.
2) Every time you get a prime, ring it.
3) Keep going until you can't go further (i.e. you're just left with primes), then write the primes out in order. If there's more than one of the same factor, you can write them as powers.

No matter which numbers you choose at each step, you'll find that the prime factorisation is exactly the same. Each number has a unique set of prime factors.

Follow the methods above to find factors and prime factors

It doesn't matter how you split the numbers when drawing a factor tree, as long as each pair of numbers multiplies to give the number above. For instance, we could have first split 420 into 21 and 20. The important thing is that you keep going until you only have prime numbers left.

LCM and HCF

Two big fancy names but don't be put off — they're both real easy. There are two methods for finding each — this page starts you off with the nice, straightforward methods.

LCM — 'Lowest Common Multiple'

GRADE 3

'Lowest Common Multiple' — sure, it sounds kind of complicated, but all it means is this:

The SMALLEST number that will DIVIDE BY ALL the numbers in question.

METHOD:
1) LIST the MULTIPLES of ALL the numbers.
2) Find the SMALLEST one that's in ALL the lists.

The LCM is sometimes called the Least (instead of 'Lowest') Common Multiple.

EXAMPLE:

Find the lowest common multiple (LCM) of 12 and 15.

Multiples of 12 are: 12, 24, 36, 48, 60, 72, 84, 96, ...
Multiples of 15 are: 15, 30, 45, 60, 75, 90, 105, ...

So the lowest common multiple (LCM) of 12 and 15 is 60.

HCF — 'Highest Common Factor'

GRADE 3

'Highest Common Factor' — all it means is this:

The BIGGEST number that will DIVIDE INTO ALL the numbers in question.

METHOD:
1) LIST the FACTORS of ALL the numbers.
2) Find the BIGGEST one that's in ALL the lists.

EXAMPLE:

Find the highest common factor (HCF) of 36, 54, and 72.

Factors of 36 are: 1, 2, 3, 4, 6, 9, 12, 18, 36
Factors of 54 are: 1, 2, 3, 6, 9, 18, 27, 54
Factors of 72 are: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

So the highest common factor (HCF) of 36, 54 and 72 is 18.

Just take care listing the factors — make sure you use the proper method (as shown on the previous page) or you'll miss one and blow the whole thing out of the water.

LCM and HCF — learn what the names mean

LCM and HCF questions shouldn't be too bad as long as you know exactly what's meant by each of the terms. Then you just multiply or divide to find the multiples or factors.

LCM and HCF

The two methods on this page are a little trickier — but you might have to use them in your exam.

LCM — Alternative Method

Grade 5

If you already know the prime factors of the numbers, you can use this method instead:

1) List all the PRIME FACTORS that appear in EITHER number.
2) If a factor appears MORE THAN ONCE in one of the numbers, list it THAT MANY TIMES.
3) MULTIPLY these together to give the LCM.

EXAMPLE:

$18 = 2 \times 3^2$ and $30 = 2 \times 3 \times 5$. Find the LCM of 18 and 30.

$18 = 2 \times 3 \times 3$ $\quad$ $30 = 2 \times 3 \times 5$

So the prime factors that appear in either number are: 2, 3, 3, 5

List 3 twice as it appears twice in 18.

LCM $= 2 \times 3 \times 3 \times 5 = 90$

HCF — Alternative Method

Grade 5

Again, there's a different method you can use if you already know the prime factors of the numbers:

1) List all the PRIME FACTORS that appear in BOTH numbers.
2) MULTIPLY these together to find the HCF.

EXAMPLE:

$180 = 2^2 \times 3^2 \times 5$ and $84 = 2^2 \times 3 \times 7$. Use this to find the HCF of 180 and 84.

$180 = 2 \times 2 \times 3 \times 3 \times 5$ $\quad$ $84 = 2 \times 2 \times 3 \times 7$

2, 2 and 3 are prime factors of both numbers, so HCF $= 2 \times 2 \times 3 = 12$

Real-Life LCM and HCF Questions

Grade 4

You might be asked a wordy real-life LCM or HCF question in your exam — these can be tricky to spot at first, but once you have done, the method's just the same.

EXAMPLE:

Maggie is making party bags. She has 60 balloons, 48 lollipops and 84 stickers. She wants to use them all. Each type of item must be distributed equally between the party bags. What is the maximum number of party bags she can make?

Factors of 60 are: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

Factors of 48 are: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

Factors of 84 are: 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84

The highest common factor (HCF) of 60, 48 and 84 is 12, so the maximum number of party bags Maggie can make is 12.

In each bag there will be $60 \div 12 = 5$ balloons, $48 \div 12 = 4$ lollipops and $84 \div 12 = 7$ stickers.

You could use the prime factorisation method here if you wanted — use whichever method's easier for you.

You might need to use these alternative methods in the exam

Lowest common multiple and highest common factor questions can be a bit intimidating but they're easy enough if you take them step by step. Keep going over the different methods until you're great at them.

Warm-up and Worked Exam Questions

These warm-up questions will test whether you've learned the facts from the last few pages.
Keep practising any you get stuck on, before moving on.

Warm-up Questions

1) Which of the following numbers are prime? 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40.
2) Explain why 27 is not a prime number.
3) Find all the factors of 40.
4) Find the prime factors of 40.
5) Find the lowest common multiple of 4 and 5.
6) Find the highest common factor of 36 and 96.

Worked Exam Questions

Time for more exam questions with the answers filled in. Understanding these solutions will help you with the exam questions that follow and in the exam itself.

1 Write down:

a) all the factors of 28,

Think of pairs of factors, for example 1 and 28, 2 and 14....

..........1, 2, 4, 7, 14, 28..........

[2 marks]

b) all the multiples of 8 which appear in the list below.

55 56 57 58 59 60 61 62 63 64 65

..........56, 64..........

[1 mark]

2 Phil is making jam.

He needs to buy mini jam jars which come in packs of 12, lids which come in packs of 16 and labels which come in packs of 36. He doesn't want to have any items left over.

Find the smallest number of packs of each item he can buy.

Multiples of 12 are: 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144, 156, ...

Multiples of 16 are: 16, 32, 48, 64, 80, 96, 112, 128, 144, 160, ...

Multiples of 36 are: 36, 72, 108, 144, 180, ...

The LCM of 12, 16 and 36 is 144, which is the minimum number of each item he needs.

The minimum number of packs of jars he needs is 144 ÷ 12 = 12 packs

The minimum number of packs of lids he needs is 144 ÷ 16 = 9 packs

The minimum number of packs of labels he needs is 144 ÷ 36 = 4 packs

..........12.......... packs of jars, ...9.......... packs of lids and ....4.......... packs of labels

[3 marks]

Exam Questions

3 Jack says, "there are no prime numbers between 100 and 110."
Is he correct? Give evidence for your answer.

[2 marks]

4 Jay thinks of a prime number. The sum of its digits is one more than a square number.
Write down one number Jay could be thinking of.

....
[2 marks]

5 Write 72 as a product of its prime factors.

Make sure your answer only uses prime numbers. Multiply them all together to check you get the number you started with.

....
[2 marks]

6 $P = 3^7 \times 11^2$ and $Q = 3^4 \times 7^3 \times 11$.

Write as the product of prime factors:

a) the LCM of P and Q,

....
[1 mark]

b) the HCF of P and Q.

....
[1 mark]

Fractions without a Calculator

These pages show you how to cope with fraction calculations without your beloved calculator.

1) Cancelling down

GRADE 2

To cancel down or simplify a fraction, divide top and bottom by the same number, till they won't go further:

EXAMPLE: **Simplify $\frac{18}{24}$.**

Cancel down in a series of easy steps — keep going till the top and bottom don't have any common factors.

$$\frac{18}{24} \underset{\div 3}{\overset{\div 3}{=}} \frac{6}{8} \underset{\div 2}{\overset{\div 2}{=}} \frac{3}{4}$$

The number on the top of the fraction is the numerator, and the number on the bottom is the denominator.

2) Mixed numbers

GRADE 2

Mixed numbers are things like $3\frac{1}{3}$, with an integer part and a fraction part.
Improper fractions are ones where the top number is larger than the bottom number.
You need to be able to convert between the two.

EXAMPLES:

1. **Write $4\frac{2}{3}$ as an improper fraction.**

1) Think of the mixed number as an addition:

$$4\frac{2}{3} = 4 + \frac{2}{3}$$

2) Turn the integer part into a fraction:

$$4 + \frac{2}{3} = \frac{12}{3} + \frac{2}{3} = \frac{12+2}{3} = \frac{14}{3}$$

2. **Write $\frac{31}{4}$ as a mixed number.**

Divide the top number by the bottom.

1) The answer gives the whole number part.

2) The remainder goes on top of the fraction.

$$31 \div 4 = 7 \text{ remainder } 3 \quad \text{so} \quad \frac{31}{4} = 7\frac{3}{4}$$

3) Multiplying

GRADE 3

Multiply top and bottom separately. Then simplify your fraction as far as possible.

EXAMPLE: **Find $\frac{8}{5} \times \frac{7}{12}$.**

Multiply the top and bottom separately: $\frac{8}{5} \times \frac{7}{12} = \frac{8 \times 7}{5 \times 12}$

Then simplify — top and bottom both divide by 4. $= \frac{56}{60} = \frac{14}{15}$

You have to know how to handle mixed numbers

Mixed numbers look difficult, but they're OK once you've converted them into normal fractions.
If you keep on practising working with fractions, you'll bag some easy marks in the exam.

Fractions without a Calculator

Here are some more tricks for dealing with fractions.

4) Dividing

GRADE 3

Turn the 2nd fraction UPSIDE DOWN and then multiply:

When you're multiplying or dividing with mixed numbers, always turn them into improper fractions first.

EXAMPLE: **Find $2\frac{1}{3} \div 3\frac{1}{2}$.**

Rewrite the mixed numbers as improper fractions: $2\frac{1}{3} \div 3\frac{1}{2} = \frac{7}{3} \div \frac{7}{2}$

Turn $\frac{7}{2}$ upside down and multiply: $= \frac{7}{3} \times \frac{2}{7} = \frac{7 \times 2}{3 \times 7}$

Simplify — top and bottom both divide by 7. $= \frac{14}{21} = \frac{2}{3}$

5) Common denominators

GRADE 4

This comes in handy for ordering fractions by size, and for adding or subtracting fractions.

You need to find a number that all the denominators divide into — this will be your common denominator. The simplest way is to find the lowest common multiple of the denominators:

EXAMPLE: **Put these fractions in ascending order of size:**

$$\frac{8}{3} \qquad \frac{5}{4} \qquad \frac{12}{5}$$

The LCM of 3, 4 and 5 is 60,
so make 60 the common denominator:

$$\frac{8}{3} \underset{\times 20}{\overset{\times 20}{=}} \frac{160}{60} \qquad \frac{5}{4} \underset{\times 15}{\overset{\times 15}{=}} \frac{75}{60} \qquad \frac{12}{5} \underset{\times 12}{\overset{\times 12}{=}} \frac{144}{60}$$

So the correct order is $\frac{75}{60}, \frac{144}{60}, \frac{160}{60}$ i.e. $\frac{5}{4}, \frac{12}{5}, \frac{8}{3}$

Don't forget to use the original fractions in the final answer.

Common denominators are really handy

You need to find a common denominator to order, add or subtract fractions (see next page). If you're ordering fractions, don't forget to turn them back into their original forms when you give your answers.

Fractions without a Calculator

6) Adding, subtracting — sort the denominators first

GRADE 4

If you're adding or subtracting mixed numbers, it usually helps to convert them to improper fractions first.

Make sure the denominators are the same (see previous page).
Add (or subtract) the top lines only.

EXAMPLE: **Calculate $2\frac{1}{5} - 1\frac{1}{2}$.**

Rewrite the mixed numbers as improper fractions: $2\frac{1}{5} - 1\frac{1}{2} = \frac{11}{5} - \frac{3}{2}$

Find a common denominator: $= \frac{22}{10} - \frac{15}{10}$

Combine the top lines: $= \frac{22-15}{10} = \frac{7}{10}$

7) Fractions of something

GRADE 2

EXAMPLE: **What is $\frac{9}{20}$ of £360?**

'$\frac{9}{20}$ of £360' means '$\frac{9}{20}$ × £360'.

Multiply by the top of the fraction and divide by the bottom.

$\frac{9}{20}$ × £360 = (£360 ÷ 20) × 9
= £18 × 9 = £162

The order that you multiply and divide in doesn't matter — just start with whatever's easiest.

8) Expressing as a Fraction

GRADE 3

EXAMPLE: **Write 180 as a fraction of 80.**

Write the first number over the second and cancel down. $\frac{180}{80} = \frac{9}{4}$

You have to learn how to handle fractions in these 8 situations

If you've learnt how to find a common denominator (p.16), then adding and subtracting fractions should be dead easy. To find a fraction of something — multiply and divide in the order that's easiest for you.

Fraction Problems

The previous three pages gave you all the tools you'll need to tackle these more pesky fraction questions. All these questions could come up on the non-calculator paper so put your calculators away.

Use the Methods You've Already Learnt

EXAMPLES:

1. **Rachel is making cupcakes. Each cupcake needs $\frac{2}{25}$ of a pack of butter. How many packs of butter will Rachel need to buy to make 30 cupcakes?**

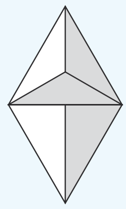
Once you've done the multiplication, convert $\frac{60}{25}$ to a mixed number to see how many packs she'll need — careful though, she can only buy a whole number of packs.

1 cupcake needs $\frac{2}{25}$ of a pack of butter,

so 30 cupcakes need: $30 \times \frac{2}{25} = \frac{60}{25} = \frac{12}{5} = 2\frac{2}{5}$ packs

2 packs won't be enough, so Rachel will need to buy 3 packs.

2. **The diamond to the right is made up of two identical equilateral triangles. The equilateral triangle at the top is split into three equal triangles. The equilateral triangle at the bottom is split into two equal triangles. Find the fraction of the diamond that is shaded.**



$\frac{2}{3}$ of the top triangle is shaded. This is $\frac{2}{3} \times \frac{1}{2} = \frac{2}{6} = \frac{1}{3}$ of the diamond.

$\frac{1}{2}$ of the bottom triangle is shaded. This is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ of the diamond.

Remember to multiply by a half because each equilateral triangle makes up half the diamond.

Find a common denominator to add the fractions. $\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$

3. **Which of the fractions, $\frac{2}{3}$ or $\frac{9}{7}$, is closer to 1?**

1) Give the fractions a common denominator. $\frac{2}{3} = \frac{2 \times 7}{3 \times 7} = \frac{14}{21}$ $\quad \frac{9}{7} = \frac{9 \times 3}{7 \times 3} = \frac{27}{21}$

2) Convert 1 into a fraction and subtract to see which fraction is closer to 1. $\frac{21}{21} - \frac{14}{21} = \frac{7}{21}$ $\quad \frac{27}{21} - \frac{21}{21} = \frac{6}{21}$ $\quad$ So $\frac{9}{7}$ is closer to 1.

4. **At the Prism School, year 10 is split into two classes, each with the same number of pupils in total. $\frac{3}{5}$ of one class are girls, and $\frac{4}{7}$ of the other class are girls. What fraction of year 10 students are girls?**

1) Divide each fraction by 2 to find the number of girls in each class as a fraction of total year 10 pupils.

Class 1 girls are: $\frac{3}{5} \div 2 = \frac{3}{10}$ of the total pupils in year 10.

Class 2 girls are: $\frac{4}{7} \div 2 = \frac{4}{14}$ of the total pupils in year 10.

2) Find a common denominator, then add the fractions.

So $\frac{3}{10} + \frac{4}{14} = \frac{21}{70} + \frac{20}{70} = \frac{41}{70}$ of year 10 pupils are girls.

Practise using the different fraction rules

You'll need the fraction rules to tackle all sorts of problems. A lot of questions require you to find a common denominator, so make sure you're comfortable with that.

Fractions, Decimals and Percentages

Fractions, decimals and percentages are three different ways of describing when you've got part of a whole thing. They're closely related and you can convert between them. These tables show some really common conversions which you should know straight off without having to work them out:

Fraction	Decimal	Percentage
$\frac{1}{2}$	0.5	50%
$\frac{1}{4}$	0.25	25%
$\frac{3}{4}$	0.75	75%
$\frac{1}{3}$	0.333333...	$33\frac{1}{3}$%
$\frac{2}{3}$	0.666666...	$66\frac{2}{3}$%
$\frac{5}{2}$	2.5	250%

Fraction	Decimal	Percentage
$\frac{1}{10}$	0.1	10%
$\frac{2}{10}$	0.2	20%
$\frac{1}{5}$	0.2	20%
$\frac{2}{5}$	0.4	40%
$\frac{1}{8}$	0.125	12.5%
$\frac{3}{8}$	0.375	37.5%

The more of those conversions you learn, the better — but for those that you don't know, you must also learn how to convert between the three types. These are the methods:

$$\text{Fraction} \xrightarrow{\text{Divide}} \text{Decimal} \xrightarrow{\times \text{ by } 100} \text{Percentage}$$

E.g. $\frac{7}{20}$ is $7 \div 20$ = 0.35 e.g. 0.35 × 100 = 35%

$$\text{Fraction} \xleftarrow[\text{The awkward one}]{} \text{Decimal} \xleftarrow[\div \text{ by } 100]{} \text{Percentage}$$

Converting decimals to fractions is awkward. To convert terminating decimals to fractions:

The digits after the decimal point go on the top, and a 10, 100, 1000, etc. on the bottom — so you have the same number of zeros as there were decimal places.

$0.6 = \frac{6}{10}$ $0.78 = \frac{78}{100}$ $0.024 = \frac{24}{1000}$ etc.

These can often be cancelled down — see p.15.

Recurring Decimals have Repeating Digits

1) Recurring decimals have a pattern of numbers which repeats forever, e.g. 0.333333... which is $\frac{1}{3}$.
2) The repeating part is usually marked with dots on top of the number.
3) If there's one dot, only one digit is repeated. If there are two dots, everything from the first dot to the second dot is the repeating bit.
4) You can convert a fraction to a recurring decimal:

E.g. $0.2\dot{5} = 0.2555555...$,
$0.\dot{2}\dot{5} = 0.25252525...$,
$0.\dot{2}6\dot{5} = 0.265265265...$

EXAMPLE: **Write $\frac{5}{11}$ as a recurring decimal.**

Just do the division, and look for the repeating pattern.

$5 \div 11 = 0.454545...$ so $\frac{5}{11} = 0.\dot{4}\dot{5}$

Fractions, decimals and percentages are interchangeable

It's important you remember that a fraction, decimal or percentage can be converted into either of the other two forms. And it's even more important that you learn how to do it.

Warm-up and Worked Exam Questions

Here's a set of warm-up questions for this section. Work through them to check you've got the hang of fractions and to limber up for the exam questions that follow.

Warm-up Questions

1) Simplify $\frac{48}{64}$ as far as possible.
2) Which of these fractions are equivalent to $\frac{1}{3}$? $\frac{2}{6}, \frac{5}{15}, \frac{9}{36}, \frac{6}{20}$
3) Work these out, then simplify your answers where possible:

   a) $\frac{2}{5} \times \frac{2}{3}$ b) $\frac{2}{5} \div \frac{2}{3}$ c) $\frac{2}{5} + \frac{2}{3}$ d) $\frac{2}{3} - \frac{2}{5}$
4) What decimal is the same as $\frac{7}{10}$?
5) What percentage is the same as $\frac{2}{3}$?
6) What fraction is the same as 0.4?
7) Write $\frac{2}{7}$ as a recurring decimal.

Worked Exam Questions

Make sure you understand what's going on in these questions before trying the next page for yourself.

1 Scott and his 4 friends eat $\frac{5}{6}$ of a pizza each. Pizzas cost £4.50 each, or 2 for £7. GRADE 4

What is the minimum amount they will have to spend on pizzas?

$5 \times \frac{5}{6} = \frac{25}{6}$

$= 4\frac{1}{6}$, so they will need 5 pizzas in total.

Cost = (2 × £7) + £4.50

= £14 + £4.50

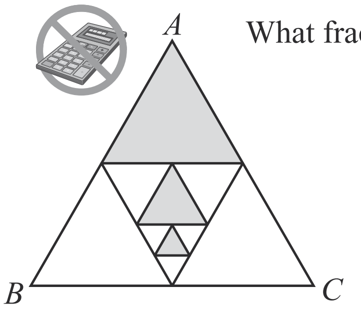
= £18.50

They can get 4 pizzas in the 2 for £7 offer.

£ 18.50

[3 marks]

2 *ABC* is an equilateral triangle. It has been divided into smaller equilateral triangles as shown below. GRADE 4



What fraction of triangle *ABC* is shaded?

Find the shaded areas as separate fractions of the whole triangle, then add them up.

Shaded regions are $\frac{1}{4}$, $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$ and $\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$

So total fraction shaded $= \frac{1}{4} + \frac{1}{16} + \frac{1}{64}$

$= \frac{16}{64} + \frac{4}{64} + \frac{1}{64}$

$= \frac{21}{64}$

$\frac{21}{64}$

[3 marks]

Exam Questions

3 The number of people at last Saturday's Norchester City game was 12 400.
Season ticket holders made up $\frac{3}{8}$ of the crowd. How many season ticket holders were there?

....
[2 marks]

4 Write the following in order of size, starting with the smallest.

Start by writing all the numbers as decimals.

65% $\qquad \frac{2}{3} \qquad$ 0.065 $\qquad \frac{33}{50}$

.... , ... , ... , ...
[2 marks]

5 Work out the following, giving your answers as mixed numbers in their simplest form:

a) $1\frac{1}{8} \times 2\frac{2}{5}$

....
[3 marks]

b) $1\frac{3}{4} \div \frac{7}{9}$

....
[3 marks]

6 Samuel, Eli, Robert and Jenny split the bill at a restaurant. Samuel pays $\frac{1}{4}$ of the bill and Eli and Robert each pay 20% of the bill. Jenny pays £17.50.

How much was the bill in total?

£ ....
[4 marks]

Rounding Numbers

You need to be able to use 3 different rounding methods.
We'll do decimal places first, but there's the same basic idea behind all three.

Decimal Places (d.p.)

GRADE 2

To round to a given number of decimal places:

1) Identify the position of the 'last digit' from the number of decimal places.

2) Then look at the next digit to the right — called the decider.

3) If the decider is 5 or more, then round up the last digit.
If the decider is 4 or less, then leave the last digit as it is.

4) There must be no more digits after the last digit (not even zeros).

If you're rounding to 2 d.p. the last digit is the second digit after the decimal point.

EXAMPLES:

1. **What is 13.72 correct to 1 decimal place?**

13.72 = 13.7

LAST DIGIT to be written (1st decimal place because we're rounding to 1 d.p.)

DECIDER

The LAST DIGIT stays the same because the DECIDER is 4 or less.

2. **What is 7.45839 to 2 decimal places?**

7.45839 = 7.46

LAST DIGIT to be written (2nd decimal place because we're rounding to 2 d.p.)

DECIDER

The LAST DIGIT rounds UP because the DECIDER is 5 or more.

Watch Out for **Pesky Nines**

If you have to round up a 9 (to 10), replace the 9 with 0, and add 1 to the digit on the left.

E.g. Round 45.698 to 2 d.p: 45.698 → 45.~~69~~ (70) → 45.70 to 2 d.p.

decider

last digit — round up

The question asks for 2 d.p. so you must put 45.70 not 45.7.

You might be asked to round off your answers in the exam

Rounding is a really important skill, and you'll be throwing easy marks away if you get it wrong.
Make sure you're completely happy with the basic method, then get plenty of practice.

Rounding Numbers

Significant Figures (s.f.)

The 1st significant figure of any number is the first digit which isn't a zero.

The 2nd, 3rd, 4th, etc. significant figures follow immediately after the 1st — they're allowed to be zeros.

	0.002309	506.07
SIG. FIGS:	1st 2nd 3rd 4th	1st 2nd 3rd 4th

To round to a given number of significant figures:

1) Find the **last digit** — if you're rounding to, say 3 s.f., then the **3rd significant figure** is the last digit.
2) Use the digit to the right of it as the **decider**, just like for d.p.
3) Once you've rounded, **fill up** with **zeros**, up to but **not beyond** the decimal point.

EXAMPLE:

Round 506.07 to 2 significant figures.

Last digit is the 2nd sig. fig.

Need one zero to fill up to decimal point.

506.07 = 510

DECIDER is 5 or more → Last digit rounds UP

To the Nearest Whole Number, Ten, Hundred etc.

You might be asked to round to the nearest whole number, ten, hundred, thousand, or million:

1) **Identify the last digit**, e.g. for the nearest **whole number** it's the **units** position, and for the '**nearest ten**' it's the **tens** position, etc.
2) **Round the last digit** and **fill in with zeros** up to the decimal point, just like for significant figures.

EXAMPLE:

Round 6751 to the nearest hundred.

Last digit is in the 'hundreds' position

Fill in 2 zeros up to decimal point.

6751 = 6800

DECIDER is 5 or more → Last digit rounds UP.

Significant figures can be a bit tricky to get your head round

Decimal places are easy, but significant figures take a bit more thinking about. Learn the method on this page for identifying significant figures, and make sure you really understand the example.

Estimating

'Estimate' doesn't mean 'take a wild guess', it means 'look at the numbers, make them a bit easier, then do the calculation'. Your answer won't be as accurate as the real thing but it's easier on your brain.

Estimating **Calculations**

GRADE 4 GRADE

1) **Round everything off to 1 significant figure.**
2) **Then work out the answer using these nice easy numbers.**
3) **Show all your working or you won't get the marks.**

Have a look at the previous page to remind yourself how to round to 1 s.f.

EXAMPLES:

1. **Estimate the value of 42.6 × 12.1.**

1) Round each number to 1 s.f.
2) Do the calculation with the rounded numbers.

$42.6 \times 12.1 \approx 40 \times 10$
$= 400$

$\approx$ means 'approximately equal to'.

You might have to say if it's an underestimate or an overestimate. Here, you rounded both numbers down, so it's an underestimate.

2. **Estimate the value of** $\dfrac{\sqrt{6242 \div 57}}{9.8 - 4.7}$.

Don't be put off by the square root, just round each number to 1 s.f. and do the calculation.

$$\frac{\sqrt{6242 \div 57}}{9.8 - 4.7} \approx \frac{\sqrt{6000 \div 60}}{10 - 5} = \frac{\sqrt{100}}{5} = \frac{10}{5} = 2$$

3. **Jo has a cake-making business. She spent £984.69 on flour last year. A bag of flour costs £1.89, and she makes an average of 5 cakes from each bag of flour. Work out an estimate of how many cakes she made last year.**

Don't panic if you get a 'real-life' estimating question — just round everything to 1 s.f. as before.

1) Estimate number of bags of flour — round numbers to 1 s.f.

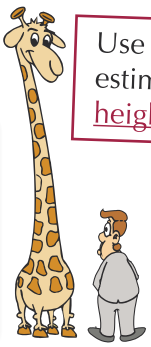
Number of bags of flour $= \dfrac{984.69}{1.89}$
$\approx \dfrac{1000}{2} = 500$

2) Multiply to find the number of cakes.

Number of cakes $\approx 500 \times 5 = 2500$

You Might Need to Estimate **Height**

GRADE 1 GRADE



Use 1.8 m as an estimate for the height of a man.

EXAMPLE:

Estimate the height of the giraffe in the picture.

In the picture the giraffe's about two and a half times as tall as the man.

Height of a man is about 1.8 m
Rough height of giraffe = 2.5 × height of man
= 2.5 × 1.8 = 4.5 m

Round the numbers first, then do the calculation like normal

If you're asked to estimate something in the exam, make sure you show all your steps (including what each number is rounded to) to prove that you didn't just use a calculator. Hassle, but it'll pay off.

Rounding Errors

Whenever a number is rounded or truncated it will have some amount of error.
The error tells you how far the actual value could be away from the rounded value.

Rounded Measurements Can Be Out By Half A Unit

GRADE 5

Whenever a measurement is rounded off to a given UNIT the actual measurement can be anything up to HALF A UNIT bigger or smaller.

EXAMPLES:

1. **A room is measured to be 9 m long to the nearest metre. What are its minimum and maximum possible lengths?**

The measurement is to the nearest 1 m, so the actual length could be up to 0.5 m bigger or smaller.

Minimum length = 9 – 0.5 = 8.5 m
Maximum length = 9 + 0.5 = 9.5 m

The actual maximum length is 9.4999... m, but it's OK to say 9.5 m instead.
If you're asked for the error interval, you can use inequalities to show the actual maximum:

2. **The mass of a cake is given as 2.4 kg to the nearest 0.1 kg. Find the interval within which *m*, the actual mass of the cake, lies.**

Minimum mass = 2.4 – 0.05 = 2.35 kg
Maximum mass = 2.4 + 0.05 = 2.45 kg

So the interval is $2.35 \text{ kg} \leq m < 2.45 \text{ kg}$

See p.51 for more on inequalities.

The actual value is greater than or equal to the minimum but strictly less than the maximum.
The actual mass of the cake could be exactly 2.35 kg,
but if it was exactly 2.45 kg it would round up to 2.5 kg instead.

Truncated Measurements Can Be A Whole Unit Out

GRADE 5

You truncate a number by chopping off decimal places. E.g. 25.765674 truncated to 1 d.p. would be 25.7

When a measurement is TRUNCATED to a given UNIT, the actual measurement can be up to A WHOLE UNIT bigger but no smaller.

If the mass of the cake in example 2 was 2.4 kg truncated to 1 d.p.
the error interval would be $2.4 \text{ kg} \leq m < 2.5 \text{ kg}$.
So even if the mass was 2.499999 kg, it would still truncate to 2.4 kg.

Learn the difference between rounding and truncating

If you're told that a measurement is rounded, the original measurement could be half a unit bigger or smaller. If a measurement is truncated, some decimal places are just chopped off the end.

Warm-up and Worked Exam Questions

Before you dive into the exam questions on the next page, have a paddle in these friendly-looking warm-up questions. If you're not sure about any of them, go back and look at the topic again.

Warm-up Questions

1) Round these numbers off to 1 decimal place:
   a) 3.24 b) 1.78 c) 2.31 d) 0.46 e) 9.76
2) Round these off to the nearest whole number:
   a) 3.4 b) 5.2 c) 1.84 d) 6.9 e) 3.26
3) Round these numbers to the stated number of significant figures:
   a) 352 to 2 s.f. b) 465 to 1 s.f. c) 12.38 to 3 s.f. d) 0.03567 to 2 s.f.
4) Round these numbers off to the nearest hundred:
   a) 2865 b) 450 c) 123
5) a) Estimate $(29.5 - 9.6) \times 4.87$ b) Is your answer an underestimate or an overestimate?
6) James has an apple with a mass of 138 g to the nearest gram.
   What is the minimum possible mass of the apple?
7) The following numbers have been rounded to 2 significant figures.
   Give the error interval for each: a) 380 b) 0.46
8) Truncate: a) 37.919 to 1 d.p. b) 2.0153 to 2 d.p.

Worked Exam Question

Look at that — an exam question with all the answers filled in. How unexpected.

1 Round the following to the given degree of accuracy.

a) Josh has 123 people coming to his party.
Write this number to the nearest 10.

123 — Last digit is in the 'tens' position
123 — Decider is less than 5
120 — 1 space to fill before the decimal point

120

[1 mark]

b) The attendance at a football match was 2568 people.
What is this to the nearest hundred?

Last digit Decider
2568
2600 — 2 spaces to fill

2600

[1 mark]

c) The population of Ulverpool is 452 529.
Round this to the nearest 100 000.

Last digit Decider
452 529
500 000 — 5 spaces to fill

500 000

[1 mark]

Exam Questions

2 The distance between two stars is 428.6237 light years.

a) Round this distance to one decimal place.

.... light years
[1 mark]

b) Round this distance to 2 significant figures.

.... light years
[1 mark]

3 A stall sells paperback and hardback books. Paperback books cost £4.95 and hardback books cost £11. One Saturday, the stall sells 28 paperback and 19 hardback books.

a) Find an estimate for the amount of money the stall made that day.
Show all your working.

£ ....
[2 marks]

b) The actual amount the stall made was £347.60.
Do you think your estimate was sensible? Explain your answer.

....

....
[1 mark]

4 Estimate the value of $\dfrac{12.2 \times 1.86}{0.19}$

You should start by rounding each number to an easier one.

....
[2 marks]

5 Joseph is weighing himself. His scales give his weight to the nearest kilogram.

According to his scales, Joseph is 57 kg.
What are the minimum and maximum weights that he could be?

Minimum weight: .... kg

Maximum weight: .... kg
[2 marks]

6 Given that $a = 3.8$ to 1 decimal place,
write down the error interval for a.

....
[2 marks]

Powers

You've already seen 'to the power 2' and 'to the power 3' — they're just 'squared' and 'cubed'. They're just the tip of the iceberg — any number can be a power if it puts its mind to it...

Powers are a very Useful Shorthand

1) Powers are 'numbers multiplied by themselves so many times':

   $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$ ('two to the power 7')

2) The powers of ten are really easy — the power tells you the number of zeros:

   $10^1 = 10$ $\quad 10^2 = 100$ $\quad 10^3 = 1000$ $\quad 10^6 = 1\,000\,000$

   to the power of 6

   6 zeros

3) Use the [$x^\blacksquare$] or [y^x] button on your calculator to find powers,
   e.g. press [3] [·] [7] [$x^\blacksquare$] [3] [=] to get $3.7^3 = 50.653$.
4) Anything to the power 1 is just itself, e.g. $4^1 = 4$.
5) 1 to any power is still 1, e.g. $1^{457} = 1$.
6) Anything to the power 0 is just 1, e.g. $5^0 = 1$, $67^0 = 1$, $x^0 = 1$.

Four Easy Rules:

1) When MULTIPLYING, you ADD THE POWERS. *e.g.* $3^4 \times 3^6 = 3^{4+6} = 3^{10}$
2) When DIVIDING, you SUBTRACT THE POWERS. *e.g.* $c^4 \div c^2 = c^{4-2} = c^2$
3) When RAISING one power to another, you MULTIPLY THE POWERS. *e.g.* $(3^2)^4 = 3^{2\times4} = 3^8$
4) FRACTIONS — Apply the power to both TOP and BOTTOM. *e.g.* $\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$

Warning: Rules 1 & 2 don't work for things like $2^3 \times 3^7$, only for powers of the same number.

EXAMPLE: $a = 5^9$ and $b = 5^4 \times 5^2$. What is the value of $\frac{a}{b}$?

1) Work out b — add the powers: $b = 5^4 \times 5^2 = 5^{4+2} = 5^6$
2) Divide a by b — subtract the powers: $\frac{a}{b} = 5^9 \div 5^6 = 5^{9-6} = 5^3 = 125$

One Trickier Rule

To find a negative power — turn it upside-down.

People have real difficulty remembering this — whenever you see a negative power you need to immediately think: "Aha, that means turn it the other way up and make the power positive".

E.g. $7^{-2} = \frac{1}{7^2} = \frac{1}{49}$, $\quad \left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^2 = \frac{5^2}{3^2} = \frac{25}{9}$

Practise these power rules — you never know when you'll need them

If you can add, subtract and multiply, there's nothing on this page you can't do — as long as you learn the rules. Try copying them over and over until you can do it with your eyes closed.

Roots

Take a deep breath, and get ready to tackle this page.

Square Roots

GRADE 2

'Squared' means 'multiplied by itself': $8^2 = 8 \times 8 = 64$

SQUARE ROOT $\sqrt{}$ is the reverse process: $\sqrt{64} = 8$

'Square Root' means 'What Number Times by Itself gives...'

EXAMPLES:

1. **What is $\sqrt{49}$?**

49 is a square number — make sure you know all the square numbers on p.1 so you can answer questions like this without a calculator.

7 times by itself gives 49: $49 = 7 \times 7$

So $\sqrt{49} = 7$

2. **What is $\sqrt{29.16}$?**

Use your calculator. Press: $\sqrt{}$ 29.16 = 5.4

3. **Find both square roots of 36.** GRADE 3

$6 \times 6 = 36$, so positive square root $= 6$

$-6 \times -6 = 36$, so negative square root $= -6$

All numbers also have a NEGATIVE SQUARE ROOT — it's just the '–' version of the normal positive one.

This little rule for multiplying roots might come in handy in your exam: $\sqrt{a} \times \sqrt{a} = a$

Be careful — this is only true if you're multiplying together two roots which are the same.

Cube Roots

GRADE 2

'Cubed' means 'multiplied by itself and then by itself again': $2^3 = 2 \times 2 \times 2 = 8$

CUBE ROOT $\sqrt[3]{}$ is the reverse process: $\sqrt[3]{8} = 2$

'Cube Root' means 'What Number Times by Itself and then by Itself Again gives...'

You need to be able to write down the cube roots of the cube numbers given on p.1 without a calculator.

To find the cube root of any other number you can use your calculator — press $\sqrt[3]{}$.

EXAMPLES:

1. **What is $\sqrt[3]{27}$?**

27 is a cube number.

3 times by itself and then by itself again gives 27: $27 = 3 \times 3 \times 3$

So $\sqrt[3]{27} = 3$

2. **What is $\sqrt[3]{4913}$?**

Press: $\sqrt[3]{}$ 4913 = 17

You can use your calculator to find any root of a number, using the $\sqrt[x]{}$ or $\sqrt[\square]{\square}$ buttons.

Make sure you know how to use your calculator to find higher order roots — the buttons might be slightly different to these ones.

Learn the difference between square roots and square numbers

Remember — a square root is a number which, when multiplied by itself, gives the number under the root symbol. When you find a square root, you'll get both a positive and a negative answer.

Standard Form

Standard form is useful for writing VERY BIG or VERY SMALL numbers in a more convenient way.

E.g. 56 000 000 000 would be 5.6×10^{10} in standard form.

0.000 000 003 45 would be 3.45×10^{-9} in standard form.

But ANY NUMBER can be written in standard form and you need to know how to do it:

What it Actually is:

A number written in standard form must always be in exactly this form:

This number must always be between 1 and 10.

(The fancy way of saying this is $1 \le A < 10$)

$$\boxed{A \times 10^n}$$

This number is just the number of places the decimal point moves.

Learn the Three Rules:

1) The front number must always be between 1 and 10.
2) The power of 10, n, is how far the decimal point moves.
3) n is positive for BIG numbers, n is negative for SMALL numbers.
(This is much better than rules based on which way the decimal point moves.)

Four Important Examples:

1 **Express 35 600 in standard form.**

1) Move the decimal point until 35 600 becomes 3.56 ($1 \le A < 10$)
2) The decimal point has moved 4 places so n = 4, giving: 10^4
3) 35 600 is a big number so n is +4, not –4

3 5 6 0 0.0

$= 3.56 \times 10^4$

2 **Express 0.0000623 in standard form.**

1) The decimal point must move 5 places to give 6.23 ($1 \le A < 10$).
So the power of 10 is 5.
2) Since 0.0000623 is a small number it must be 10^{-5} not 10^{+5}

0.0 0 0 0 6 2 3

$= 6.23 \times 10^{-5}$

3 **Express 4.95×10^{-3} as an ordinary number.**

1) The power of 10 is negative, so it's a small number — the answer will be less than 1.
2) The power is –3, so the decimal point moves 3 places.

0 0 0 4.9 5 $\times 10^{-3}$

= 0.00495

4 **Which is the largest number in the following list?** 9.5×10^8 2.7×10^5 3.6×10^8 5.6×10^6

1) Compare the powers first.

9.5×10^8 and 3.6×10^8 have the biggest powers so one of them is the largest.

2) Then, compare the front numbers.

9.5 is greater than 3.6

So 9.5×10^8 is the largest number.

Standard Form

You might be asked to add, subtract, multiply or divide using numbers in standard form without using a calculator.

Multiplying and Dividing

GRADE 5

1) Rearrange to put the front numbers and the powers of 10 together.
2) Multiply or divide the front numbers, and use the power rules (see p.28) to multiply or divide the powers of 10.
3) Make sure your answer is still in standard form.

EXAMPLES:

1. **Find $(2 \times 10^3) \times (6 \times 10^5)$ without using a calculator. Give your answer in standard form.**

$(2 \times 10^3) \times (6 \times 10^5)$

Multiply front numbers and powers separately

$= (2 \times 6) \times (10^3 \times 10^5)$

$= 12 \times 10^{3+5}$ Add the powers (see p.28)

$= 12 \times 10^8$

Not in standard form so convert it — divide the number by 10...

$= 1.2 \times 10^9$ ... and multiply the power by 10.

2. **Calculate $(2 \times 10^5) \div (4 \times 10^{10})$ without using a calculator. Give your answer in standard form.**

$(2 \times 10^5) \div (4 \times 10^{10})$

$= \frac{2 \times 10^5}{4 \times 10^{10}} = \frac{2}{4} \times \frac{10^5}{10^{10}}$

Divide front numbers and powers separately

$= 0.5 \times 10^{5-10}$ Subtract the powers (see p.28)

$= 0.5 \times 10^{-5}$

Not in standard form so convert it — multiply the number by 10...

$= 5 \times 10^{-6}$ ... and divide the power by 10.

Adding and Subtracting

GRADE 5

1) Make sure the powers of 10 are the same.
2) Add or subtract the front numbers.
3) Convert the answer to standard form if necessary.

EXAMPLE:

Calculate $(2.8 \times 10^4) + (6.6 \times 10^4)$ without using a calculator. Give your answer in standard form.

1) Check that both powers of 10 are equal. $(2.8 \times 10^4) + (6.6 \times 10^4)$
2) Then add the front numbers. $= (2.8 + 6.6) \times 10^4$
3) This is in standard form, so you don't need to convert it. $= 9.4 \times 10^4$

To put standard form numbers into your calculator, use the EXP or the $\times 10^x$ button.

E.g. enter 2.67×10^{15} by pressing 2.67 EXP 15 = or 2.67 $\times 10^x$ 15 = .

Remember, n tells you how far the decimal point moves

You don't have to use the method above, you can add and subtract numbers in standard form by writing them as normal numbers, adding or subtracting as usual, then converting the answer back to standard form.

Warm-up and Worked Exam Questions

I know that you'll be champing at the bit to get into the exam questions, but these warm-up questions are invaluable for getting the basic facts straight first.

Warm-up Questions

1) a) Find $3^3 + 4^2$ without a calculator.
   b) Use your calculator to find 6.2^3.
   c) Write ten thousand as a power of 10.
2) Simplify: a) $4^2 \times 4^3$ b) $\frac{7^6}{7^3}$ c) $(q^2)^4$
3) Without using a calculator, find: a) $\frac{6^3 \times 6^5}{6^6}$ b) 2^{-4}
4) Find a) $\sqrt{196}$ b) $\sqrt[3]{9261}$ c) $\sqrt[7]{2187}$
5) The volume of a cube is 1.728 cm³. Find the length of one of its sides, in cm.
6) Work out $\sqrt[3]{19.34} + (1.3 + 2.5)^2$. Write down the full calculator display.
7) Write a) 4.32×10^8 as an ordinary number.
   b) 0.000387 in standard form.
8) Which of these numbers is the smallest? 2.6×10^{-3} 1.9×10^{-3} 3.4×10^{-2}
9) Work out the following. Give your answers in standard form.
   a) $(9 \times 10^7) \div (3 \times 10^3)$
   b) $(6.7 \times 10^{10}) - (4.8 \times 10^{10})$

Worked Exam Question

With the answers written in, it's very easy to just skim over this worked exam question. But that's not really going to help you, so take the time to make sure you've really understood it.

1 The table on the right shows the masses of four different particles. GRADE 5

Particle	Mass (g)
Particle A	2.1×10^{-7}
Particle B	8.6×10^{-8}
Particle C	1.4×10^{-6}
Particle D	3.2×10^{-7}

a) Which particle is the heaviest?

Look at the power first. –6 moves the decimal point fewer places to the left than the others.

C
[1 mark]

b) What is the mass of particle C?
Give your answer as an ordinary number.

$1.4 \times 10^{-6} = 0.0000014$ g

Move the decimal point 6 places.

0.0000014 g
[1 mark]

c) How much more does particle D weigh than particle A?
Give your answer in standard form.

$(3.2 \times 10^{-7}) - (2.1 \times 10^{-7}) = (3.2 - 2.1) \times 10^{-7}$
$= 1.1 \times 10^{-7}$ g

The power of 10 is the same for particles D and A, so it's just a simple subtraction.

1.1×10^{-7} g
[2 marks]

Exam Questions

2 A square has an area of 6.25 cm². Find the length of one side of the square.

.... cm
[1 mark]

3 $A = 4.834 \times 10^9$, $B = 2.4 \times 10^5$, $C = 5.21 \times 10^3$

a) Write A as an ordinary number.

....
[1 mark]

b) Put A, B and C in order from smallest to largest.

.... , ... , ...
[1 mark]

4 Simplify the expression $\frac{3^4 \times 3^7}{3^6}$. Leave your answer in index form.

....
[2 marks]

5 Work out the value of:

a) $6^5 \div 6^3$

....
[1 mark]

b) $(2^4 \times 2^7) \div (2^3 \times 2^2)^2$

....
[2 marks]

6 Light travels at approximately 2×10^5 miles per second.
The distance from the Earth to the Sun is approximately 9×10^7 miles.

How long will it take light to travel this distance?

Use the formula:
time (s) = distance (miles) ÷ speed (miles/s)

.... seconds
[2 marks]

Revision Questions for Section One

Well, that wraps up Section One — time to put yourself to the test and find out how much you really know.

- Try these questions and tick off each one when you get it right.
- When you've done all the questions for a topic and are completely happy with it, tick off the topic.

Calculators are only allowed in questions 2, 7, 9, 10, 15 and 22. Sorry.

Arithmetic (p1-6)

1) What are square numbers? Write down the first ten of them.`
2) Using the numbers 2, 4 and 5, and +, –, × and ÷, what is the smallest possible positive number you can make? You can use each number/operation a maximum of once. You may also use brackets.
3) Tickets for a show cost £12 each. A senior's ticket is half price. A child's ticket is a third of the full price. How much does it cost for a family of 2 adults, 2 children and 1 senior to watch the show?
4) Find: a) £1.20 × 100 b) £150 ÷ 300
5) Work out: a) 51 × 27 b) 338 ÷ 13 c) 3.3 × 19 d) 4.2 ÷ 12
6) Find: a) $-10 - 6$ b) $-35 \div -5$ c) $-4 + -5 + 22 - -7$

Types of Number, Factors and Multiples (p9-12)

7) Find all the prime numbers between 40 and 60 (there are 5 of them).
8) What are multiples? Find the first six multiples of: a) 10 b) 4
9) Express each of these as a product of prime factors: a) 210 b) 1050
10) Find: a) the HCF of 42 and 28 b) the LCM of 8 and 10

Fractions and Decimals (p15-19)

11) Work out without a calculator: a) $\frac{25}{6} \div \frac{8}{3}$ b) $\frac{2}{3} \times 4\frac{2}{5}$ c) $\frac{5}{8} + \frac{9}{4}$ d) $\frac{2}{3} - \frac{1}{7}$
12) Calculate a) $\frac{4}{7}$ of 560 b) $\frac{2}{5}$ of £150
13) Amy, Brad and Cameron are all playing a video game. Amy has completed $\frac{5}{8}$ of the game, Brad has completed $\frac{7}{11}$ of the game and Cameron has completed $\frac{15}{22}$ of the game. Who has the largest fraction of the game left to complete?
14) Write: a) 0.04 as: (i) a fraction (ii) a percentage b) 65% as: (i) a fraction (ii) a decimal
15) a) What is a recurring decimal? b) Write $\frac{2}{9}$ as a recurring decimal.

Rounding and Estimating (p22-25)

16) Round: a) 17.65 to 1 d.p. b) 6743 to 2 s.f. c) 3 643 510 to the nearest million.
17) Estimate the value of a) $\frac{17.8 \times 32.3}{6.4}$ b) $\frac{96.2 \times 7.3}{0.463}$
18) Give the error intervals for x and y if $x = 200$ when rounded to 1 s.f.

Powers and Roots (p28-29)

19) If $f = 7^6 \times 7^4$ and $g = 7^5$, what is $f \div g$?
20) What is the value of 5^{-2}? Give your answer as a fraction.
21) Find without using a calculator: a) $\sqrt{121}$ b) $\sqrt[3]{64}$ c) $8^2 - 2^3$ d) 100 000 as a power of ten.
22) Use a calculator to find: a) 7.5^3 b) $\sqrt{23.04}$ c) $\sqrt[3]{512}$ d) $\sqrt[5]{161051}$

Standard Form (p30-31)

23) What are the three rules for writing numbers in standard form?
24) Write: a) 3 560 000 000 in standard form b) 2.75×10^{-6} as an ordinary number.
25) Calculate: a) $(3.2 \times 10^6) \div (1.6 \times 10^3)$ b) $(5 \times 10^{11}) + (7 \times 10^{11})$
Give your answers in standard form.

Algebra — Simplifying

Algebra really terrifies so many people. But honestly, it's not that bad.
Make sure you understand and learn these basics for dealing with algebraic expressions.

Terms

GRADE 2

Before you can do anything else with algebra, you must understand what a term is:

A TERM is a collection of numbers, letters and brackets, all multiplied/divided together

Terms are separated by + and – signs. Every term has a + or – attached to the front of it.

If there's no sign in front of the first term, it means there's an invisible + sign.

$4x^2 \quad + 5x \quad - 2y \quad + 6y^2 \quad + 4$

'x^2' term 'x' term 'y' term 'y^2' term 'number' term

Simplifying or 'Collecting Like Terms'

GRADE 2

To simplify an algebraic expression made up of all the same terms, just add or subtract them.

EXAMPLES:

1. **Simplify $q + q + q + q + q$**

Just add up all the q's:

'q' just means '1q'. $q + q + q + q + q = 5q$

2. **Simplify $4t + 5t - 2t$**

Again, just combine the terms — don't forget there's a '–' before the 2t:

$4t + 5t - 2t = 7t$

If you have a mixture of different terms, it's a bit more tricky. To simplify an algebraic expression like this, you combine 'like terms' (e.g. all the x terms, all the y terms, all the number terms etc.).

EXAMPLE:

Simplify $2x - 4 + 5x + 6$

Invisible + sign — number terms — x-terms

$2x \; -4 \; +5x \; +6 \; = \; \underbrace{2x \; +5x}_{7x} \; \underbrace{-4 \; +6}_{+2} \; = \; 7x + 2$

1) Put bubbles round each term — be sure you capture the +/– sign in front of each.
2) Then you can move the bubbles into the best order so that like terms are together.
3) Combine like terms.

EXAMPLE:

Simplify $4 + \sqrt{3} - 1 + 3\sqrt{3}$

Just treat $\sqrt{3}$-terms like x-terms — don't combine them with the number terms.

GRADE 5

Invisible + sign — $\sqrt{3}$-terms — number-terms

$4 \; +\sqrt{3} \; -1 \; +3\sqrt{3} \; = \; \underbrace{4 \; -1}_{3} \; \underbrace{+\sqrt{3} \; +3\sqrt{3}}_{+4\sqrt{3}} \; = 3 + 4\sqrt{3}$

You need to know what terms are — and how to collect like ones

Terms are collections of numbers and letters separated by + and – signs. When you collect like terms together, you combine x terms, or y terms, or xy terms, or x^2 terms or y^2 terms or...

Algebra — Multiplying and Dividing

Multiplying algebra is a lot like multiplying numbers — here are a few rules to get you started.

Multiplying and Dividing Letters

GRADE 3

Watch out for these combinations of letters in algebra that regularly catch people out:

1) abc means $a \times b \times c$ and $3a$ means $3 \times a$. The ×'s are often left out to make it clearer.

2) gn^2 means $g \times n \times n$. Note that only the n is squared, not the g as well.

3) $(gn)^2$ means $g \times g \times n \times n$. The brackets mean that BOTH letters are squared.

4) Powers tell you how many letters are multiplied together — so $r^6 = r \times r \times r \times r \times r \times r$.

5) $\frac{a}{b}$ means $a \div b$. Use the power rules from p.28 if you're dividing powers of the same letter.

EXAMPLES:

Careful — k times itself 4 times is k^4, not 4k (4k means k + k + k + k or 4 × k).

1. Simplify $k \times k \times k \times k$

You have 4 k's multiplied together:

$k \times k \times k \times k = k^4$

2. Simplify $2p \times 3q \times 5$

Multiply the numbers together, then the letters together:

$2p \times 3q \times 5 = 2 \times 3 \times 5 \times p \times q = 30pq$

3. Simplify $6m^2 \div 8m$

Write as a fraction then simplify: $6m^2 \div 8m = \frac{6m^2}{8m} = \frac{3}{4}m$

$m^2 \div m = m$

Leave the number in front of the letter as a fraction not a decimal.

Multiplying Brackets

GRADE 3

The key thing to remember about multiplying brackets is that the thing outside the brackets multiplies each separate term inside the brackets.

EXAMPLE:

Expand the following:

a) $3(2x + 5)$

$= (3 \times 2x) + (3 \times 5)$

$= 6x + 15$

b) $-4(3y - 2)$

$= (-4 \times 3y) + (-4 \times -2)$

$= -12y + 8$

c) $2e(e - 4)$

$= (2e \times e) + (2e \times -4)$

$= 2e^2 - 8e$

EXAMPLE:

Expand $x(2x + 1) + y(y - 4) + 3x(y + 2)$

1) Expand each bracket separately.

$x(2x + 1) + y(y - 4) + 3x(y + 2)$

$= 2x^2 + x + y^2 - 4y + 3xy + 6x$

2) Group together like terms.

$= 2x^2 + x + 6x + 3xy + y^2 - 4y$

3) Simplify the expression.

$= 2x^2 + 7x + 3xy + y^2 - 4y$

Watch out when multiplying letters together

In the cases where multiplying and dividing letters gets a bit tricky, just remember the five points above. Remember the power rules too — when multiplying, add powers, and when dividing, subtract powers.

Multiplying Double Brackets

Double brackets are a bit more tricky than single brackets — this time, you have to multiply everything in the first bracket by everything in the second bracket.

Use the FOIL Method to Multiply Out Double Brackets

There's a handy way to multiply double brackets — it's called the FOIL method and works like this:

First — multiply the first term in each bracket together

Outside — multiply the outside terms (i.e. the first term in the first bracket by the second term in the second bracket)

Inside — multiply the inside terms (i.e. the second term in the first bracket by the first term in the second bracket)

Last — multiply the second term in each bracket together

When multiplying double brackets, you get 4 terms — and 2 of them usually combine to leave 3 terms.

EXAMPLES:

1. **Expand and simplify $(x + 3)(x + 8)$**

$$\begin{aligned}
& \qquad\qquad\quad \text{F} \qquad\qquad\quad \text{O} \qquad\qquad \text{I} \qquad\qquad \text{L} \\
(x + 3)(x + 8) &= (x \times x) + (x \times 8) + (3 \times x) + (3 \times 8) \\
&= x^2 + 8x + 3x + 24 \\
&= x^2 + 11x + 24
\end{aligned}$$

The two x-terms combine together.

2. **Expand and simplify $(n - 2)(2n + 7)$**

$$\begin{aligned}
& \qquad\qquad\quad \text{F} \qquad\qquad\quad \text{O} \qquad\qquad \text{I} \qquad\qquad \text{L} \\
(n - 2)(2n + 7) &= (n \times 2n) + (n \times 7) + (-2 \times 2n) + (-2 \times 7) \\
&= 2n^2 + 7n - 4n - 14 \\
&= 2n^2 + 3n - 14
\end{aligned}$$

Write Out Squared Brackets as Double Brackets

Always write out squared brackets as two brackets (to avoid mistakes) — then multiply them out using the FOIL method above.

EXAMPLE:

Expand and simplify $(3x + 2)^2$

$$\begin{aligned}
(3x + 2)^2 &= (3x + 2)(3x + 2) \\
&= 9x^2 + 6x + 6x + 4 \\
&= 9x^2 + 12x + 4
\end{aligned}$$

Write out the expression as two brackets, then use the FOIL method.

DON'T make the mistake of thinking that $(3x + 2)^2 = 9x^2 + 4$ (this is wrong wrong wrong).

Use the FOIL method to make sure you don't miss out any terms

When multiplying out double brackets, don't rush — you'll make mistakes and throw away easy marks. Remember to keep an eye out for minus signs too, and combine like terms.

Factorising

Right, now you know how to expand brackets, it's time to put them back in. This is known as factorising.

Factorising — Putting Brackets In

GRADE 4

This is the exact reverse of multiplying out brackets. You have to look for common factors — numbers or letters that go into every term. Here's the method to follow:

1) Take out the biggest number that goes into all the terms.
2) For each letter in turn, take out the highest power (e.g. x, x^2, etc.) that will go into EVERY term.
3) Open the bracket and fill in all the bits needed to reproduce each term.
4) Check your answer by multiplying out the bracket and making sure it matches the original expression.

EXAMPLES:

1. Factorise $8y + 4$

Biggest number that goes into 8 and 4

Decide what you need to multiply 4 by to get $8y$ and 4.

$4(2y + 1)$

Check: $4(2y + 1) = 8y + 4$ ✓

2. Factorise $3x^2 + 6x$

Biggest number that goes into 3 and 6

Highest power of x that will go into both terms

$3x(x + 2)$

Check: $3x(x + 2) = 3x^2 + 6x$ ✓

REMEMBER: The bits taken out and put at the front are the common factors. The bits inside the bracket are what's needed to get back to the original terms if you multiply the bracket out again.

D.O.T.S. — The Difference Of Two Squares

GRADE 5

The 'difference of two squares' (D.O.T.S. for short) is where you have 'one thing squared' take away 'another thing squared'. There's a quick and easy way to factorise it — just use the rule below:

$$a^2 - b^2 = (a + b)(a - b)$$

EXAMPLE:

Factorise:

a) $t^2 - 1$ — $t^2 - 1 = (t + 1)(t - 1)$
Don't forget that 1 is a square number.

b) $s^2 - 64$ — $s^2 - 64 = (s + 8)(s - 8)$
$64 = 8^2$, so in the formula above, $b = 8$.

c) $25m^2 - 1$ — $25m^2 - 1 = (5m + 1)(5m - 1)$
Here you have to remember that it's 5m, not just m.

d) $9p^2 - 16q^2$ — $9p^2 - 16q^2 = (3p + 4q)(3p - 4q)$
This time you had to spot that 9 and 16 are square numbers.

Factorising is the opposite of multiplying out brackets

There's no excuse for making mistakes when factorising — you can check your answer by multiplying the brackets out again. Do it right, and you'll get back to the original expression.

Warm-up and Worked Exam Questions

It's time to test your basic algebra skills — have a go at these warm-up questions to see what you know. If you get any wrong, go back over this section, then try again.

Warm-up Questions

1) Simplify the following expressions:
   a) $6b + 8b - 3b - b$ b) $5x + y - 2x + 7y$ c) $6 + 5\sqrt{5} + 3 - 2\sqrt{5}$
2) Simplify the following expressions:
   a) $5r \times -2s \times 6$ b) $7(3m - 2)$ c) $4p(p + 2q)$
3) Show that $5(x + 8) + 2(x - 12) = 7x + 16$.
4) Expand and simplify:
   a) $(x + 6)(x - 5)$ b) $(2y - 1)(y + 9)$ c) $(x - 3)^2$ d) $(4y + 5)^2$
5) Factorise the following expressions:
   a) $12x + 30$ b) $6y + 15y^2$ c) $x^2 - 25$ d) $36x^2 - 49y^2$

Worked Exam Questions

I've gone through these questions and written in answers, just like you'll do in the exam. They should really help with the questions which follow, so don't say I never do anything for you.

1 Circle the simplified version of $4s - 3s + 9s$. (Grade 2)

They're all s-terms so just add/subtract the numbers: $4 - 3 + 9 = 10$.

$16s$ $12s$ $10s$ $-8s$ $11s$

[1 mark]

2 Expand and simplify where possible. (Grade 3)

a) $3(x - 1) + 5(x + 2)$

Multiply out the brackets first, then collect like terms.

$$(3 \times x) + (3 \times -1) + (5 \times x) + (5 \times 2) = 3x - 3 + 5x + 10$$
$$= 8x + 7$$

$8x + 7$

[2 marks]

b) $4a(a + 2b)$

$$(4a \times a) + (4a \times 2b) = 4a^2 + 8ab$$

$4a^2 + 8ab$

[1 mark]

c) $9 - 3(x + 2)$

$$9 + (-3 \times x) + (-3 \times 2) = 9 - 3x - 6$$
$$= 3 - 3x$$

Careful with the minus signs here.

$3 - 3x$

[2 marks]

Exam Questions

3 Simplify the following.

a) $p + p + p + p$

....
[1 mark]

b) $m + 3m - 2m$

....
[1 mark]

c) $7r - 2p - 4r + 6p$

....
[2 marks]

4 Factorise the expression below.

$6x + 3$

....
[1 mark]

5 Expand and simplify the following:

a) $(x + 2)(x + 4)$

....
[2 marks]

b) $(y + 3)(y - 3)$

....
[2 marks]

c) $(2z - 1)(z - 5)$

....
[2 marks]

6 Factorise the following expressions.

Use the special rule for factorising the difference of two squares.

a) $x^2 - 49$

....
[2 marks]

b) $9x^2 - 100$

....
[2 marks]

c) $y^2 - m^2$

....
[2 marks]

Solving Equations

'Solving equations' basically means 'find the value of x (or whatever letter is used) that makes the equation true'. To do this, you usually have to rearrange the equation to get x on its own.

The 'Common Sense' Approach

GRADE 2

The trick here is to realise that the unknown quantity 'x' is just a number and the 'equation' is a cryptic clue to help you find it.

EXAMPLE:

Solve the equation $3x + 4 = 46$.

This just means 'find the value of x'.

This is what you should say to yourself:

'Something + 4 = 46', hmmm, so that 'something' must be 42.

So that means $3x = 42$, which means '3 × something = 42'.

So it must be $42 \div 3 = 14$, so $x = 14$.

If you were writing this down in an exam question, just write down the bits in blue.

In other words don't think of it as algebra, but as 'find the mystery number'.

The 'Proper' Way

GRADE 2

The 'proper' way to solve equations is to keep rearranging them until you end up with '$x =$' on one side. There are a few important points to remember when rearranging.

Golden Rules

1) Always do the SAME thing to both sides of the equation.
2) To get rid of something, do the opposite.
   The opposite of + is – and the opposite of – is +.
   The opposite of × is ÷ and the opposite of ÷ is ×.
3) Keep going until you have a letter on its own.

EXAMPLES:

1\. **Solve $x + 7 = 11$.**

The opposite of +7 is –7

$x + 7 = 11$

(–7) $x + 7 - 7 = 11 - 7$

$x = 4$

This means 'take away 7 from both sides'.

2\. **Solve $x - 3 = 7$.**

The opposite of –3 is +3

$x - 3 = 7$

(+3) $x - 3 + 3 = 7 + 3$

$x = 10$

3\. **Solve $5x = 15$.**

$5x$ means $5 \times x$, so do the opposite — divide both sides by 5

$5x = 15$

(÷5) $5x \div 5 = 15 \div 5$

$x = 3$

4\. **Solve $\frac{x}{3} = 2$.**

$\frac{x}{3}$ means $x \div 3$, so do the opposite — multiply both sides by 3

$\frac{x}{3} = 2$

(×3) $\frac{x}{3} \times 3 = 2 \times 3$

$x = 6$

That's it — all the steps you need to solve any of these equations

It's always good to know the proper way to solve equations, just in case you get thrown a curveball in the exam and they give you a real nightmare of an equation to solve.

Solving Equations

You're not done with solving equations yet — not by a long shot.

Two-Step Equations

If you come across an equation like $4x + 3 = 19$ (where there's an x-term and a number on the same side), use the methods from the previous page to solve it — just do it in two steps:

1) Add or subtract the number first.
2) Multiply or divide to get 'x = '.

EXAMPLE: **Solve the equation $4x - 3 = 17$.**

$4x - 3 = 17$ — The opposite of –3 is +3, so add 3 to both sides.

(+3) $4x - 3 + 3 = 17 + 3$

$4x = 20$ — The opposite of ×4 is ÷4, so divide both sides by 4.

(÷4) $4x \div 4 = 20 \div 4$

$x = 5$

Equations with an 'x' on Both Sides

For equations like $2x + 3 = x + 7$ (where there's an x-term on each side), you have to:

1) Get all the x's on one side and all the numbers on the other.
2) Multiply or divide to get 'x = '.

EXAMPLE: **Solve the equation $3x + 5 = 5x + 7$.**

$3x + 5 = 5x + 7$ — To get the x's on only one side, subtract $3x$ from each side.

($-3x$) $3x + 5 - 3x = 5x + 7 - 3x$

$5 = 2x + 7$ — Now subtract 7 to get the numbers on the other side.

(–7) $5 - 7 = 2x + 7 - 7$

$-2 = 2x$ — The opposite of ×2 is ÷2, so divide both sides by 2.

(÷2) $-2 \div 2 = 2x \div 2$

$-1 = x$

Don't be put off by the fact that the x ends up on the right, not the left. $-1 = x$ is exactly the same as $x = -1$.

Equations with Brackets

If the equation has brackets in, you have to multiply out the brackets (see p.36) before solving it as above.

EXAMPLE: **Solve the equation $5x + 3 = 4(x + 2)$.**

$5x + 3 = 4(x + 2)$ — Multiply out the brackets.

$5x + 3 = 4x + 8$ — To get the x's on only one side, subtract $4x$ from each side.

($-4x$) $5x + 3 - 4x = 4x + 8 - 4x$

$x + 3 = 8$ — The opposite of +3 is –3, so subtract 3 from each side.

(–3) $x + 3 - 3 = 8 - 3$

$x = 5$

You can use the same methods to solve all sorts of equations

If your equation has brackets, multiply them out before solving it as normal.
You can check your answer by putting it back into the original equation to see if it works.

Expressions, Formulas and Functions

Before we get started, there are a few definitions you need to know:

1) EXPRESSION — a collection of terms (see p.35). Expressions DON'T have an = sign in them.
2) EQUATION — an expression with an = sign in it (so you can solve it)
3) FORMULA — a rule that helps you work something out (it will also have an = sign in it).
4) FUNCTION — an expression that takes an input value, processes it and produces an output value.

Putting Numbers into Formulas

GRADE 2

You might be given a formula and asked to work out its value when you put in certain numbers.
All you have to do here is follow this method.

1) Write out the formula.
2) Write it again, directly underneath, but substituting numbers for letters on the RHS (right-hand side).
3) Work it out in stages. Use BODMAS (see p.1) to work things out in the right order.
   Write down values for each bit as you go along.
4) DO NOT attempt to do it all in one go on your calculator — you're more likely to make mistakes.

EXAMPLE:

The formula for converting from Celsius (C) to Fahrenheit (F) is $F = \frac{9}{5}C + 32$.
Use this formula to convert –10 °C into Fahrenheit.

$F = \frac{9}{5}C + 32$ — 1) Write out the formula.

$F = \frac{9}{5} \times -10 + 32$ — 2) Write it again, substituting numbers for letters on the RHS.

$F = -18 + 32$ — 3) Use BODMAS to work things out in the right order — do the multiplication first, then do the addition.

$F = 14$ so -10 °C $= 14$ °F

Be careful when substituting negative numbers into a formula — just do it step-by-step.

Functions Produce Outputs from Inputs

GRADE 2

1) A function takes an input, processes it (e.g. multiplies it by 5 and adds 2) and outputs a value.
2) If you have to use a function machine, just put in the number, follow the steps and see what comes out.
3) If you're given the output and have to find the input, use the function machine in reverse.

EXAMPLE:

The function machine below represents the function 'multiply by 5 and add 2'.

$$x \longrightarrow \boxed{\times 5} \longrightarrow \boxed{+\,2} \longrightarrow y$$

If this was an equation, it'd be written as $y = 5x + 2$.

a) Find the value of y when $x = 5$.

Just put 5 into the machine: $5 \xrightarrow{\times 5} 25 \xrightarrow{+2} 27$. So $y = 27$.

b) Find the value of x when $y = 42$.

This time, put $y = 42$ into the machine and work backwards:

$42 \xrightarrow{-2} 40 \xrightarrow{\div 5} 8$. So $x = 8$.

Don't forget to reverse each step as well — so +2 becomes –2 and ×5 becomes ÷5.

Function machines are just a series of operations

Given the input, it's easy to find the output of a function machine — just do the operations step-by-step.
Finding the input if you're given the output is trickier — work through backwards and reverse each step.

Formulas and Equations from Words

Making expressions or formulas from words can be a bit confusing as you're given a lot of information in one go. You just have to go through it slowly and carefully and extract the maths from it.

Make Expressions or Formulas from Given Information

GRADE 3

Here are some of examples of how to use the information to write expressions and formulas.

EXAMPLE:

Tiana is x years old. Leah is 5 years younger than Tiana. Martin is 4 times as old as Tiana. Find a simplified expression for the sum of their ages in terms of x.

Tiana's age is x

Leah's age is $x - 5$ — Leah is 5 years younger, so subtract 5

Martin's age is $4 \times x = 4x$ — 4 times older

The sum of their ages is:

$x + (x - 5) + 4x = 6x - 5$

If you'd been told the sum of their ages, you'd have to set your expression equal to the sum and solve it to find x.

EXAMPLE:

In rugby union, tries score 5 points and conversions score 2 points. A team scores a total of P points, made up of t tries and c conversions. Write a formula for P in terms of t and c.

Tries score 5 points — t tries will score $5 \times t = 5t$ points

Conversions score 2 points — c conversions will score $2 \times c = 2c$ points

So total points scored are $P = 5t + 2c$

Because you're asked for a formula, you must include the 'P = ' bit to get full marks (i.e. don't just put 5t + 2c).

Use Your Expression to Solve Equations

GRADE 4

Sometimes, you might be asked to use an expression to solve an equation.

EXAMPLE:

A zoo has x zebras and four times as many lemurs. The difference between the number of zebras and the number of lemurs is 45. How many zebras does the zoo have?

The zoo has x zebras and $4 \times x = 4x$ lemurs.

The difference is $4x - x = 3x$, so $3x = 45$, which means $x = 15$. — Once you've formed the equation, you need to solve it to find the value of x.

So the zoo has **15 zebras**.

EXAMPLE:

Will, Naveed and Camille give some books to charity. Naveed gives 6 more books than Will, and Camille gives 7 more books than Naveed. Between them, they give away 46 books. How many books did they give each?

Let the number of books Will gives be x.

Then Naveed gives $x + 6$ books

and Camille gives $(x + 6) + 7 = x + 13$ books

So in total they give $x + x + 6 + x + 13 = 3x + 19$ books

So $3x + 19 = 46$ — You're told this in the question.

$3x = 27$

$x = 9$

So Will gives **9 books**,

Naveed gives $9 + 6 =$ **15 books** and

Camille gives $15 + 7 =$ **22 books**.

Writing expressions and formulas isn't hard once you get the idea

Exam questions might not tell you to write an equation or formula — sometimes you have to come up with one for yourself to be able to answer the question, so make sure you get lots of practice.

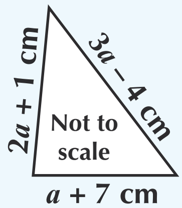
Formulas and Equations from Diagrams

Formulas and equations can even sneak into area and perimeter questions
— have a look at page 112 if you need to brush up on those topics.

Use **Shape Properties** to Find **Formulas** and **Equations**

In some questions, you'll need to use what you know about shapes (e.g. side lengths or areas) to come up with a formula or an equation to solve.

EXAMPLE:



a) Write a formula for *P*, the perimeter of the triangle below, in terms of *a*.

Form an expression for the perimeter:

$P = (a + 7) + (2a + 1) + (3a - 4)$

$P = 6a + 4$

b) If the triangle has a perimeter of 58 cm, find the value of *a*.

$P = 58$, so set your formula equal to 58 and solve to find a:

$6a + 4 = 58$

$6a = 54$

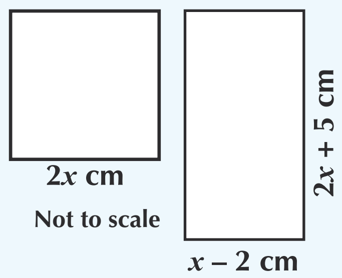
$a = 9$

Compare **Dimensions** of **Two Shapes** to Find Equations

You might get a question that involves two shapes with related areas or perimeters — you'll have to use this fact to find side lengths or missing values.

EXAMPLE:

The perimeter of the rectangle is the same as the perimeter of the square. Find the value of *x*.



Perimeter of square $= 2x + 2x + 2x + 2x = 8x$ cm

Perimeter of rectangle $= (2x + 5) + (x - 2) + (2x + 5) + (x - 2)$
$= 6x + 6$ cm

Set the perimeter of the rectangle equal to the perimeter of the square and solve:

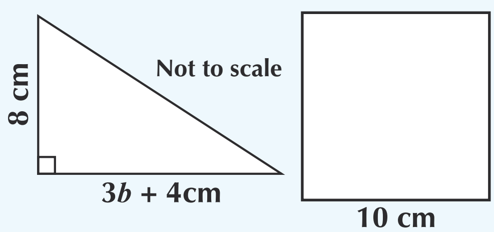
$8x = 6x + 6$

$2x = 6$

$x = 3$

EXAMPLE:

The area of the square is the same as the area of the triangle. Find the length of the base of the triangle.



Area of triangle: $\frac{1}{2} \times (3b + 4) \times 8 = 12b + 16$ cm^2

Area of square: $10^2 = 100$ cm^2

Set the areas equal to each other and solve:

$12b + 16 = 100$

$12b = 84$

$b = 7$

So base length $= (3 \times 7) + 4 = 25$ cm

Formulas and equations can crop up in lots of different topics

Don't be put off by the shapes — once you've turned the information into an equation, it's just normal algebra. As long as you can find the perimeter and area of simple shapes you won't have much trouble.

Rearranging Formulas

The subject of a formula is the letter on its own before the = (so x is the subject of $x = 2y + 3z$).

Changing the Subject of a Formula

GRADE 5

Rearranging formulas means making a different letter the subject, e.g. getting '$y =$ ' from '$x = 3y + 2$'. Fortunately, you can use the same methods that you used for solving equations (see p.41-42) — here's a quick reminder:

Golden Rules

1) Always do the SAME thing to both sides of the formula.
2) To get rid of something, do the opposite.
   The opposite of + is – and the opposite of – is +.
   The opposite of × is ÷ and the opposite of ÷ is ×.
3) Keep going until you have the letter you want on its own.

EXAMPLE:

Rearrange $a = 3b + 4$ to make b the subject of the formula.

$a = 3b + 4$ — The opposite of +4 is –4, so take away 4 from both sides.

(–4) $a - 4 = 3b + 4 - 4$

$a - 4 = 3b$ — The opposite of ×3 is ÷3, so divide both sides by 3.

(÷3) $(a - 4) \div 3 = 3b \div 3$

$\frac{a-4}{3} = b$ OR $b = \frac{a-4}{3}$

Careful here — you divide the whole side by 3, not just one term.

EXAMPLE:

Rearrange $p = 5(q + 2)$ to make q the subject of the formula.

$p = 5q + 10$ — Multiply out the brackets.

(–10) $p - 10 = 5q + 10 - 10$ — The opposite of +10 is –10, so take away 10 from both sides.

$p - 10 = 5q$

(÷5) $(p - 10) \div 5 = 5q \div 5$ — The opposite of ×5 is ÷5, so divide both sides by 5.

$\frac{p-10}{5} = q$ OR $q = \frac{p-10}{5}$

EXAMPLE:

Rearrange $m = \frac{n}{4} - 7$ to make n the subject of the formula.

$m = \frac{n}{4} - 7$ — The opposite of –7 is +7, so add 7 to both sides.

(+7) $m + 7 = \frac{n}{4} - 7 + 7$

$m + 7 = \frac{n}{4}$ — The opposite of ÷4 is ×4, so multiply both sides by 4.

(×4) $(m + 7) \times 4 = \frac{n}{4} \times 4$

$4(m + 7) = n$ OR $n = 4m + 28$

Just remember — the subject is the letter on its own

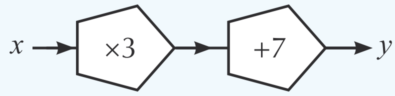
Rearranging formulas is really just like solving equations — so if you learn the method for one, you know the method for the other. All you have to do is remember the golden rules and you'll be set.

Warm-up and Worked Exam Questions

I know it was a pretty tough section, but being able to rearrange and solve equations and formulas is likely to come in handy in the exam. See how you fare with these warm-up questions.

Warm-up Questions

1) Solve these equations:
   a) $x + 2 = 8$ b) $x - 6 = 8$ c) $4x = 12$ d) $\frac{x}{5} = 3$
2) a) Solve $6x - 5 = 3x + 10$ b) Solve $4(y - 2) = 2y + 6$
3) $v = u + at$. Find the value of v when $u = 5$, $a = 4$ and $t = 6$.
4) Use the function machine on the right to find the value of:
   a) y when $x = 6$ b) x when $y = 40$

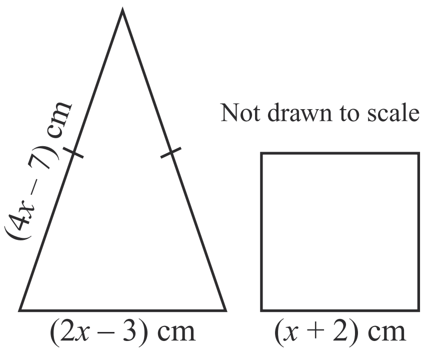


5) Ali, Ben and Joe sell 73 raffle tickets between them. Ben sells twice as many tickets as Ali, and Joe sells 8 more tickets than Ben. How many tickets does each person sell?
6) Three positive whole numbers have a sum of 48. The second number is five times the first, and the third number is double the second number. What are the three numbers?
7) A regular pentagon has sides of length x cm. A square with sides of length $x + 4$ cm has the same perimeter as the pentagon. Find the value of x.
8) Rearrange $u = \frac{v}{3} - 2$ to make v the subject of the formula.
9) Rearrange $c = 6d - 12$ to make d the subject of the formula.

Worked Exam Question

I'd like an exam question, and the answers written in — and a surprise. Two out of three's not bad.

1 The perimeter of the isosceles triangle below is double the perimeter of the square. Find the length of the base of the triangle. GRADE 5



Perimeter of triangle = $(4x - 7) + (4x - 7) + (2x - 3)$
$= 10x - 17$ cm

Work out the perimeter of both shapes.

Perimeter of square = $4 \times (x + 2) = 4x + 8$ cm

Perimeter of triangle = 2 × perimeter of square

$10x - 17 = 2 \times (4x + 8)$

Use the information from the question to make your equation.

$10x - 17 = 8x + 16$

$2x = 33$

$x = 16.5$ cm

Solve your equation to find x.

So base of triangle = $(2 \times 16.5) - 3 = 30$ cm

Don't forget to use the value of x to find the length of the base of the triangle.

..........30.......... cm
[4 marks]

Exam Questions

2 $S = 4m^2 + 2.5n$

a) Calculate the value of S when $m = 2$ and $n = 10$.

....
[2 marks]

b) Calculate the value of S when $m = 6.5$ and $n = 4$.

....
[2 marks]

3 Solve the following equations.

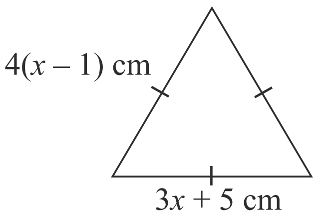
a) $40 - 3x = 17x$

$x =$ ....
[2 marks]

b) $2y - 5 = 3y - 12$

$y =$ ....
[2 marks]

4 Find the length of one side of this equilateral triangle.



.... cm
[3 marks]

5 Rearrange the formula $\frac{a+2}{3} = b - 1$ to make a the subject.

....
[2 marks]

6 At a football match between Redwood Rovers and Whitewater Wanderers, there were three times as many Redwood fans as Whitewater fans. The difference between the number of fans for each team was 7000.

How many fans were there in total?

....
[3 marks]

Sequences

Sequences are lists of numbers or shapes that follow a rule. You need to be able to spot what the rule is.

Finding the Rule for Number Sequences

The trick to finding the rule for number sequences is to write down what you have to do to get from one number to the next in the gaps between the numbers. There are 2 main types to look out for:

1) Add or subtract the same number

These are known as arithmetic sequences.

E.g. $2 \xrightarrow{+3} 5 \xrightarrow{+3} 8 \xrightarrow{+3} 11 \xrightarrow{+3} 14 \xrightarrow{+3} \ldots$ $30 \xrightarrow{-6} 24 \xrightarrow{-6} 18 \xrightarrow{-6} 12 \xrightarrow{-6} \ldots$

The RULE: 'Add 3 to the previous term' 'Subtract 6 from the previous term'

2) Multiply or divide by the same number each time

These are known as geometric sequences.

E.g. $2 \xrightarrow{\times 3} 6 \xrightarrow{\times 3} 18 \xrightarrow{\times 3} 54 \xrightarrow{\times 3} \ldots$ $40\,000 \xrightarrow{\div 10} 4000 \xrightarrow{\div 10} 400 \xrightarrow{\div 10} 40 \xrightarrow{\div 10} \ldots$

The RULE: 'Multiply the previous term by 3' 'Divide the previous term by 10'

Sometimes you might get sequences that follow a different rule — e.g. you might have to add or subtract a changing number each time, or add together the two previous terms (see the examples below).

EXAMPLE: **Find the next two terms in each of the following sequences.**

a) 1, 3, 6, 10, 15, ...

'The number you add on increases by one each time' (i.e. +2, +3, +4, ...) so the next two terms are:

15 + 6 = 21
21 + 7 = 28

This is the sequence of triangular numbers.

b) 1, 1, 2, 3, 5, ...

The rule is 'add together the two previous terms', so the next two terms are:

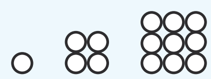
3 + 5 = 8
5 + 8 = 13

This is known as the Fibonacci sequence.

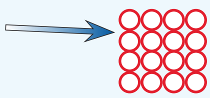
Finding the Rule for Shape Sequences

If you have a sequence of shape patterns, you need to be able to continue the sequence. You might also have to find the rule for the sequence to work out how many shapes there'll be in a later pattern.

EXAMPLE: **On the right, there are some patterns made of circles.**
a) Draw the next pattern in the sequence.
b) Work out how many circles there will be in the 10th pattern.



a) Just continue the sequence — the circles make a square pattern.



b) Find the rule for the number of circles: there's 1^2 = 1 circle in the first pattern, 2^2 = 4 circles in the second pattern, 3^2 = 9 circles in the third pattern etc. The rule is 'square the number of the pattern'. So in the 10th pattern, there'll be 10^2 = 100 circles.

Always write the change in the gaps between the numbers

It's the most straightforward way to spot the pattern — you'll see straight away if the difference is the same, changing by a certain amount, or multiplying. Read on for more about sequences.

Sequences

You might be asked to "find an expression for the *n*th term of a sequence" — this is a rule with n in, like $5n - 3$. It gives every term in a sequence when you put in different values for n.

Finding the nth Term of a Sequence

This method works for sequences with a common difference — where you add or subtract the same number each time.

EXAMPLE:

Find an expression for the *n*th term of the sequence that starts 5, 8, 11, 14, ...

n:	1	2	3	4
term:	5	8	11	14

$5 \xrightarrow{+3} 8 \xrightarrow{+3} 11 \xrightarrow{+3} 14$

3n:	3	6	9	12
	↓+2	↓+2	↓+2	↓+2
term:	5	8	11	14

So the expression for the nth term is 3n + 2

1. Find the common difference. It's 3, so this tells you '$3n$' is in the formula.
2. List the values of $3n$.
3. Work out what you have to add or subtract to get from $3n$ to the term. So it's +2.
4. Put '$3n$' and '+2' together.

Check your formula by putting the first few values of n back in:

$n = 1$ gives $3n + 2 = 3 + 2 = 5$ ✓

$n = 2$ gives $3n + 2 = 6 + 2 = 8$ ✓

Deciding if a Term is in a Sequence

You might be given the *n*th term and asked if a certain value is in the sequence. The trick here is to set the expression equal to that value and solve to find n. If n is a whole number, the value is in the sequence.

EXAMPLE:

A sequence is given by the rule $6n - 2$.

Have a look at p.41-42 for more on solving equations.

a) Find the 6th term in the sequence.

Just put n = 6 into the expression:

$(6 \times 6) - 2 = 36 - 2$
$= 34$

b) Is 45 a term in this sequence?

Set it equal to 45... $6n - 2 = 45$

$6n = 47$ ...and solve for n.

$n = 47 \div 6 = 7.8333...$

n is not a whole number, so 45 is not in the sequence $6n - 2$.

It might be even easier to decide if a number is in a sequence or not — for example, if the sequence was all odd numbers, there's no way that an even number could be in the sequence. You just have to use your common sense — e.g. if all the terms in the sequence ended in 3 or 8, 44 would not be in the sequence.

Follow the steps above to find the nth term of a sequence

Learn the steps above for finding the *n*th term. With it, you can easily find the 500th term, or the 500 000th — but without it, you'll need a dozen extra sheets of paper and a spare pen.

Inequalities

Inequalities are a bit tricky, but once you've learned the tricks involved, most of the algebra for them is identical to ordinary equations (have a look back at pages 41-42 if you need a reminder).

The Inequality Symbols

$>$ means '**Greater than**'	$\geq$ means '**Greater than or equal to**'
$<$ means '**Less than**'	$\leq$ means '**Less than or equal to**'

REMEMBER — the one at the BIG end is BIGGEST so $x > 4$ and $4 < x$ both say: 'x is greater than 4'.

EXAMPLE: **x is an integer such that $-4 < x \leq 3$. Write down all possible values of x.**

Work out what each bit of the inequality is telling you:

$-4 < x$ means 'x is greater than -4',
and $x \leq 3$ means 'x is less than or equal to 3'.

Remember, integers are just whole numbers (+ve and –ve, including 0).

Now just write down all the values that x can take: $-3, -2, -1, 0, 1, 2, 3$

-4 isn't included because of the $<$ but 3 is included because of the $\leq$.

You Can Show Inequalities on Number Lines

Drawing inequalities on a number line is dead easy — all you have to remember is that you use an open circle (○) for $>$ or $<$ and a coloured-in circle (●) for $\geq$ or $\leq$.

EXAMPLE: **Show the inequality $-4 < x \leq 3$ on a number line.**

Open circle because –4 isn't included.

Closed circle because 3 is included.

Algebra with Inequalities

Solve inequalities like regular equations but WITH ONE BIG EXCEPTION:

Whenever you MULTIPLY OR DIVIDE by a NEGATIVE NUMBER, you must FLIP THE INEQUALITY SIGN.

EXAMPLES:

1. Solve $3x - 2 \leq 13$.

Just solve it like an equation — but leave the inequality sign in your answer:

(+2) $3x - 2 + 2 \leq 13 + 2$
$3x \leq 15$
(÷3) $3x \div 3 \leq 15 \div 3$
$x \leq 5$

2. Solve $2x + 7 > x + 11$.

Again, solve it like an equation:

(–7) $2x + 7 - 7 > x + 11 - 7$
$2x > x + 4$
(–x) $2x - x > x + 4 - x$
$x > 4$

3. Solve $9 - 2x > 15$.

Watch out for the sign change:

(–9) $9 - 2x - 9 > 15 - 9$
$-2x > 6$
(÷–2) $-2x \div -2 < 6 \div -2$
$x < -3$

The $>$ has turned into a $<$, because we divided by a negative number.

Treat inequalities like equations

Learn the golden rules for solving equations and you'll be able to solve inequalities too. Just remember the extra rule about flipping the inequality sign if you multiply or divide by a negative number.

Warm-up and Worked Exam Questions

Sequences and inequalities — love them or hate them, you have to do them — it's just a case of learning the method and practising lots of questions. So let's start with some warm-up questions...

Warm-up Questions

1) Write down the next two terms in each of these sequences:
   a) 2, 6, 10, 14 b) 1, 3, 9, 27 c) 2, 3, 5, 8, 12
2) A sequence starts 3, 6, 12, ... There are two possible rules for this sequence. Write down both possible rules and find the next two terms of the sequence in each case.
3) For each of the sequences, say whether it is an arithmetic or geometric sequence and explain why. a) 5, 9, 13, 17... b) 4, 16, 64, 256...
4) A sequence starts 9, 15, 21, 27...
   a) Find an expression for the nth term of the sequence.
   b) Use your expression to find the 8th term in the sequence.
   c) Is 62 a term in the sequence? Explain your answer.
5) n is an integer such that $-1 \le n < 5$. Write down all the possible values of n.
6) Solve the following inequalities: a) $4x + 3 < 27$ b) $4x \ge 18 - 2x$

Worked Exam Question

Work through the question below and give all the questions on the next page a good go.

1 The first four terms in a sequence are 2, 9, 16, 23, …

a) Find the nth term of the sequence.

$$2 \xrightarrow{+7} 9 \xrightarrow{+7} 16 \xrightarrow{+7} 23$$

n =	1	2	3	4
7n =	7	14	21	28
	–5 ↓	–5 ↓	–5 ↓	–5 ↓
term =	2	9	16	23

The common difference is 7, so 7n is in the formula.

You have to subtract 5 to get to the term.

So the expression for the nth term is 7n – 5

7n – 5

[2 marks]

b) What is the 30th term of the sequence?

30th term = (7 × 30) – 5 = 205

205

[1 mark]

c) Is 55 a term in this sequence? Explain your answer.

If 55 is a term in the sequence, then 7n – 5 = 55, 7n = 60 so n = 8.571...

n is not a whole number, so 55 is not a term in the sequence.

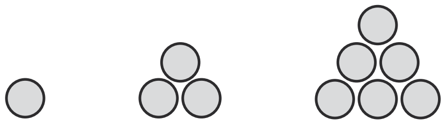
[2 marks]

Exam Questions

2 n is an integer. List all the possible values of n that satisfy the inequality $-3 \leq n < 2$.

....
[2 marks]

3 The patterns in the sequence below represent the first three triangle numbers.



a) Draw the next pattern in the sequence.

[1 mark]

b) How many circles are in the tenth pattern in the sequence? Give a reason for your answer.

....

....
[2 marks]

4 To find the next term in the sequence below, you add together the two previous terms.
Fill in the gaps to complete the sequence.

3	....	7	....	....	29

[2 marks]

5 A quadratic sequence starts 2, 6, 12, 20, …
Find the next term in the sequence.

Find the pattern in the differences between each pair of terms and use this to find the next term.

....
[2 marks]

6 p and q are integers. $p \leq 45$ and $q > 25$.
What is the largest possible value of $p - q$?

....
[2 marks]

Quadratic Equations

A quadratic equation is one where the highest power is x^2.
The standard format for a quadratic equation is $x^2 + bx + c = 0$.

You can **Factorise Quadratic Equations**

GRADE 5

1) You can solve quadratic equations by factorising.
2) 'Factorising a quadratic' means 'putting it into 2 brackets'.

Factorising Quadratics

1) ALWAYS rearrange into the STANDARD FORMAT: $x^2 + bx + c = 0$.
2) Write down the TWO BRACKETS with the x's in: $(x \quad)(x \quad) = 0$.
3) Then find 2 numbers that MULTIPLY to give 'c' (the number term) but also ADD/SUBTRACT to give 'b' (the number in front of the x term).
4) Fill in the +/– signs and make sure they work out properly.
5) As an ESSENTIAL CHECK, EXPAND the brackets to make sure they give the original equation.

Ignore any minus signs at this stage.

3) As well as factorising a quadratic, you might be asked to solve the equation. This just means finding the values of x that make each bracket 0 (see example below).

EXAMPLE: **Solve $x^2 - x = 12$.**

1) $x^2 - x - 12 = 0$
$(b = -1,\ c = -12)$

2) $(x \quad)(x \quad) = 0$

3)
1×12 Add/subtract to give: 13 or 11
2×6 Add/subtract to give: 8 or 4
3×4 Add/subtract to give: 7 or 1

$(x \quad 3)(x \quad 4) = 0$

This is what we want.

4) $(x + 3)(x - 4) = 0$

5) Check:
$(x + 3)(x - 4) = x^2 - 4x + 3x - 12$
$= x^2 - x - 12$ ✓

6) $(x + 3) = 0 \Rightarrow x = -3$
$(x - 4) = 0 \Rightarrow x = 4$

1) Rearrange into the standard format.

2) Write down two brackets with x's in.

3) Find the right pair of numbers that multiply to give c (= 12), and add or subtract to give b (= 1) (remember, we're ignoring the +/– signs for now).

4) Now fill in the +/– signs so that 3 and 4 add/subtract to give –1 (= b).

5) ESSENTIAL check — EXPAND the brackets to make sure they give the original expression.

But we're not finished yet — we've only factorised it, we still need to...

6) SOLVE THE EQUATION by setting each bracket equal to 0.

To help you work out which signs you need, look at c.

- If c is positive, the signs will be the same — both positive or both negative.
- If c is negative the signs will be different — one positive and one negative.

Factorising quadratics is not easy — but it is important

Make sure you learn the method, then it's a case of practise, practise, practise. People often get the signs the wrong way round, so make sure you always check your answer by expanding the brackets.

Simultaneous Equations

Simultaneous equations might sound a bit scary, but they're just a pair of equations that you have to solve at the same time. You have to find values of x and y that work in both equations.

Six Steps for Simultaneous Equations

GRADE 5 GRADE

EXAMPLE: **Solve the simultaneous equations $2x + 4y = 6$**
$4x + 3y = -3$

Both your equations should be in the form ax + by = c, where a, b and c are numbers.

1. Label your equations (1) and (2).

$2x + 4y = 6$ — (1)
$4x + 3y = -3$ — (2)

2. Match up the numbers in front of either the x's or y's in both equations. You may need to multiply one or both equations by a suitable number. Relabel the equations (3) and (4) if you need to change them.

(1) × 2: $4x + 8y = 12$ — (3)
$4x + 3y = -3$ — (2)

You don't need to change equation 2 for this example.

3. Add or subtract the two equations to eliminate the terms with the same number in front.

(3) – (2):
$$\begin{array}{r} 4x + 8y = 12 \\ -\ \ 4x + 3y = -3 \\ \hline 0x + 5y = 15 \end{array}$$

If the numbers have the same sign (both +ve or both –ve) then subtract.
If the numbers have opposite signs (one +ve and one –ve) then add.

4. Solve the resulting equation.

$5y = 15 \Rightarrow \underline{y = 3}$

5. Substitute the value you've found back into equation (1) and solve it.

Sub $y = 3$ into (1): $2x + (4 \times 3) = 6 \Rightarrow 2x + 12 = 6 \Rightarrow 2x = -6 \Rightarrow \underline{x = -3}$

6. Substitute both these values into equation (2) to make sure it works.
If it doesn't then you've done something wrong and you'll have to do it all again.

Sub x and y into (2): $(4 \times -3) + (3 \times 3) = -12 + 9 = -3$, which is right, so it's worked.
So the solutions are: $x = -3$, $y = 3$

It doesn't matter if you eliminate the x's or y's... do whatever's easiest

It might just be me, but I think simultaneous equations are quite fun... well, maybe not fun... but quite satisfying. Anyway, it doesn't matter whether you like them or not — you have to learn how to do them.

Proof

I'm not going to lie — proof questions can look a bit terrifying. But there are a couple of tricks you can use that make them a bit less scary.

Prove Statements are True or False

1) The most straightforward proofs are ones where you're given a statement and asked if it's true or false.
2) To show that it's false, all you have to do is find one example that doesn't work.
3) Showing that something is true is a bit trickier — you might have to do a bit of rearranging to show that two things are equal, or show that one thing is a multiple of a certain number.

EXAMPLE:

Find an example to show that the statement below is not correct.
"The difference between two prime numbers is always even."

GRADE 3

2 and 5 are both prime, so try them:

$5 - 2 = 3$, which is odd —
so the statement is not correct.

It was easy to find an example for this one — but sometimes you might have to try a few different numbers to find a pair that doesn't work.

EXAMPLE:

Prove that $(n + 3)^2 - (n - 2)^2 \equiv 5(2n + 1)$.

GRADE 5

Take one side of the equation and play about with it until you get the other side:

$$\begin{aligned} \text{LHS: } (n + 3)^2 - (n - 2)^2 &\equiv n^2 + 6n + 9 - (n^2 - 4n + 4) \\ &\equiv n^2 + 6n + 9 - n^2 + 4n - 4 \\ &\equiv 10n + 5 \\ &\equiv 5(2n + 1) = \text{RHS} \ ✓ \end{aligned}$$

See p.38 for a reminder on factorising.

$\equiv$ is the identity symbol, and means that two things are identically equal to each other. So $a + b \equiv b + a$ is true for all values of a and b (unlike an equation, which is only true for certain values).

Show that One Thing is a Multiple of Another

GRADE 5

1) To show that one thing is a multiple of a particular number (let's say 5), you need to rearrange the thing you're given to get it into the form 5 × a whole number, which means it's a multiple of 5.
2) If it can't be written as 5 × a whole number, then it's not a multiple of 5.

EXAMPLE:

$a = 3(b + 9) + 5(b - 2) + 3$.
Show that a is a multiple of 4 for any whole number value of b.

$a = 3(b + 9) + 5(b - 2) + 3$
$= 3b + 27 + 5b - 10 + 3$ — Expand the brackets...
$= 8b + 20$ — ... simplify...
$= 4(2b + 5)$ — ... and factorise.

2b + 5 is a whole number because b is a whole number.

a can be written as 4 × something (where the something is $2b + 5$)
so it is a multiple of 4.

3) It's always a good idea to keep in mind what you're aiming for — here, you're trying to write the expression for a as '4 × a whole number', so you'll need to take out a factor of 4 at some point.

Proof questions aren't as bad as they look

If you're asked to prove that two things are equal, rearranging should do the trick. If you're asked to prove something is wrong, just find an example of it not working. Always keep in mind what you're aiming for.

Warm-up and Worked Exam Questions

A mixed bag of tricky topics to end the algebra section — the only way you're going to get your head around them is to do some practice questions. Start by warming up with these questions.

Warm-up Questions

1) Factorise the following expressions:
   a) $x^2 + 2x - 15$ b) $x^2 - 2x - 3$ c) $x^2 + 7x + 12$
2) Solve the following equations:
   a) $x^2 + 7x + 10 = 0$ b) $x^2 + 5x - 14 = 0$ c) $x^2 - 5x + 3 = -3$
3) Solve the simultaneous equations: $x + y = 3$ and $4x + 3y = 10$.
4) Solve the simultaneous equations: $2x + 4y = 2$ and $5x - 3y = 18$.
5) For each of the following statements, find an example to prove that they are false.
   a) All square numbers end in 1, 4, 6 or 9.
   b) The product of two prime numbers is always odd.
6) Prove that $(x + 2)^2 + (x - 2)^2 = 2(x^2 + 4)$ for all values of x.

Worked Exam Questions

That's enough warming up, it's time for the real thing. Take a look at these worked examples first and then turn over and have a go at some practice questions yourself.

1 The equation $x^2 - 9x + 20 = 0$ is an example of a quadratic equation. GRADE 5

a) Fully factorise the expression $x^2 - 9x + 20$.

Number pairs that multiply to give 20 are:
1 × 20, 2 × 10, 4 × 5

–4 and –5 add to give – 9

$x^2 - 9x + 20 = (x - 4)(x - 5)$

$(x - 4)(x - 5)$ *[2 marks]*

b) Use your answer to part a) to solve the equation $x^2 - 9x + 20 = 0$.

$(x - 4)(x - 5) = 0$
So, $x - 4 = 0$ or $x - 5 = 0$
$x = 4$ or $x = 5$

$x =$ 4 or $x =$ 5 *[1 mark]*

2 Solve this pair of simultaneous equations. GRADE 5

$4x + 3y = 16$ ①
$4x + 2y = 12$ ②

①–②: $4x + 3y = 16$
$- \underline{4x + 2y = 12}$
$y = 4$

Eliminate the x term by subtracting the equations

Substitute $y = 4$ into equation ①.

$4x + (3 \times 4) = 16$
$4x = 4$ so $x = 1$

Check your answer by putting $x = 1$ and $y = 4$ into ①

$(4 \times 1) + (3 \times 4) = 16$ ✓

$x =$ 1 $y =$ 4 *[2 marks]*

Exam Questions

3 For each statement below, write down an example to show that the statement is incorrect.

a) There are no factors of 48 between 15 and 20.

....

[1 mark]

b) The sum of two square numbers is always odd.

....

[1 mark]

c) All numbers that end in an 8 are multiples of either 4, 6 or 8.

....

[1 mark]

4 Solve this pair of simultaneous equations.

$x + 3y = 11$

$3x + y = 9$

$x =$ .... $y =$ ....

[3 marks]

5 Solve the equation $x^2 + 4x - 12 = 0$.

$x =$ .... or $x =$ ....

[3 marks]

6 q is a whole number. Show that $2(18 + 3q) + 3(3 + q)$ is a multiple of 9.

[3 marks]

Revision Questions for Section Two

There was a lot of nasty algebra in that section — let's see how much you remember.

- Try these questions and tick off each one when you get it right.
- When you've done all the questions for a topic and are completely happy with it, tick off the topic.

Algebra (p35-38)

1) Simplify: a) $e + e + e$ b) $6f + 7f - f$
2) Simplify: a) $2x + 3y + 5x - 4y$ b) $11a + 2 - 8a + 7$
3) Simplify: a) $m \times m \times m$ b) $p \times q \times 7$ c) $2x \times 9y$
4) Expand: a) $6(x + 3)$ b) $-3(3x - 4)$ c) $x(5 - x)$
5) Expand and simplify $4(3 + 5x) - 2(7x + 6)$
6) Expand and simplify: a) $(x + 2)(2x - 5)$ b) $(5y + 2)^2$
7) What is factorising?
8) Factorise: a) $8x + 24$ b) $18x^2 + 27x$ c) $36x^2 - 81y^2$

Solving Equations (p41-42)

9) Solve: a) $x + 9 = 16$ b) $x - 4 = 12$ c) $6x = 18$
10) Solve: a) $4x + 3 = 19$ b) $3x + 6 = x + 10$ c) $3(x + 2) = 5x$

Expressions, Functions and Formulas (p43-46)

11) $Q = 5r + 6s$. Work out the value of Q when $r = -2$ and $s = 3$.
12) A function takes a number, doubles it and subtracts 8.
What is the result when 11 is put in the machine?
13) Tony and Robbie have the same number of marbles. Nadia has 26 marbles.
Between them, they have 100 marbles. How many marbles does Tony have?
14) A rectangle measures $2x$ cm by $7x$ cm. An equilateral triangle has the same perimeter as the rectangle. Find the length of one side of the triangle in terms of x.
15) Rearrange the formula $W = 4v + 5$ to make v the subject.

Sequences and Inequalities (p49-51)

16) For each of the following sequences, find the next term and write down the rule you used.
a) 3, 10, 17, 24, ... b) 1, 4, 16, 64, ... c) 2, 5, 7, 12, ...
17) Find an expression for the nth term of the sequence that starts 4, 10, 16, 22, ...
18) Is 34 a term in the sequence given by the expression $7n - 1$?
19) Write the following inequalities out in words: a) $x > -7$ b) $x \leq 6$
20) $0 < k \leq 7$. Find all the possible integer values of k.
21) Solve the following inequalities: a) $x + 4 < 14$ b) $3x + 5 \leq 26$.

Quadratic Equations (p54)

22) Factorise the following quadratic expressions: a) $x^2 + 10x + 16$ b) $x^2 - 6x - 7$
23) Solve the quadratic equation $x^2 + 3x - 18 = 0$.

Simultaneous Equations and Proof (p55-56)

24) Solve the following pair of simultaneous equations: $4x + 5y = 33$ and $-x + 3y = 13$
25) Find an example to show that the statement
"if a number ends in a 9, it must divide by 3" is not true.
26) Show that x is never a multiple of 5 if $x = 3(y + 2) + 2(y + 6)$ for any whole number y.

Coordinates and Midpoints

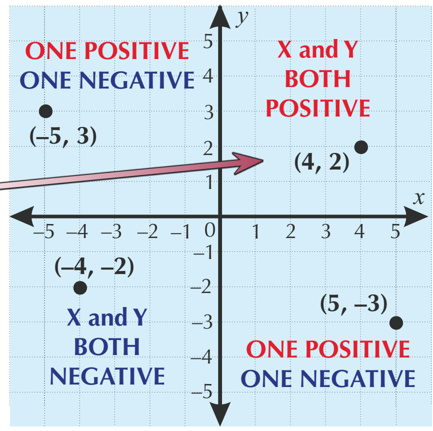
What could be more fun than points in one quadrant? Points in four quadrants, that's what...

The Four Quadrants

A graph has four different quadrants (regions).

The top-right region is the easiest because ALL THE COORDINATES IN IT ARE POSITIVE.

You have to be careful in the other regions though, because the x- and y- coordinates could be negative, and that makes life much more difficult.



THREE IMPORTANT POINTS ABOUT COORDINATES:

1) The coordinates are always in ALPHABETICAL ORDER, x then y. **(x, y)**
2) x is always the flat axis going ACROSS the page. In other words 'x is a...cross'.
3) Remember it's always IN THE HOUSE (→) and then UP THE STAIRS (↑) so it's ALONG first and then UP, i.e. x-coordinate first, and then y-coordinate

The Midpoint of a Line

The 'MIDPOINT OF A LINE SEGMENT' is the POINT THAT'S BANG IN THE MIDDLE of it.

Finding the coordinates of a midpoint is pretty easy.
LEARN THESE THREE STEPS...

1) **Find the average of the x-coordinates.**
2) **Find the average of the y-coordinates.**
3) **Plonk them in brackets.**

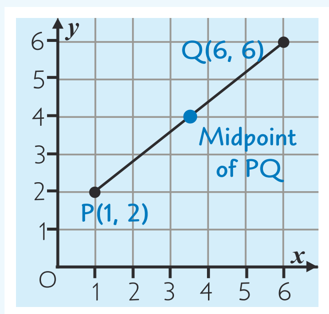
EXAMPLE:

P and Q have coordinates (1, 2) and (6, 6). Find the midpoint of the line PQ.

Average of x-coordinates = $\frac{1+6}{2} = 3.5$

Average of y-coordinates = $\frac{2+6}{2} = 4$

Coordinates of midpoint = (3.5, 4)



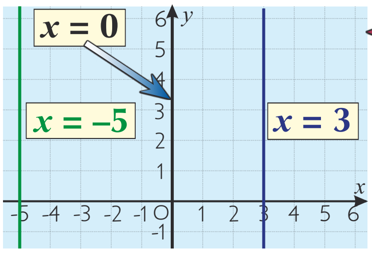
Coordinates should always be written as (x, y)

Learn the three points for getting x and y the right way round and the three easy steps for finding the midpoint of a line segment. Then close the book and write them all down.

Straight-Line Graphs

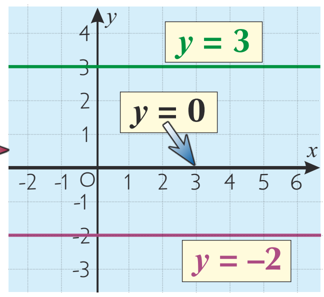
Here are the basic straight-line graphs — you need to be able to draw them and give their equations.

Vertical and **Horizontal** lines: ‘x = a’ and ‘y = a’

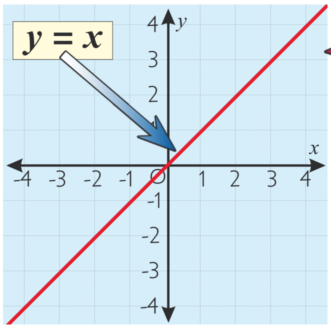


$x = a$ is a vertical line through ‘a’ on the x-axis

$y = a$ is a horizontal line through ‘a’ on the y-axis

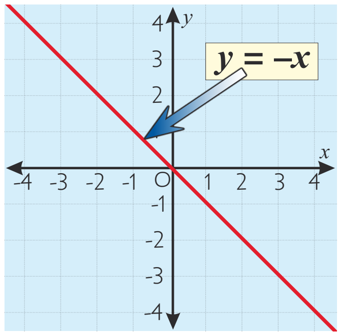


The **Main Diagonals**: ‘y = x’ and ‘y = –x’



‘$y = x$’ is the main diagonal that goes UPHILL from left to right.

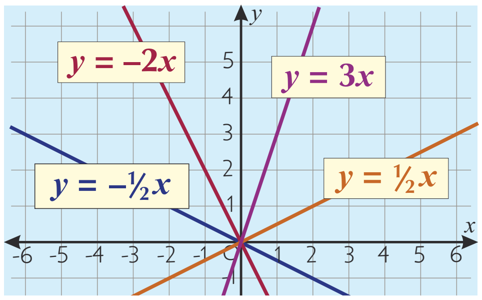
‘$y = -x$’ is the main diagonal that goes DOWNHILL from left to right.



Other **Lines** Through the Origin: ‘y = ax’ and ‘y = –ax’

$y = ax$ and $y = -ax$ are the equations for A SLOPING LINE THROUGH THE ORIGIN.

The value of ‘a’ (known as the gradient) tells you the steepness of the line. The bigger ‘a’ is, the steeper the slope. A MINUS SIGN tells you it slopes DOWNHILL.



Learn to **Spot Straight Lines** from their **Equations**

All straight-line equations just contain ‘something x, something y and a number’.

Straight lines:	
$x - y = 0$	$y = 2 + 3x$
$2y - 4x = 7$	$4x - 3 = 5y$

NOT straight lines:	
$y = x^3 + 3$	$\frac{1}{y} + \frac{1}{x} = 2$
$x^2 = 4 - y$	$xy + 3 = 0$

There’s more on x^2 graphs on page 68.

Get it straight — which lines are straight (and which aren’t)

The graphs $y = ax$ and $y = -ax$ are diagonals like $y = x$ and $y = -x$. They’re steeper or flatter depending on the value of ‘a’. Make sure you can spot when an equation will be a straight line.

Drawing Straight-Line Graphs

You might be asked to DRAW THE GRAPH of an equation in the exam.
This EASY METHOD will net you the marks every time:

1) **Choose 3 values of x and draw up a table.**
2) **Work out the corresponding y-values.**
3) **Plot the coordinates, and draw the line.**

You might get lucky and be given a table in an exam question. Don't worry if it contains 5 or 6 values.

Doing the 'Table of Values'

GRADE 3

EXAMPLE: **Draw the graph of $y = 2x - 3$ for values of x from –2 to 4.**

1) Choose 3 easy x-values for your table:

Use x-values from the grid you're given.
Avoid negative ones if you can.

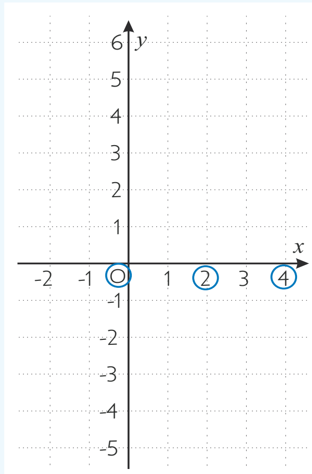
x	0	2	4
y			

2) Find the y-values by putting each x-value into the equation:

x	0	2	4
y	–3	1	5

When $x = 0$,
$y = 2x - 3$
$= (2 \times 0) - 3 = -3$

When $x = 4$,
$y = 2x - 3$
$= (2 \times 4) - 3 = 5$



Plotting the Points and Drawing the Graph

GRADE 3

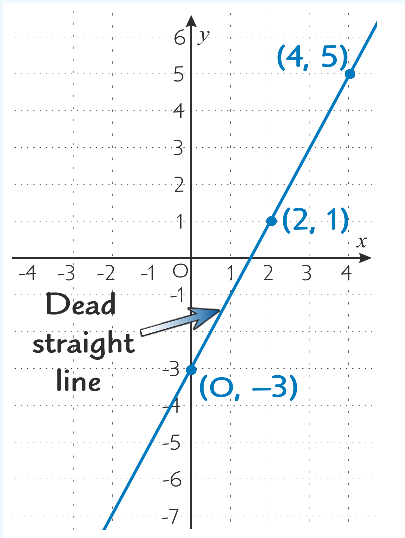
EXAMPLE: **...continued from above.**

3) PLOT EACH PAIR of x- and y- values from your table.

The table gives the coordinates (0, –3), (2, 1) and (4, 5).

Now draw a STRAIGHT LINE through your points.

If one point looks a bit wacky, check 2 things:
– the y-values you worked out in the table
– that you've plotted the points properly.



Spot and plot a straight line — then check it looks right

In the exam you might get an equation like $3x + y = 5$ to plot, making finding the y-values a bit trickier.
Just substitute in the x-value and find the y-value that makes the equation true.
E.g. when $x = 1$, $3x + y = 5 \rightarrow (3 \times 1) + y = 5 \rightarrow 3 + y = 5 \rightarrow y = 2$.
Or you can rearrange the equation to get y on its own if you find that easier.

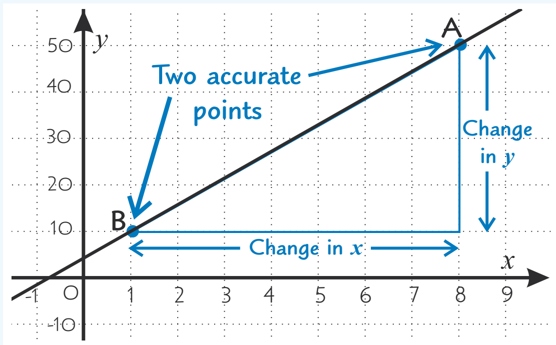
Straight-Line Graphs — Gradients

Time to hit the slopes. Well, find them anyway...

Finding the **Gradient**

The gradient of a line is a measure of its slope. The bigger the number, the steeper the line.

EXAMPLE: **Find the gradient of the straight line shown.**



1) Find two accurate points and complete the triangle.

Choose easy points with positive coordinates.

Two points that can be read accurately are:
Point A: (8, 50) Point B: (1, 10)

2) Find the change in y and the change in x.

Change in y = 50 – 10 = 40
Change in x = 8 – 1 = 7

Make sure you subtract the x-coordinates the SAME WAY ROUND as you do the y-coordinates.
E.g. y-coord. of pt A – y-coord. of pt B and x-coord. of pt A – x-coord. of pt B

3) LEARN this formula, and use it:

$$\text{GRADIENT} = \frac{\text{CHANGE IN Y}}{\text{CHANGE IN X}}$$

Gradient = $\frac{40}{7}$ = 5.71 (to 2 d.p.)

Make sure you get the formula the right way up.
Remember it's VERy HOt — VERtical over HOrizontal.

4) Check the sign's right.

If it slopes uphill left → right ( ) then it's positive.
If it slopes downhill left → right ( ) then it's negative.

As the graph goes uphill, the gradient is positive. So the gradient is 5.71 (not -5.71).

1) In real life, gradients are often given as a ratio or a percentage.
2) You'll often see them on road signs to describe the steepness of hills and slopes.
3) E.g. if the gradient = $\frac{1}{4}$, it has a ratio of 1 : 4, a percentage of 25% and you say it's "1 in 4" (for every 4 units you move horizontally, you move 1 unit vertically).

Gradient = change in y over change in x

It's really important that you get to grips with this method for finding gradients. Learn the four steps, then test yourself by writing them down. Gradients crop up in a few different types of graph question — for example, when you're asked to find the equation of a straight line (coming up on the next page).

Straight-Line Graphs — y =mx + c

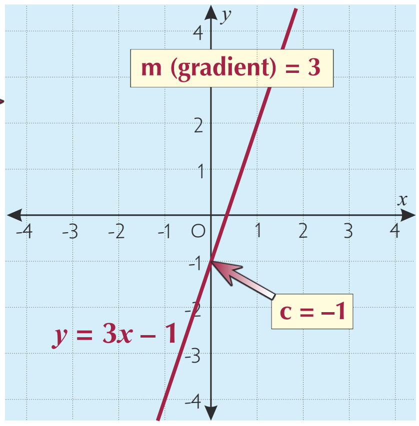
This sounds a bit scary, but give it a go and you might like it.

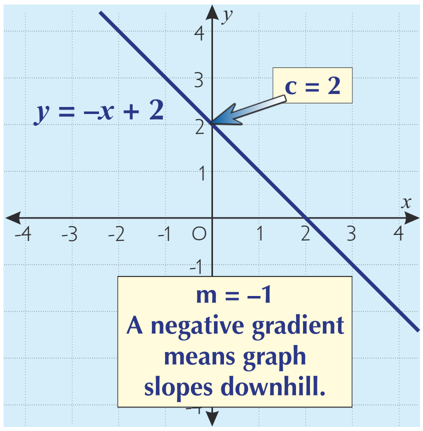
y = mx + c is the Equation of a **Straight Line**

$y = mx + c$ is the general equation for a straight-line graph, and you need to remember:

'm' is equal to the GRADIENT of the graph
'c' is the value WHERE IT CROSSES THE Y-AXIS and is called the Y-INTERCEPT.

'm' and 'c' are always just numbers — so $y = 3x - 1$ and $y = -x + 2$ are in $y = mx + c$ form.





You might have to rearrange a straight-line equation to get it into this form:

Straight line:		Rearranged into '$y = mx + c$'	
$y = 2 + 3x$	→	$y = 3x + 2$	(m = 3, c = 2)
$x - y = 4$	→	$y = x - 4$	(m = 1, c = –4)
$4 - 3x = y$	→	$y = -3x + 4$	(m = –3, c = 4)

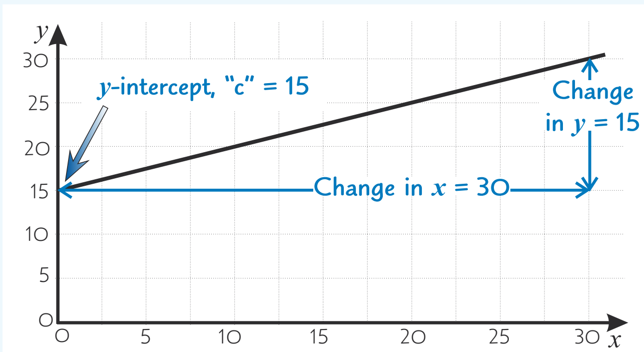
WATCH OUT: people mix up 'm' and 'c' when they get something like $y = 5 + 2x$.
Remember, 'm' is the number in front of the 'x' and 'c' is the number on its own.

Finding the **Equation** of a Straight-Line **Graph**

EXAMPLE:

Find the equation of the line on the graph in the form $y = mx + c$.

1. Find 'm' (gradient) $\quad m = \frac{\text{change in } y}{\text{change in } x} = \frac{15}{30} = \frac{1}{2}$
   It's an uphill graph, so the gradient is positive.
2. Read off 'c' (y-intercept) $\quad c = 15$
3. Use these to write the equation in the form $y = mx + c$. $\quad y = \frac{1}{2}x + 15$



'm' is the gradient and 'c' is the y-intercept

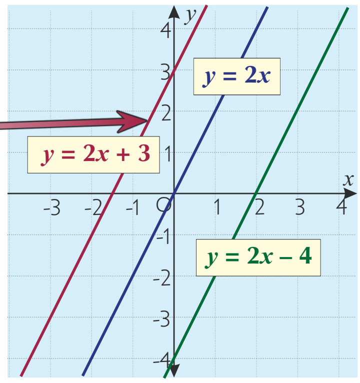
The key thing to remember is that 'm' is the number in front of the x, and 'c' is the number on its own. Remember that and you'll be able to find the equation of any straight line they throw at you.

Using y = mx + c

This page covers some of the awkward questions you might get asked about straight lines.

Parallel Lines Have the **Same Gradient**

Parallel lines all have the same gradient, which means their $y = mx + c$ equations all have the same value of m.
So the lines: $y = 2x + 3$, $y = 2x$ and $y = 2x - 4$ are all parallel.



EXAMPLE:

Line J has a gradient of –3. Find the equation of Line K, which is parallel to Line J and passes through point (2, 3).

1) First find the 'm' value for Line K.

Lines J and K are parallel so their gradients are the same $\Rightarrow$ m = –3

2) Substitute the value for 'm' into $y = mx + c$ to give you the 'equation so far'.

$y = -3x + c$

3) Substitute the x and y values for the given point on Line K and solve for 'c'.

When $x = 2$, $y = 3$:
$3 = (-3 \times 2) + c \Rightarrow 3 = -6 + c$
$c = 9$

4) Write out the full equation.

$y = -3x + 9$

Finding the **Equation** of a Line **Through Two Points**

If you're given two points on a line you can find the gradient, then you can use the gradient and one of the points to find the equation of the line. It's a bit tricky, but try to follow the method used in this example.

EXAMPLE:

Find the equation of the straight line that passes through (–2, 9) and (3, –1). Give your answer in the form $y = mx + c$.

1) Use the two points to find 'm' (gradient).

$m = \frac{\text{change in } y}{\text{change in } x} = \frac{-1-9}{3-(-2)} = \frac{-10}{5} = -2$

So $y = -2x + c$

2) Substitute one of the points into the equation you've just found.

Substitute (–2, 9) into eqn: $9 = (-2 \times -2) + c$
$9 = 4 + c$

3) Rearrange the equation to find 'c'.

$c = 9 - 4$
$c = 5$

4) Write out the full equation.

$y = -2x + 5$

Sometimes you'll be asked to give your equation in other forms, such as $ax + by + c = 0$.
Just rearrange your $y = mx + c$ equation to get it in this form.

Parallel lines have the same gradient

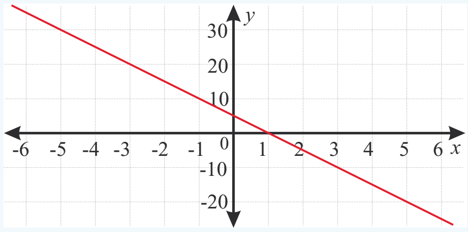
To check that lines are parallel, rearrange each equation into the form $y = mx + c$ and then compare their values of m — if they're the same then the lines are parallel, if they're different then they aren't parallel.

Warm-up and Exam Questions

In the exam, you'll have to know straight-line graphs like the back of your hand. If you struggle with any of the warm-up questions, go back over the section again before you go any further.

Warm-up Questions

1) a) Plot point A(–3, 2) and point B(3, 5) on a grid.
   b) Find the coordinates of the midpoint of AB.
2) Say whether the graph of each of the following equations will be a straight line.
   a) $2y = -x + 7$ b) $y = 4x2 - 1$ c) $x = 3y$ d) $x + 2y = 2 - 3x$
3) Draw the graph of $y = x + 4$ for values of x from –6 to 2.
4) Draw the graph of $y + 3x = 2$ for values of x from –2 to 2.
5) Find the gradient of the line shown on the right.
6) What is the gradient of the line with equation $y = 4 - 2x$?
7) a) Line Q goes through (0, 5) and (4, 7).
   Find the equation of Line Q in the form $y = mx + c$.
   b) Line R is parallel to line Q. It intersects the y-axis at (0, 10).
   Write down the equation of line R in the form $y = mx + c$.



Worked Exam Question

You know the routine by now — work carefully through this question and make sure you understand it. Then it's on to the real test of doing some exam questions for yourself.

1 Use the grid for the questions below.

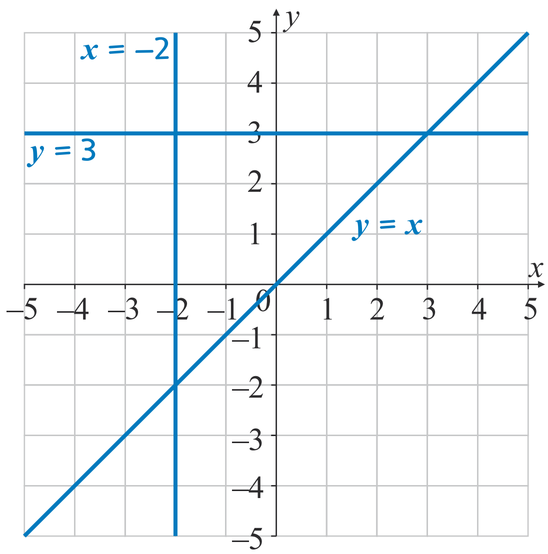
a) Draw and label the following lines.

$y = 3$ A horizontal line through 3 on the y-axis.

$x = -2$ A vertical line through –2 on the x-axis.

$y = x$ The diagonal with gradient 1, going through the origin.

[3 marks]



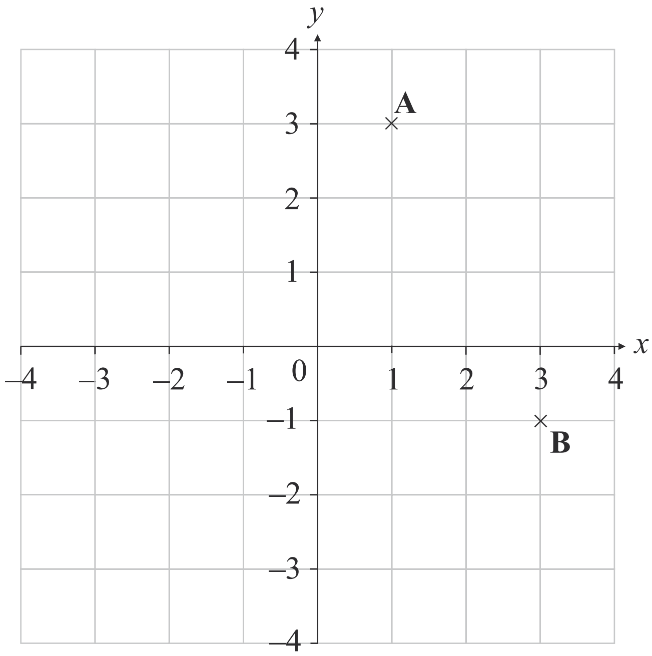
b) What are the coordinates of the point where the lines $y = 3$ and $y = x$ meet?

(.......3......., ...3.......)

[1 mark]

Exam Questions

2 Points **A** and **B** have been plotted on the grid below.



a) Write down the coordinates of the midpoint of the line segment **AB**.

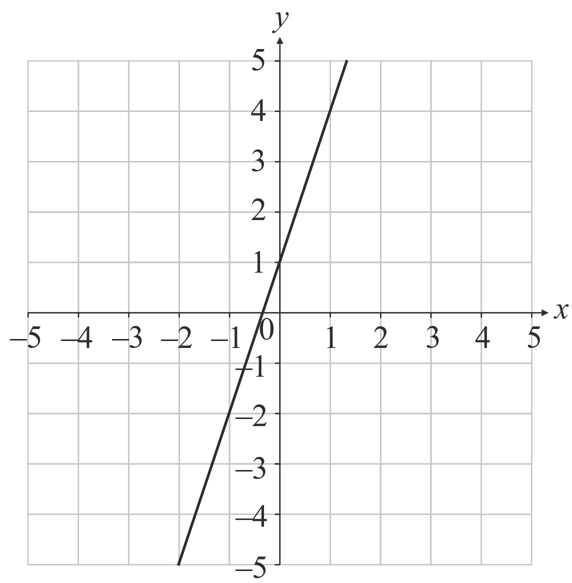
(..............., ...)
[2 marks]

b) Point **C** has the coordinates (0, –1). Given that line **AB** and line **CD** have the same midpoint, find the coordinates of point **D**.

(..............., ...)
[2 marks]

3 Look at the graph on the right.
Find the equation of the straight line.
Give your answer in the form $y = mx + c$.

....
[3 marks]

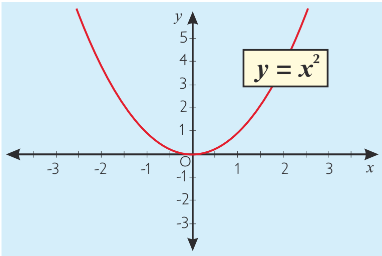


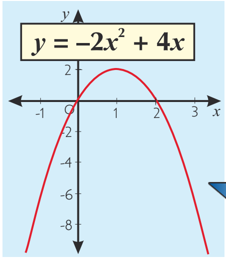
4 $y = 4x - 3$ is the equation of a straight line.
Find the equation of the line parallel to $y = 4x - 3$ that passes through the point (–1, 0).

....
[3 marks]

Quadratic Graphs

Enough of straight lines. You now get to move on to lovely, smooth curves. Quadratic ones to be precise.





Quadratic graphs are of the form y = anything with x^2 (but not higher powers of x).

They all have the same symmetrical bucket shape.

If the x^2 bit has a '–' in front of it then the bucket is upside down.

Plotting Quadratics

EXAMPLE: **Complete the table of values for the equation $y = x^2 + 2x - 3$ and then plot the graph.**

x	–4	–3	–2	–1	0	1	2
y	5	0	–3	–4	–3	0	5

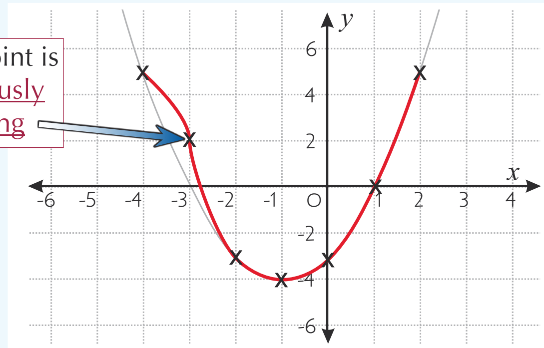
1) Substitute each x-value into the equation to get each y-value.

   E.g. $y = (-4)^2 + (2 \times -4) - 3 = 5$

2) Plot the points and join them with a completely smooth curve.

NEVER EVER let one point drag your graph off in some ridiculous direction. When a graph is generated from an equation, you never get spikes or lumps.

This point is obviously wrong



Finding the Turning Point

You could be asked to find the turning point of a quadratic, so here's a nifty method to do just that.

EXAMPLE: **Plot the graph of the equation $y = -x^2 - x + 2$, labelling its turning point with its coordinates.**

x	–3	–2	–1	0	1	2
y	–4	0	2	2	0	–4

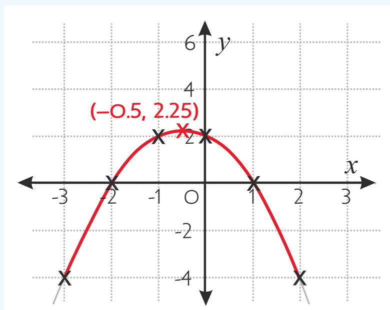
Plot the graph, then use symmetry to find the turning point of the curve:

1) Choose two points on the curve where the y-value is the same. At $x = -1$ and $x = 0$, $y = 2$

2) The x-coordinate of the turning point is halfway between these two values. x-coordinate of the turning point $= -0.5$

3) Put the x-coordinate back into the equation to find the y-coordinate. $y = -(-0.5)^2 - (-0.5) + 2$
$= 2.25$

The turning point is (–0.5, 2.25).



Quadratic graphs have an x^2 term, but no higher powers of x

Learn the method for drawing quadratic graphs and practise drawing some smooth curves. You can find their turning points by using the symmetry of the curve — follow the nifty little method shown above.

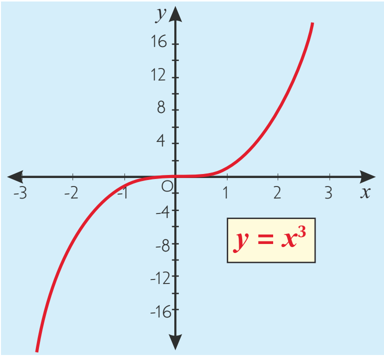
Harder Graphs

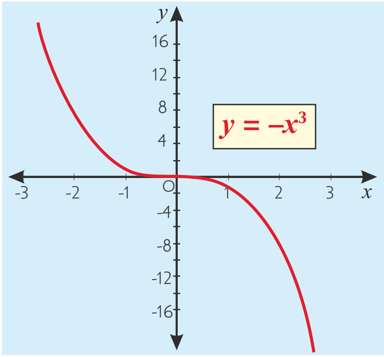
Graphs come in all sorts of shapes, sizes and wiggles — here are just a couple:

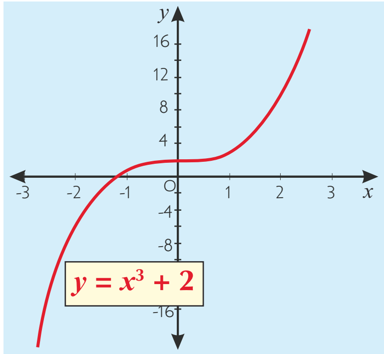
x^3 (Cubic) Graphs

GRADE 5

1) Cubic graphs have the form y = anything with x^3 (but no higher powers of x).
2) All x^3 graphs have a wiggle in the middle.







EXAMPLE:

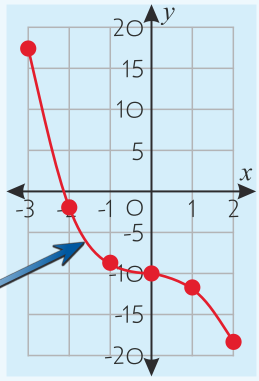
Draw the graph of $y = -x^3 - 10$ for values of x between -3 and $+2$.

Start by making a table of values.

x	-3	-2	-1	0	1	2
$y = -x^3 - 10$	17	-2	-9	-10	-11	-18

E.g. $y = -(-2)^3 - 10 = -2$

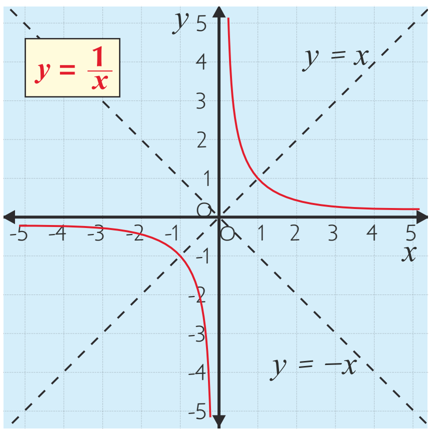
Plot the points and join them with a lovely smooth curve. Don't use your ruler — that would be a trifle daft.



1/x (Reciprocal) Graphs

GRADE 5

These appear as two symmetrical curves — one in the top right, one in the bottom left. They never touch the axes.
They're symmetrical about the lines $y = x$ and $y = -x$.



If you're not sure what a graph is supposed to look like in the exam, remember you can always use a table of values — even if it's just to refresh your memory.

Learn these two different types of graph

Cubic graphs and reciprocals can look pretty confusing, but all you really need to do is recognise the shape and be able to sketch them. Remember that the general equation for a reciprocal graph is $y = A/x$.

Solving Equations Using Graphs

You can plot graphs to find solutions (or approximate solutions) to simultaneous equations and other equations. Plot the equations you want to solve and the solution lies where the lines intersect.

Solving **Simultaneous Equations**

See p.55 for more on simultaneous equations.

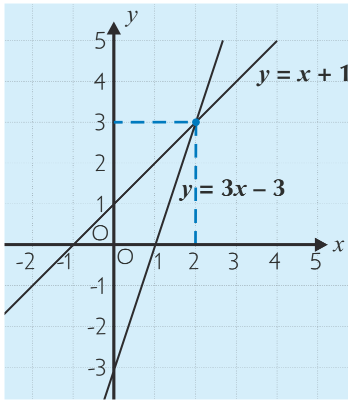
If you want to solve a pair of simultaneous equations with a graph, it's just a matter of plotting them both on a graph and writing down where they cross.

EXAMPLES:

1. **Use the graph to the right to solve the simultaneous equations $y = 3x - 3$ and $y = x + 1$.**

Read off the x and y values where the two lines intersect.

$x = 2$, $y = 3$



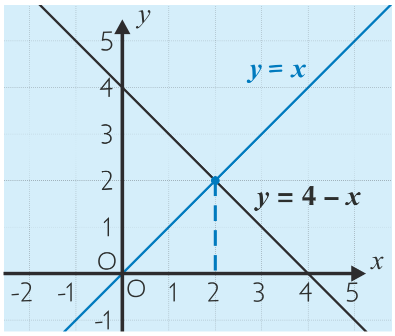
2. **The graph of $y = 4 - x$ is shown to the right. Use the graph to find the solution to $4 - x = x$.**

Each side of the equation $4 - x = x$ represents a line. These lines are $y = 4 - x$ and $y = x$.

Draw the line $y = x$ on the graph, then read off the x-coordinate where it crosses $y = 4 - x$.

The solution is $x = 2$.

At the point where the lines cross, both sides of the equation are equal, so this is the solution.



Solving **Quadratic Equations**

EXAMPLE:

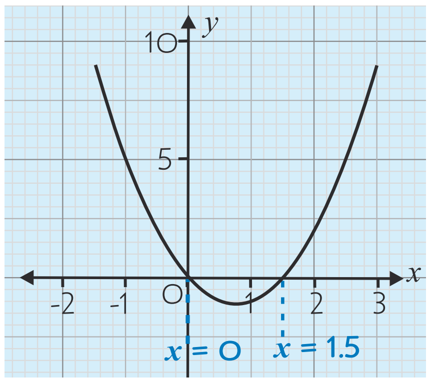
Use the graph of $y = 2x^2 - 3x$ (on the right) to find both roots of the equation $2x^2 - 3x = 0$.

The left-hand side of the equation $2x^2 - 3x = 0$ represents the curve $y = 2x^2 - 3x$, and the right-hand side represents the line $y = 0$ (the x-axis).

Read off the x-values where the curve crosses the x-axis — these are the solutions or roots.

The roots are $x = 0$ and $x = 1.5$.

Quadratic equations usually have 2 roots (see p.54).



The solutions lie where the graphs intersect

Draw both graphs on the same axes, then see where the two cross. The x and y values at these points are the solutions to your equations. Try and be as accurate as possible with your drawings.

Distance-Time Graphs

Here is another type of graph for you to learn about... You can easily identify a distance-time graph by the axis labels — distance on the y-axis and time on the x-axis. Surprising isn't it?

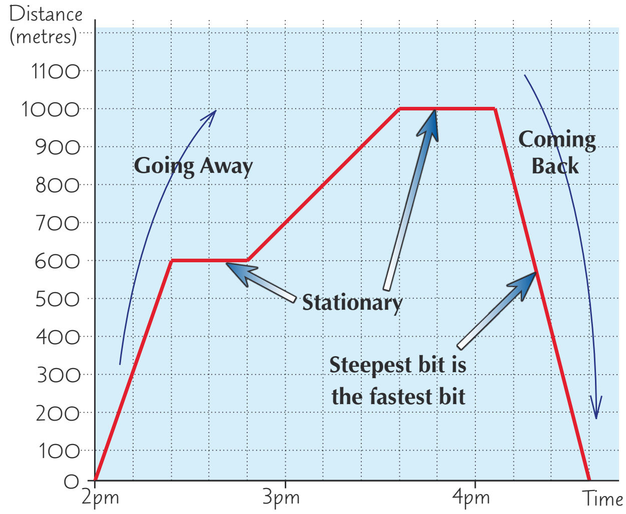
Distance-Time Graphs

GRADE 3

Distance-time graphs can look a bit awkward at first, but they're not too bad once you get your head around them.

Just remember these 4 important points:

1) At any point, **GRADIENT = SPEED.**
2) The **STEEPER** the graph, the **FASTER** it's going.
3) **FLAT SECTIONS** are where it is **STOPPED.**
4) If the gradient's negative, it's **COMING BACK.**



EXAMPLE: **Henry went out for a ride on his bike. After a while he got a puncture and stopped to fix it. This graph shows the first part of Henry's journey.**

a) **What time did Henry leave home?**

He left home at the point where the line starts. At 8:15

b) **How far did Henry cycle before getting a puncture?**

The horizontal part of the graph is where Henry stopped. 12 km

c) **What was Henry's speed before getting a puncture?**

Using the speed formula (p.98) is the same as finding the gradient.

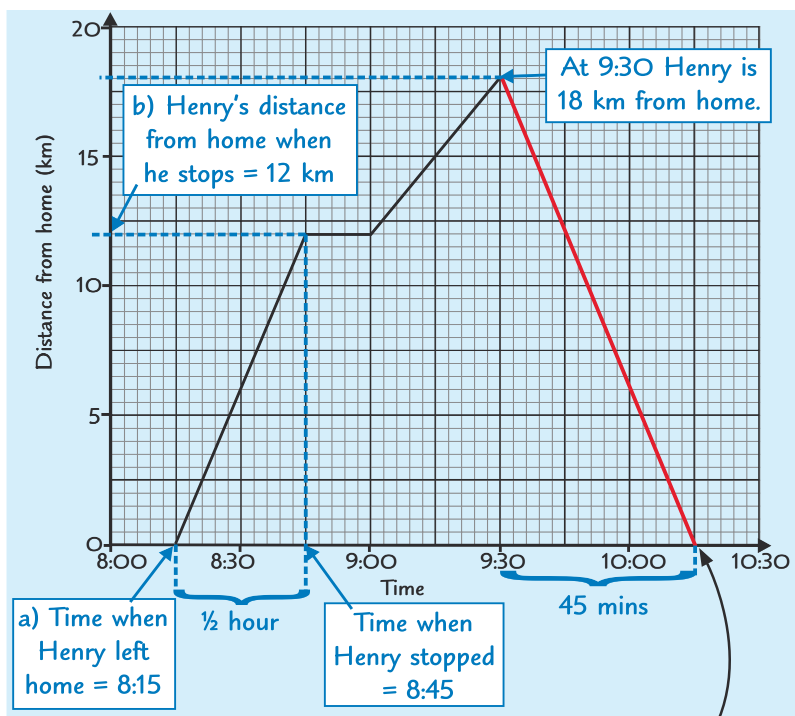
$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{12\text{ km}}{0.5\text{ hours}}$$
$$= 24\text{ km/h}$$

d) **At 9:30 Henry turns round and cycles home at 24 km/h. Complete the graph to show this.**

You have to work out how long it will take Henry to cycle the 18 km home:

$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{18\text{ km}}{24\text{ km/h}} = 0.75\text{ hours}$$
$$0.75 \times 60\text{ mins} = 45\text{ mins}$$

Convert the decimal time to minutes.



45 minutes after 9:30 is 10:15, so that's the time Henry gets home. Now you can complete the graph.

The gradient of a distance-time graph equals the speed

Learn the four important details about distance-time graphs, then cover the page and write them down. Exam questions on this topic can look a bit daunting, but spot the key details and you'll be fine.

Real-Life Graphs

In the exam you might get a graph which converts something like £ to dollars or mph to km/h.

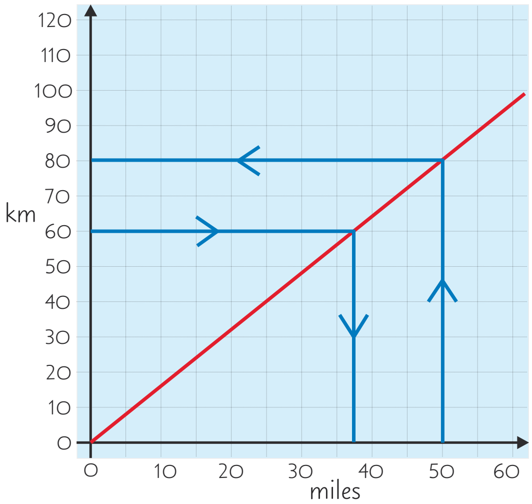
Conversion Graphs are Easy to Use

GRADE 2

METHOD FOR USING CONVERSION GRAPHS:

1. Draw a line from a value on one axis.
2. When you hit the LINE, change direction and go straight to the other axis.
3. Read off the value from this axis. The two values are equivalent.

Here are a couple of straightforward examples: This graph converts between miles and kilometres



Miles and km are directly proportional. See p.83-84.

EXAMPLE: **How many miles is 60 km?**

1. Draw a line across from '60' on the 'km' axis.
2. When it hits the line, go down to the 'miles' axis.
3. Read off the answer: 37.5 miles

EXAMPLE: **How many km is 150 miles?** GRADE 3

150 miles is way off the graph, so find a point which will make calculating easy. Use 50 miles, then multiply by 3.

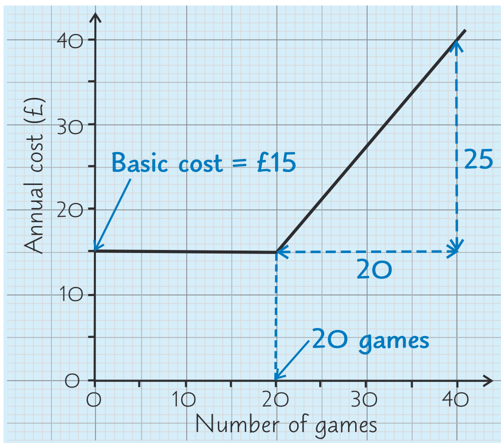
1. Draw a line up from '50' on the 'miles' axis.
2. When it hits the line, go across to the 'km' axis.
3. Read off the answer and multiply by 3 to find 150 miles. $80 \times 3 = 240$ km

Graphs Can Show How Much You'll Pay

GRADE 3

Graphs are great for showing how much you'll be charged for using a service or buying multiple items.

EXAMPLE: **A company hires out computer games to its members. The graph shows how much it charges.**



a) How many games are included in the basic annual cost?

20

The first section of the graph is horizontal. You pay £15 if you rent between 0 and 20 games. It's only if you hire more than 20 games that you pay more.

b) Estimate the cost per game for additional games.

Gradient of sloped section = cost per game

$$\frac{\text{vertical change}}{\text{horizontal change}} = \frac{25}{20} = £1.25 \text{ per game}$$

Learn how to convert graph questions into marks

These real-life graph questions aren't too bad, as long as you learn the three-step method above. Always leave your working lines showing on the graph — they might get you a mark in your exam.

Real-Life Graphs

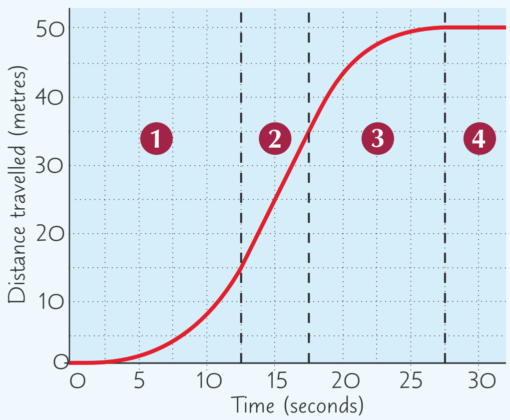
Graphs can tell you just about anything. I can't show you all the possibilities due to health and safety concerns about back injuries resulting from the weight of the book. But here are some useful examples.

Gradients Show the **Rate of Change**

Basically, gradients always show you the '<u>change in something per something else</u>'.

The gradient is always **(y-axis UNITS) PER (x-axis UNITS)**

The graph below shows the distance travelled by a remote control car.



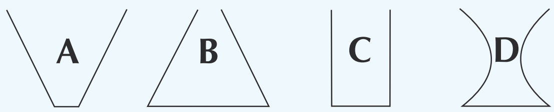
The <u>gradient</u> of the <u>graph</u> tells you the <u>speed</u> of the remote control car (see p.71).

1. In this section of the graph the gradient is <u>increasing</u> (i.e. the graph is getting steeper) so the speed of the car is increasing — it's <u>accelerating</u>.
2. Here, the gradient doesn't change so the car is travelling at a <u>constant speed</u>.
3. The gradient is <u>decreasing</u> (i.e. the graph is getting flatter) so the speed of the car is decreasing — it's <u>decelerating</u>.
4. The gradient is flat so the car has <u>stopped</u>.

Graphs Can Show **Changes with Time**

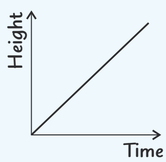
EXAMPLE:

Four different-shaped glasses are shown on the right. Juice is poured into each glass at a constant rate.



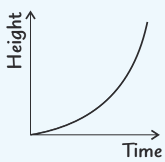
Each graph below shows how the height of juice in one glass changes. Match each graph to the correct glass.

A <u>steeper</u> slope means that the juice height is changing <u>faster</u>.



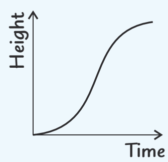
Glass C

Glass C has <u>straight vertical sides</u>, so the juice height increases steadily.



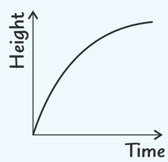
Glass B

Glass B is <u>widest at the bottom</u>, so the juice height increases <u>slowly at first</u>.



Glass D

Glass D is <u>narrowest in the middle</u>, so the height will increase <u>fastest</u> in the <u>middle part</u> of the graph.



Glass A

Glass A is <u>narrowest at the bottom</u>, so the height will increase <u>fastest</u> at the start of the graph.

The gradient tells you the number of y-axis units per x-axis unit

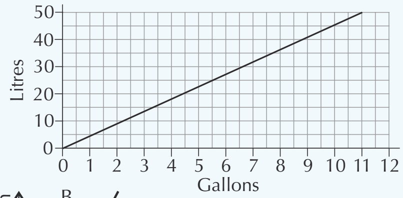
If the question talks about 'rate' or 'something per something else', it's a big clue that the gradient's involved. Make sure you're happy with interpreting graphs by looking at how the gradient changes.

Warm-up and Worked Exam Questions

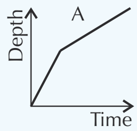
There were a few tough pages in that section — unfortunately the exam questions can be quite tough too. These warm-up questions will get you in shape to tackle the practice exam questions on the next page.

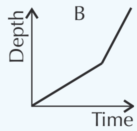
Warm-up Questions

1) a) Draw the graph of $y = x^2 + 3x - 7$ for values of x between –3 and 3.
   b) Find the turning point of the graph $y = x^2 + 3x - 7$.
2) Draw the graph of $y = x^3 + 8$ from $x = -2$ to $x = 2$.
3) By plotting the graph, find the solutions to the equations $y = 2x - 4$ and $y = x + 2$.
4) a) Using the graph on page 71, how long did Henry stop for?
   b) What was Henry's speed after he had repaired the puncture, before he turned back home?
5) Use the graph opposite to answer the questions below.
   a) A water tank holds 8 gallons. How many litres is this?
   b) Approximately how many gallons of water would fit into a 20 litre container?



6) A bath is being filled using only the hot tap. After 3 minutes the cold tap is also turned on. Which graph, A or B, shows how the depth of water in the bath changes over time?



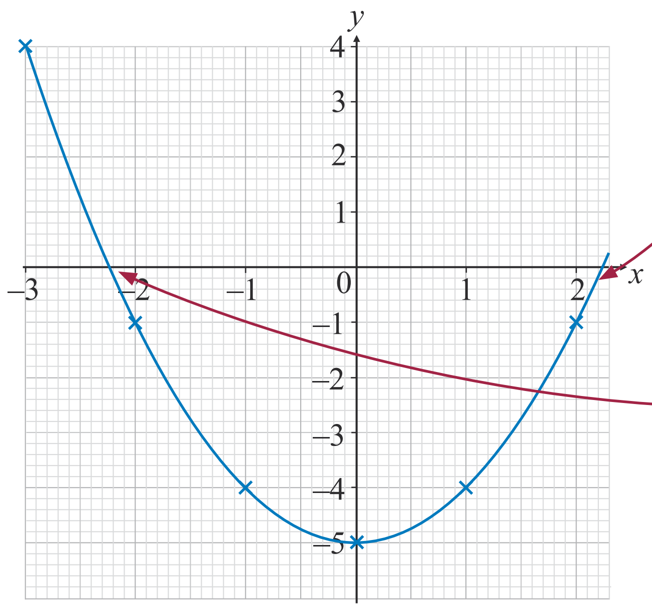


Worked Exam Question

Wow, an exam question with the answer written in. Make the most of it... you know what's coming.

1 A table of values for $y = x^2 - 5$ is shown below. GRADE 5

x	–3	–2	–1	0	1	2
y	4	–1	–4	–5	–4	–1



a) Draw the graph of $y = x^2 - 5$ on the grid.

Make sure you join the points with a smooth curve.

[2 marks]

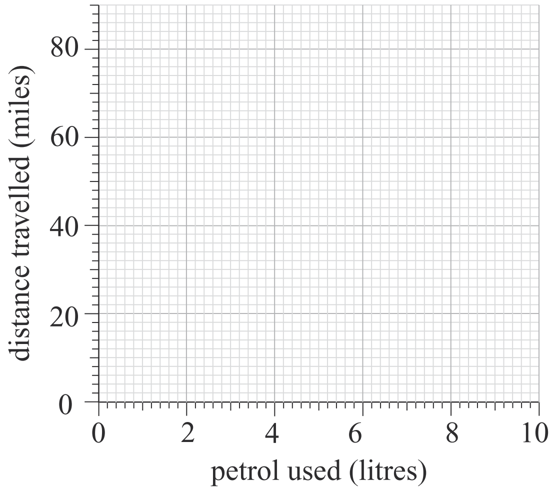
b) Use your graph to estimate the negative solution to the equation $x^2 - 5 = 0$.
Give your answer to 1 decimal place.

Find the x-coordinate where the graph crosses the x-axis.

x = ...–2.2......
[1 mark]

Exam Questions

2 The table to the right shows how much petrol a car used on 3 journeys. (Grade 3)



Distance (miles)	16	40	72
Petrol used (litres)	2	5	9

a) Using the table above, draw a conversion graph on the grid to the left.

[2 marks]

b) Find the gradient of the line.

....

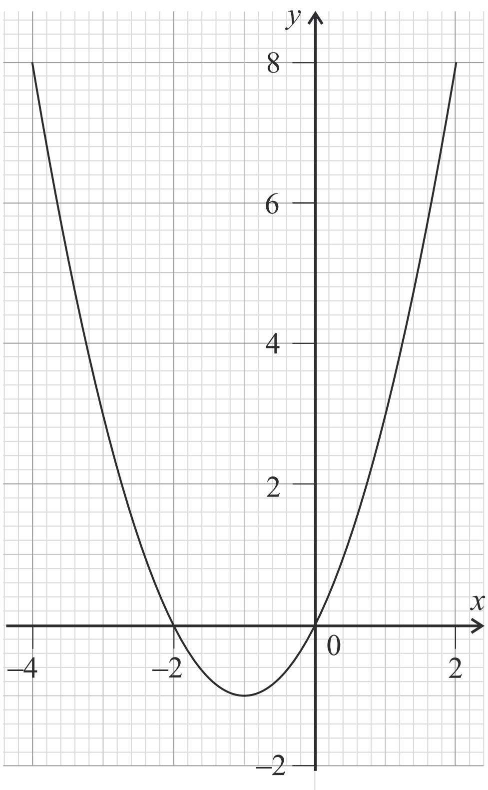
[2 marks]

c) What does the gradient of the line represent?

....

[1 mark]

3 The graph of $y = x^2 + 2x + c$ is shown below. (Grade 5)



a) Circle the correct value of c.

–2 1 0 –1 –4

[1 mark]

b) Circle both solutions of $x^2 + 2x = 0$.

–1 –2 3 0 1

[1 mark]

c) Write down the coordinates of the turning point of the curve.

(............., ...)

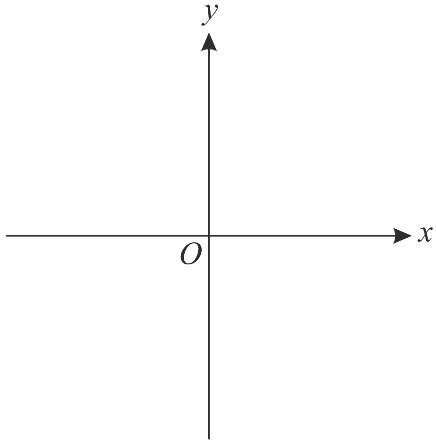
[1 mark]

Exam Questions

4 Sketch the following graphs on the axes below.
Label the points where they intersect the axes.

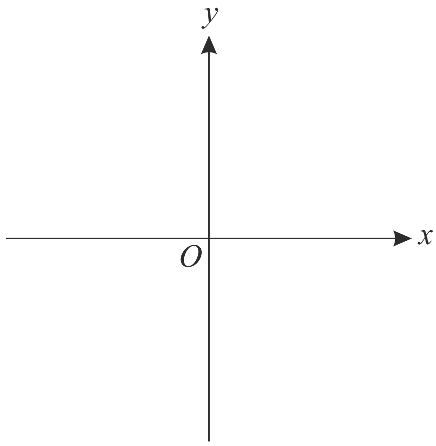
When you're sketching a curve just draw the general shape, but make sure the important bits are in the right place.

a) $y = -x^2$



[2 marks]

b) $y = x^3$



[2 marks]

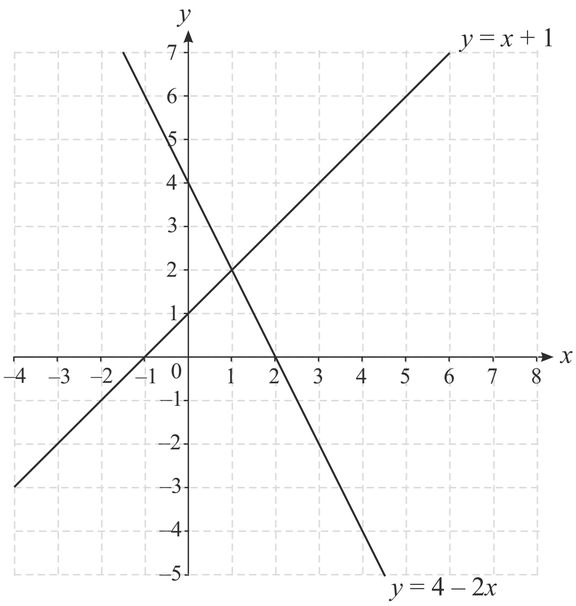
5 The diagram below shows the lines $y = x + 1$ and $y = 4 - 2x$.

a) Use the diagram to solve $x + 1 = 4 - 2x$

$x =$ ....
[1 mark]

b) By drawing another straight line, solve these simultaneous equations:

$y = x + 1$
$3y = x + 9$



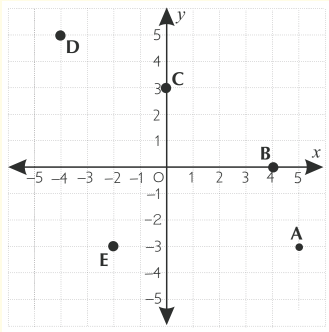
$x =$ .... $y =$ ....
[3 marks]

Revision Questions for Section Three

Well, that wraps up Section Three — time to put yourself to the test and find out how much you really know.

- Try these questions and tick off each one when you get it right.
- When you've done all the questions for a topic and are completely happy with it, tick off the topic.

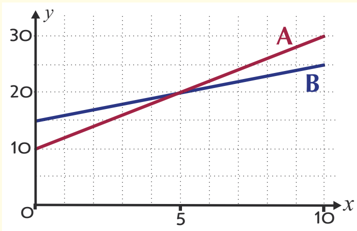
Coordinates and Midpoints (p60)



1) Give the coordinates of points A to E in the diagram on the right.
2) Find the midpoint of a line segment with endpoints B and C.

Straight-Line Graphs (p61-65)

3) Draw these lines on a grid: a) $y = -x$, b) $y = -4$, c) $x = 2$
4) By making a table of values, draw the graph of $y = -4x - 2$.
5) a) Find the gradient of line A on the right.
   b) Find the equation of line B on the right, giving your answer in the form $y = mx + c$.
6) Find the equation of the line passing through (4, 2) which is parallel to $y = 2x - 1$.
7) Find the equation of the line passing through (3, –6) and (6, –3).

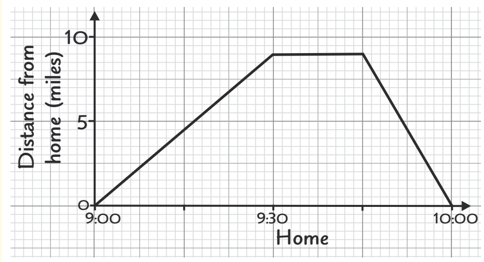


Quadratic and Harder Graphs (p68-70)

8) Describe the shapes of the graphs $y = x^2 - 8$ and $y = -x^2 + 2$.
9) a) Create and complete a table of values for values of x between –3 and 1 for the equation $y = x^2 + 3x$.
   b) Plot the graph of $y = x^2 + 3x$, labelling the turning point of the curve with its coordinates.
10) Describe in words and with a sketch the forms of these graphs:
    a) $y = ax^3$ b) $y = 1/x$
11) Plot the graph $y = 2x - 1$ and use it to find the solution to $5 = 2x - 1$.

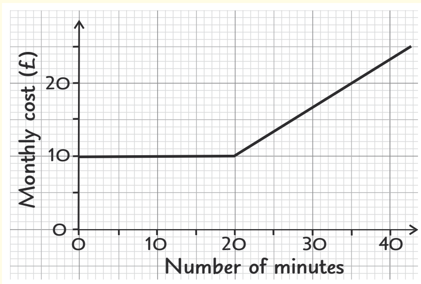
Distance-Time Graphs (p71)

12) What does a horizontal line mean on a distance-time graph?
13) The graph on the right shows Ben's car journey to the supermarket and home again.
    a) Did he drive faster on his way to the supermarket or on his way home?
    b) How long did he spend at the supermarket?



Real-Life Graphs (p72-73)

14) On a straight-line graph, how would you find the rate of change?
15) This graph shows the monthly cost of a mobile phone contract.
    a) How many minutes does the basic monthly fee include?
    b) Mary uses her phone for 35 minutes one month. What will her bill be?
    c) Stuart is charged £14 one month. Estimate how long he used his phone for.
    d) Estimate the cost per minute for additional minutes. Give your answer to the nearest 1p.



Ratios

Ratios are a pretty important topic — they can crop up in all sorts of questions, so you need to be prepared. If you understand the examples on the next three pages, you'll have a fighting chance in the exams.

Reducing Ratios to their Simplest Form

To reduce a ratio to a simpler form, divide all the numbers in the ratio by the same thing (a bit like simplifying a fraction — see p.15). It's in its simplest form when there's nothing left you can divide by.

EXAMPLE: **Write the ratio 15 : 18 in its simplest form.**

For the ratio 15:18, both numbers have a factor of 3, so divide them by 3.

We can't reduce this any further. So the simplest form of 15:18 is **5:6**.

$$15:18 \xrightarrow{\div 3} = 5:6$$

A handy trick for the calculator papers — use the fraction button

If you enter a fraction with the ⊟ or $a^b/_c$ button, the calculator automatically cancels it down when you press =. So for the ratio 8 : 12, just enter $\frac{8}{12}$ as a fraction, and you'll get the reduced fraction $\frac{2}{3}$. Now you just change it back to ratio form, i.e. 2 : 3.

The More Awkward Cases:

1) If the ratio contains decimals or fractions — multiply

For fractions, multiply by a number that gets rid of both denominators.

EXAMPLE: **Simplify the ratio 2.4 : 3.6 as far as possible.**

1) Multiply both sides by 10 to get rid of the decimal parts.
2) Now divide to reduce the ratio to its simplest form.

$$2.4:3.6 \xrightarrow{\times 10} = 24:36 \xrightarrow{\div 12} = 2:3$$

2) If the ratio has mixed units — convert to the smaller unit

EXAMPLE: **Reduce the ratio 24 mm : 7.2 cm to its simplest form.**

1) Convert 7.2 cm to millimetres.
2) Simplify the resulting ratio. Once the units on both sides are the same, get rid of them for the final answer.

24 mm : 7.2 cm
= 24 mm : 72 mm
$\xrightarrow{\div 24}$ = 1:3

3) To get to the form 1 : n or n : 1 — just divide

EXAMPLE: **Reduce 5 : 54 to the form 1 : n.**

Divide both sides by 5:

$$5:54 \xrightarrow{\div 5} = 1:\frac{54}{5} \text{ or } 1:10.8$$

Simplifying ratios is a lot like simplifying fractions

You have to divide both sides of the ratio by the same number (just like you'd divide the top and bottom of a fraction by the same number) — remember this and you'll be a simplifying master.

Ratios

Another page on ratios coming up — it's more interesting than the first but not as exciting as the next one...

Scaling Up Ratios

GRADE 3

If you know the ratio between parts and the actual size of one part, you can scale the ratio up to find the other parts.

EXAMPLE:

Mortar is made from mixing sand and cement in the ratio 7 : 2. How many buckets of mortar will be made if 21 buckets of sand are used in the mixture?

You need to multiply by 3 to go from 7 to 21 on the left-hand side (LHS) — so do that to both sides:

sand : cement

$$\times 3 \left(\begin{array}{c} 7:2 \\ = \\ 21:6 \end{array} \right) \times 3$$

So 21 buckets of sand and 6 buckets of cement are used.

Amount of mortar made = 21 + 6 = 27 buckets

The two parts of a ratio are always in direct proportion (see p.83-84). So in the example above, sand and cement are in direct proportion, e.g. if the amount of sand doubles, the amount of cement doubles.

Writing Ratios as Fractions

GRADE 3

1) To write one part as a fraction of another part — put one number over the other

E.g. if apples and oranges are in the ratio 2 : 9 then we say there are $\frac{2}{9}$ as many apples as oranges or $\frac{9}{2}$ times as many oranges as apples.

2) To write one part as a fraction of the total — add up the parts to find the total, then put the part you want over the total.

E.g. a pie dough is made by mixing flour, butter and water in the ratio 3 : 2 : 1.
The total number of parts is $3 + 2 + 1 = 6$.
So $\frac{3}{6} = \frac{1}{2}$ of the dough is flour, $\frac{2}{6} = \frac{1}{3}$ is butter and $\frac{1}{6}$ is water.

Part : Whole Ratios

GRADE 3

You might come across a ratio where the LHS is included in the RHS — these are called part : whole ratios.

EXAMPLE:

Mrs Miggins owns tabby cats and ginger cats.
The ratio of tabby cats to the total number of cats is 3 : 5.

a) What fraction of Mrs Miggins' cats are tabby cats?

The ratio tells you that for every 5 cats, 3 are tabby cats.

$$\frac{\text{part}}{\text{whole}} = \frac{3}{5}$$

b) What is the ratio of tabby cats to ginger cats?

3 in every 5 cats are tabby, so 2 in every 5 are ginger.

5 – 3 = 2

For every 3 tabby cats there are 2 ginger cats.

tabby : ginger = 3 : 2

c) Mrs Miggins has 12 tabby cats. How many ginger cats does she have?

Scale up the ratio from part b) to find the number of ginger cats.

tabby : ginger

$$\times 4 \left(\begin{array}{c} 3:2 \\ = \\ 12:8 \end{array} \right) \times 4$$

There are 8 ginger cats

Ratios

If you were worried I was running out of great stuff to say about ratios then worry no more...

Proportional Division

Grade 4

In a proportional division question a TOTAL AMOUNT is split into parts in a certain ratio.
The key word here is PARTS — concentrate on 'parts' and it all becomes quite painless:

EXAMPLE: **Jess, Mo and Greg share £9100 in the ratio 2:4:7. How much does Mo get?**

1) ADD UP THE PARTS:
The ratio 2:4:7 means there will be a total of 13 parts: 2 + 4 + 7 = 13 parts

2) DIVIDE TO FIND ONE "PART":
Just divide the total amount by the number of parts: £9100 ÷ 13 = £700 (= 1 part)

3) MULTIPLY TO FIND THE AMOUNTS:
We want to know Mo's share, which is 4 parts: 4 parts = 4 × £700 = £2800

Watch out for pesky proportional division questions that don't give you the total amount.
You can't just follow the method above, you'll have to be a bit more crafty.

EXAMPLE: **A baguette is cut into 3 pieces. The second piece is twice as long as the first and the third piece is five times as long as the first.**

a) Find the ratio of the lengths of the 3 pieces. Give your answer in its simplest form.

If the first piece is 1 part,
then the second piece is 1 × 2 = 2 parts
and the third piece is 1 × 5 = 5 parts.
So the ratio of the lengths = 1:2:5.

b) The first piece is 28 cm smaller than the third piece. How long is the second piece?

1) Work out how many parts 28 cm makes up. 28 cm = 3rd piece – 1st piece
= 5 parts – 1 part = 4 parts

2) Divide to find one part. 28 cm ÷ 4 = 7 cm

3) Multiply to find the length of the 2nd piece. 2nd piece = 2 parts = 2 × 7 cm = 14 cm

You need to know how to answer all kinds of ratio questions

You should understand how to simplify ratios, scale them up and deal with part:whole ratios. You'll also need to learn the three steps for proportional division. If you're stuck on a question, it's often really helpful to think about what one 'part' is and take it from there...

Warm-up and Worked Exam Questions

You'll have to know ratios inside out for your exam — try these quick warm-up questions first and then you'll be ready to tackle those tricky exam practice questions on the next page.

Warm-up Questions

1) Write these ratios in their simplest forms:
   a) 4 : 8 b) 12 : 27 c) 1.2 : 5.4 d) $\frac{8}{3} : \frac{7}{6}$ e) 0.5 litres : 400 ml
2) Reduce 5 : 22 to the form 1 : n.
3) A recipe uses flour and sugar in the ratio 3 : 2. How much flour do you need if you're using 300 g of sugar?
4) A nursery group has 12 girls and 6 boys.
   a) Write the ratio of girls to boys. b) What fraction of the class are girls?
5) Sarah collects mugs. The ratio of red mugs to the total number of mugs is 6 : 15. Given that Sarah has 50 mugs, how many of them are red?
6) Divide £2400 in the ratio 5 : 7.
7) Divide 180 in the ratio 3 : 4 : 5.

Worked Exam Question

I'm sure you're raring to get stuck into those exam practice questions — before you do, here's one I answered earlier. Read through it carefully and follow the working.

1 Brian is making a fruit punch. He mixes apple juice, pineapple juice and cherryade in the ratio 4 : 3 : 7.

a) What fraction of the fruit punch is pineapple juice? GRADE 3

$4 + 3 + 7 = 14$ parts in total

3 of the 14 parts are pineapple juice

$\frac{3}{14}$

[1 mark]

b) He makes 700 ml of fruit punch. What volume of each drink does he use? GRADE 4

1 part = $700 \div (4 + 3 + 7)$
$= 700 \div 14 = 50$ ml

First find how many ml 1 part is equal to.

Multiply to find the amounts of each juice — apple juice is 4 parts, pineapple juice is 3 parts and cherryade is 7 parts.

Apple juice: 50 ml × 4 = 200 ml
Pineapple juice: 50 ml × 3 = 150 ml
Cherryade: 50 ml × 7 = 350 ml

Apple juice: ......200...... ml

Pineapple juice: ......150...... ml

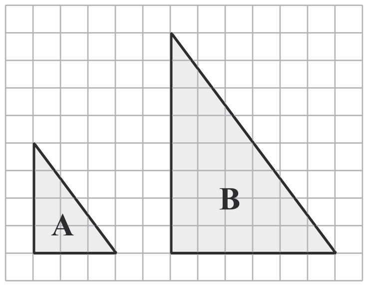
Cherryade: ......350...... ml

[3 marks]

Exam Questions

2 The grid on the right shows two shapes, A and B.

Give the following ratios in their simplest form.



a) Shortest side of shape A : shortest side of shape B

.... *[2 marks]*

b) Area of shape A : area of shape B

.... *[3 marks]*

3 Last month a museum received £21 000 in donations. After taking off the cost of monthly bills, the museum spent the remaining money on new exhibitions.

The ratio of bills to donations was 5 : 14. How much did they spend on new exhibitions?

£ .... *[3 marks]*

4 Mr Appleseed's Supercompost is made by mixing soil, compost and grit in the ratio 4 : 3 : 1. Soil costs £8 per 40 kg, compost costs £15 per 25 kg and grit costs £12 per 15 kg.

What is the total cost of materials for 16 kg of Mr Appleseed's Supercompost?

Start by working out how much of each material is needed for 16 kg of Supercompost.

£ .... *[5 marks]*

Direct Proportion Problems

Direct proportion problems all involve amounts that increase or decrease together.

Learn the **Golden Rule** for **Proportion** Questions

GRADE 3

There are lots of exam questions which at first sight seem completely different but in fact they can all be done using the GOLDEN RULE...

Divide for ONE, then Times for ALL

EXAMPLE:

5 pints of milk cost £1.30. How much will 3 pints cost?

The GOLDEN RULE tells you to:
Divide the price by 5 to find how much FOR ONE PINT,
then multiply by 3 to find how much FOR 3 PINTS.

1 pint: £1.30 ÷ 5 = 0.26 = 26p
3 pints: 26p × 3 = 78p

EXAMPLE:

Emma is handing out some leaflets. She gets paid per leaflet she hands out.
If she hands out 300 leaflets she gets £2.40.
How many leaflets will she have to hand out to earn £8.50?

Divide by £2.40 to find how many leaflets she has to hand out to earn £1.

To earn £1: 300 ÷ £2.40 = 125 leaflets

Multiply by £8.50 to find how many leaflets she has to hand out to earn £8.50.

To earn £8.50: 125 × £8.50 = 1062.5
So she'll need to hand out 1063 leaflets.

You need to round your answer up because 1062 wouldn't be enough.

Scaling **Recipes** Up or Down

GRADE 3

EXAMPLE:

Judy is making orange and pineapple punch using the recipe shown on the right.
She wants to make enough to serve 20 people.
How much of each ingredient will Judy need?

Fruit Punch (serves 8)
800 ml orange juice
140 g fresh pineapple

The GOLDEN RULE tells you to divide each amount by 8 to find how much FOR ONE PERSON, then multiply by 20 to find how much FOR 20 PEOPLE.

So for 1 person you need:
800 ml ÷ 8 = 100 ml orange juice ⇒
140 g ÷ 8 = 17.5 g pineapple ⇒

And for 20 people you need:
20 × 100 ml = 2000 ml orange juice
20 × 17.5 g = 350 g pineapple

Divide for one, then times for all... divide for one, then times for all... divide for one...

Memorise this golden rule — it'll help make direct proportion questions a whole lot easier. Examiners love recipe questions so learn how to use this golden rule to scale the ingredients up and down.

Direct Proportion Problems

There are many types of direct proportion question — here are another couple for you to learn.

Best Buy Questions

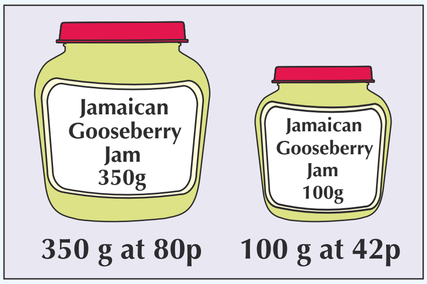
GRADE 3

A slightly different type of direct proportion question is comparing the 'value for money' of 2 or 3 similar items. For these, follow the second GOLDEN RULE...

Divide by the PRICE in pence (to get the amount per penny)

EXAMPLE:

The local 'Supplies 'n' Vittals' stocks two sizes of Jamaican Gooseberry Jam, as shown on the right. Which of these represents better value for money?



Follow the GOLDEN RULE —
divide by the price in pence to get the amount per penny.

In the 350 g jar you get 350 g ÷ 80p = 4.38 g per penny
In the 100 g jar you get 100 g ÷ 42p = 2.38 g per penny

The 350 g jar is better value for money, because you get more jam per penny.

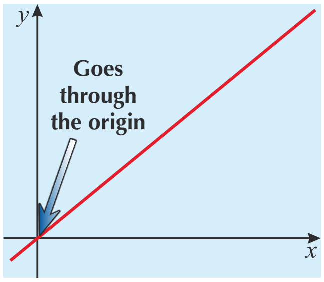
In some cases it might be easier to divide by the weight to get the cost per gram.
If you're feeling confident then you can do it this way — if not, the golden rule always works.

Graphing Direct Proportion

GRADE 4

Two things are in direct proportion if, when you plot them on a graph, you get a straight line through the origin.

Remember, the general equation for a straight line through the origin is $y = Ax$ (see p.61) where A is a number.
All direct proportions can be written as an equation in this form.



EXAMPLE:

The amount of petrol, p litres, a car uses is directly proportional to the distance, d km, that the car travels. The car used 12 litres of petrol on a 160 km journey.

a) **Write an equation in the form $p = Ad$ to represent this direct proportion.**

1) Put the values of $p = 12$ and $d = 160$ into the equation to find the value of A.

$12 = A \times 160$

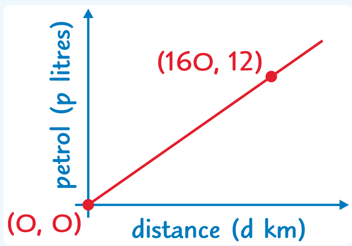
$A = \frac{12}{160}$

$A = 0.075$

2) Put the value of A back into the equation.

$p = 0.075d$

b) **Sketch the graph of this direct proportion, marking two points on the line.**



Direct proportion graphs are always straight lines through the origin

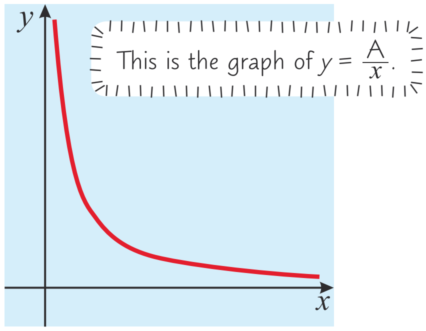
Graphing direct proportions is an important skill but don't forget those best buy questions — in the exam it doesn't matter if you find the amount per penny or price per unit, just do whichever is easiest.

Inverse Proportion Problems

Here's a trickier type of proportion — but once you've learnt this page you'll be an expert.

Graphing Inverse Proportion

When two things are in inverse proportion, one increases as the other decreases.



On the graph you can see that as the value of x increases, the value of y decreases. E.g. if x is doubled, y is halved, or if x is multiplied by 5, y is divided by 5.

The general equation for inverse proportion is $y = \frac{A}{x}$.

EXAMPLE:

Circle each of the equations below that show that s is inversely proportional to t.

$s = \frac{3}{t}$ $\quad 9s = t \quad$ $t = \frac{1}{s}$ $\quad s = \frac{3}{t^2} \quad s = \frac{3}{t} + 7 \quad$ $\frac{s}{5} = \frac{1}{t}$

$s = \frac{3}{t}$ $\quad$ $s = \frac{1}{t}$ $\quad$ $s = \frac{5}{t}$

Check which equations can be written in the form $s = \frac{A}{t}$.

Solving Inverse Proportion Questions

On page 83 you saw the 'divide and times' method for direct proportions. Well, inverse proportions are the opposite so you have to:

TIMES for ONE, then DIVIDE for ALL

EXAMPLES:

1. **It takes 3 farmers 10 hours to plough a field. How long would it take 6 farmers?**

Multiply by 3 to find how long it would take 1 farmer. $\quad 10 \times 3 = 30$ hours for 1 farmer

Divide by 6 to find how long it would take 6 farmers. $\quad 30 \div 6 = 5$ hours for 6 farmers

Another way of looking at this question is that there are twice as many farmers, so it will take half as long ($10 \div 2 = 5$ hours).

2. **4 bakers can decorate 100 cakes in 5 hours. If 5 bakers work at the same rate, how much quicker would they decorate 100 cakes?**

Multiply by 4 to find how long it would take 1 baker. $\quad$ 5 hours $\times 4 = 20$ hours for 1 baker

Divide by 5 to find how long it would take 5 bakers. $\quad 20 \div 5 = 4$ hours for 5 bakers

So 5 bakers are $5 - 4 = 1$ hour quicker than 4.

Think of inverse proportions as the opposite of direct proportions

Learn how to identify inverse proportions by their graphs and equations — then it's a simple matter of practising the 'times for one, then divide for all' method until you're blue in the face.

Warm-up and Worked Exam Questions

Question pages take up a large proportion of this section, but that's just because we think they'll be really useful. Give these warm-up questions a shot when you're ready.

Warm-up Questions

1) If three chocolate bars cost 96p, how much will four of the bars cost?
2) Seven pencils cost £1.40.
   a) How much will four pencils cost?
   b) What is the maximum number of pencils you could buy for £6.50?
3) Marmalade can be bought in 3 different sizes: 250 g (£1.25), 350 g (£2.10) or 525 g (£2.50). Which size is best value for money?
4) It takes 2 carpenters 6 hours to make a bookcase. How long would it take 8 carpenters to make a bookcase?
5) Give two features of a direct proportion graph.
6) Sketch the graph that shows that x is inversely proportional to y.

Worked Exam Questions

I've worked through one direct proportion and one inverse proportion question for you. Go through both questions carefully and you'll be ready to tackle the next page.

1 Joanna gets paid the same hourly rate whenever she works.
In the first week of July Joanna worked for 28 hours and got paid £231.
In each of the next 3 weeks of July, she worked for 25 hours.
How much will Joanna get paid in total for the 4 weeks she worked in July?

GRADE 3

Each hour Joanna gets paid: £231 ÷ 28 = £8.25

In the next three weeks she works 25 × 3 = 75 hours

So in total she gets paid £231 + (75 × £8.25) = £849.75

£ 849.75

[2 marks]

2 Circle the **two** equations below that show that f is inversely proportional to g.

GRADE 4

$f = g^2$ $\quad$ $f = g + 5$ $\quad$ $fg = 7$ $\quad$ $f = \frac{g}{5}$ $\quad$ $g = \frac{3}{f}$

Look for equations which can be rearranged to have the form $f = \frac{A}{g}$, where A is a number.

[2 marks]

Exam Questions

3 Brown sauce can be bought in three different sizes.
The price of each is shown on the right.
Which size of bottle is the best value for money?

.... ml
[3 marks]

4 A football coach buys a bottle of water for each child in a football club.
All the bottles of water are the same price. There are 42 boys in the club.
He spends £52.50 on water for the boys. He spends £35 on water for the girls.
How many girls are there in the football club?

....
[2 marks]

5 A ship has enough food to cater for 250 people for 6 days.

a) For how many days can it cater for 300 people?

.... days
[2 marks]

b) How many more people can it cater for on a 2-day cruise than on a 6-day cruise?

.... people
[3 marks]

6 Bryn and Richard have just finished playing a game.
The ratio of Bryn's points to Richard's was 5 : 2.

a) On a set of axes, draw a graph that could be used to work out Bryn's points if you know Richard's points.

[2 marks]

b) Richard scored 22 points. How many points did Bryn score?

.... points
[1 mark]

Percentages

You're going to see 6 different types of percentage question on the next three pages.
The first few shouldn't give you too much trouble. Especially if you remember:

1) 'Per cent' means 'out of 100', so 20% means '20 out of 100' = $\frac{20}{100}$.
2) To work out the percentage OF something replace the word OF with a multiplication (×).

Six Different Question Types

Type 1 — "Find x% of y"

GRADE 3

If you have a calculator, turn the percentage into a decimal, then multiply.

EXAMPLE: **Find 18% of £4.**

Change 18% to a decimal and multiply.

18% of £4
= 18% × £4
= 0.18 × £4 = £0.72

Replace 'of' with '×'.

If you don't have a calculator, you can use this clever method instead:

EXAMPLE: **Find 135% of 600 kg.**

You can also find 1% by dividing by 100.

100% = 600 kg

1) Find 10% by dividing by 10: 10% = 600 ÷ 10 = 60 kg
2) Find 5% by dividing 10% by 2: 5% = 60 ÷ 2 = 30 kg
3) Use these values to make 135%: 135% = 100% + (3 × 10%) + 5%
= 600 + (3 × 60) + 30 = 810 kg

Type 2 — "Express x as a percentage of y"

GRADE 3

Divide *x* by *y*, then multiply by 100.

EXAMPLES: 1. **Give 36p as a percentage of 80p.**

Divide 36p by 80p, then multiply by 100: $\frac{36}{80} \times 100 = 45\%$

If you don't have a calculator you'll have to simplify the fraction (see p.15).

2. **Farmer Littlewood measured the width of his prized pumpkin at the start and end of the month. At the start of the month it was 84 cm wide and at the end of the month it was 1.32 m wide. Give the width at the end of the month as a percentage of the width at the start.**

1) Make sure both amounts are in the same units. 1.32 m = 132 cm

2) Divide 132 cm by 84 cm, then multiply by 100: $\frac{132}{84} \times 100 = 157.14\%$ (2 d.p.)

Percentages are one of the most useful things you'll ever learn

Whenever you open a newspaper, see an advert, watch TV or do a maths exam paper you will see percentages. It's really important you get confident with using them — so practise.

Percentages

Type 3 — New Amount After a % Increase or Decrease

GRADE 3

There are two different ways of finding the new amount after a percentage increase or decrease:

1) Find the % then Add or Subtract.

Find the % of the original amount. Add this on to (or subtract from) the original value.

EXAMPLE: **A dress has increased in price by 30%. It originally cost £40. What is the new price of the dress?**

1) Find 30% of £40: 30% of £40 = 30% × £40 = 0.3 × 40 = £12

2) It's an increase, so add on to the original: £40 + £12 = £52

2) The Multiplier Method

This time, you first need to find the multiplier — the decimal that represents the percentage change.

E.g. 5% increase is 1.05 (= 1 + 0.05) 26% decrease is 0.74 (= 1 – 0.26)

Then you just multiply the original value by the multiplier and voilà — you have the answer.

A % decrease has a multiplier less than 1,
a % increase has a multiplier greater than 1.

EXAMPLE: **A hat is reduced in price by 20% in the sales. It originally cost £12. What is the new price of the hat?**

1) Find the multiplier: 20% decrease = 1 – 0.20 = 0.8

2) Multiply the original value by the multiplier: £12 × 0.8 = £9.60

Type 4 — Simple Interest

GRADE 3

Compound interest is covered on page 92.

Simple interest means a certain percentage of the original amount only is paid at regular intervals (usually once a year). So the amount of interest is the same every time it's paid.

EXAMPLE: **Regina invests £380 in an account which pays 3% simple interest each year. How much interest will she earn in 4 years?**

1) Work out the amount of interest earned in one year: 3% = 3 ÷ 100 = 0.03
3% of £380 = 0.03 × £380 = £11.40

2) Multiply by 4 to get the total interest for 4 years: 4 × £11.40 = £45.60

Learn how to solve these simple question types

Before you move on to the trickier types on the next page, you need to be confident with the first four types of percentage questions. So, cover the page and practise.

Percentages

Watch out for these trickier types of percentage question — they'll often include lots of real-life context.

Type 5 — Finding the Percentage Change

GRADE 4

1) This is the formula for giving a change in value as a percentage — LEARN IT, AND USE IT:

$$\text{Percentage 'Change'} = \frac{\text{'CHANGE'}}{\text{ORIGINAL}} \times 100$$

2) Typical questions will ask 'Find the percentage increase/profit/error' or 'Calculate the percentage decrease/loss/discount', etc.

EXAMPLE:

Tariq buys a plain plate for £2. He paints it, then sells it at a craft fair for £3.75. Find his profit as a percentage.

1) Here the 'change' is profit, so the formula looks like this: $\text{percentage profit} = \frac{\text{profit}}{\text{original}} \times 100$

2) Work out the profit (amount made – original cost) profit = £3.75 – £2 = £1.75

3) Calculate the percentage profit: $\text{percentage profit} = \frac{£1.75}{£2} \times 100 = 87.5\%$

Type 6 — Finding the Original Value

GRADE 4

This is the type that most people get wrong — but only because they don't recognise it as this type, and don't apply this simple method:

1) Write the amount in the question as a percentage of the original value.
2) Divide to find 1% of the original value.
3) Multiply by 100 to give the original value (= 100%).

EXAMPLE:

A house increases in value by 10% to £165 000. Find what it was worth before the rise.

Note: The new, not the original value is given.

1) An increase of 10% means £165 000 represents 110% of the original value.

2) Divide by 110 to find 1% of the original value.

3) Then multiply by 100.

$$\begin{aligned} £165\,000 &= 110\% \\ &\downarrow \div 110 \\ £1500 &= 1\% \\ &\downarrow \times 100 \\ £150\,000 &= 100\% \end{aligned}$$

If it was a decrease of 10%, then you'd put '£165 000 = 90%' and divide by 90 instead of 110.

So the original value was £150 000

Always set them out exactly like this example. The trickiest bit is deciding the top % figure on the right-hand side — the 2nd and 3rd rows are always 1% and 100%.

Learn the 6 different types of percentage question

If you learn how to use the percentage change formula then you'll breeze through those questions on the exam. To find the original value remember to divide to find 1% and then multiply to find 100%.

Percentages

This page is all about tackling tricky percentage questions. Once you know the basic types (pages 88-90), have a look at these examples and try to follow them step by step.

Working with Percentages

GRADE 5 GRADE

1) Sometimes there isn't a set method you can follow to answer percentage questions.
2) You'll have to use what you've learnt on the last few pages and do a bit of thinking for yourself.

EXAMPLES:

1. 80% of the members of a gym are male.
35% of the male members are aged 40 and over.
What percentage of gym members are males under 40 years old?

1) The percentage of male members under 40 is: 100% – 35% = 65%
2) The percentage of gym members that are male and under 40 is: 65% of 80% = 0.65 × 80% = 52%

It's just like finding x% of y — but this time the y is a percentage too.

2. Yohan sells scarves. Each scarf costs £1.50 to make. One day he sold 500 scarves.
He sold 90% of them for £2 each and the rest at £1 each.
How much profit did Yohan make?

1) To make 500 scarves, it costs him: 500 × £1.50 = £750
2) Work out how many were sold at each price. 90% of 500 = 0.9 × 500 = 450 at £2 each
   10% of 500 = 0.1 × 500 = 50 at £1 each
3) Work out the amount he made from selling them. (450 × £2) + (50 × £1) = £900 + £50 = £950
4) Calculate his profit. £950 – £750 = £200

3. The number of visitors to a theme park in 2012 was 250 000. In 2013, the number of visitors was 12% lower than in 2012. In 2014, the number of visitors was 15% higher than in 2013. How many visitors were there in 2014?

1) First, work out the number of visitors in 2013. 100% – 12% = 88%
   88% of 250 000 = 0.88 × 250 000 = 220 000
2) Then work out the number of visitors in 2014. Make sure you use the value you've just found. 100% + 15% = 115%
   115% of 220 000 = 1.15 × 220 000 = 253 000

Percentage questions can crop up in all shapes and sizes

The key thing is not to panic if you see an odd percentages question in your exam.
It will always be a case of using one or more of the methods you've seen on the previous pages.

Compound Growth and Decay

One more sneaky % type for you... In compound growth/decay the amount added on/taken away changes each time — it's a percentage of the new amount, rather than the original amount.

Compound Growth and Decay

Grade 5

Compound interest is a popular context for these questions — it means the interest is added on each time, and the next lot of interest is calculated using the new total rather than the original amount.

EXAMPLE: **Daniel invests £10 000 in a savings account which pays 4% compound interest per annum. How much money will there be in his account after 3 years?**

1) Work out the multiplier: Multiplier = 4% increase = 1.04

2) Find the amount in the savings account each year until you get to 3 years.

After 1 year: £10 000 × 1.04 = £10 400
After 2 years: £10 400 × 1.04 = £10 816
After 3 years: £10 816 × 1.04 = £11 248.64

'Per annum' just means 'each year'.

Compound decay (depreciation) questions are about things that decrease in value or number over time.

EXAMPLE: **Susan has just bought a car for £6500. The car depreciates by 8% each year. How many years will it be before the car is worth less than £5000?**

1) Work out the multiplier: 8% decrease = 1 – 0.08 = 0.92

2) Calculate the value of the car each year — stop when the value drops below £5000.

After 1 year: £6500 × 0.92 = £5980
After 2 years: £5980 × 0.92 = £5501.60
After 3 years: £5501.60 × 0.92 = £5061.472
After 4 years: £5061.472 × 0.92 = £4656.55424

So it will be 4 years before the car is worth less than £5000.

The Formula

Grade 5

If you're feeling confident with compound growth and decay then learn this formula. It'll speed things up...

$$N = N_0 \times (\text{multiplier})^n$$

Amount after n days/hours/years → N
Initial amount → N_0
Percentage change multiplier → multiplier
Number of days/hours/years → n

EXAMPLE: **The number of bacteria in a sample increases at a rate of 40% each day. Initially, there were 1000 bacteria. How many bacteria will there be after 10 days? Give your answer to the nearest 1000.**

1) Work out the multiplier: Multiplier = 40% increase = 1.4

2) Put an initial value of 1000, a time of 10 days and the multiplier into the formula. → $1000 \times 1.4^{10} = 28\,925.465...$

So after 10 days there will be 29 000 bacteria (to the nearest 1000).

Compound growth and decay — percentages applied again and again

What this method does is to get the original value, increase it by the percentage, then increase that amount by the percentage, then take that amount and increase it by the percentage, then... get it?

Warm-up and Worked Exam Questions

Have a go at these warm-up questions and see how you get on — the exam questions will be a bit more tricky, so it's important that you can do these first.

Warm-up Questions

1) Calculate 34% of £50.
2) What is 37 out of 50 as a percentage?
3) Find 15% of 60 (without using a calculator).
4) A suit costs £120 during a sale. Once the sale is over, the price of the suit rises by 15%. What is the new price of the suit?
5) You pay £200 into a bank account that pays 2.5% simple interest per year. How much money will be in the account after one year?
6) What is the percentage decrease when 1200 g is decreased to 900 g?
7) A junk shop is having a 20% off sale. A lobster statue is £4.88 in the sale. How much was the lobster statue before the sale?
8) £3000 is invested at 3% compound interest (per year). Work out how much money is in the account at the end of 4 years, to the nearest penny.

Worked Exam Question

Study this worked exam question well — then have a go at some for yourself on the next page.

1 Oli and Ben each have a bank account that pays 8% simple interest per annum.
They each deposit an amount of money and don't pay in or take out any other money.

a) Oli deposits £2000 in the account. How much will be in the account after 3 years? (Grade 3)

8% = 0.08

Each year he gets £2000 × 0.08 = £160 interest

After 3 years he'll have £2000 + (3 × £160) = £2480

£ ......2480......

[2 marks]

b) After the first year, Ben had £702 in his account.
How much money did he originally put in the account? (Grade 4)

£702 = 108%

÷ 108 ÷ 108

£6.50 = 1%

× 100 × 100

£650 = 100%

£ ......650......

[3 marks]

Exam Questions

2 Jane owns a fashion shop.
Jane sells a pair of jeans for £33.25 plus VAT at 20%.
How much does she sell the pair of jeans for? (Grade 3)

£ ....
[2 marks]

3 Franz always spends £2.40 a week on packs of football stickers.
The stickers normally cost 40p per pack but this week they are 40% cheaper. (Grade 4)

How many more packs of stickers can he get this week than in a normal week?

....
[4 marks]

4 A pet rescue shelter houses cats and dogs. The ratio of cats : dogs is 3 : 7.
40% of the cats are black and 50% of the dogs are black. (Grade 5)
What percentage of the animals at the shelter are black?

.... %
[4 marks]

5 Mrs Burdock borrows £750 to buy a sofa.
She is charged 6% compound interest per annum. (Grade 5)
If Mrs Burdock doesn't pay back any of the money for 3 years, how much will she owe?
Give your answer to the nearest penny.

£ ....
[3 marks]

Unit Conversions

A nice easy page for a change — just some facts to learn.

Metric and Imperial Units

GRADE 1

COMMON METRIC CONVERSIONS

1 cm = 10 mm	1 tonne = 1000 kg
1 m = 100 cm	1 litre = 1000 ml
1 km = 1000 m	1 litre = 1000 cm^3
1 kg = 1000 g	1 cm^3 = 1 ml

COMMON IMPERIAL CONVERSIONS

1 Yard = 3 Feet   1 Foot = 12 Inches
1 Gallon = 8 Pints
1 Stone = 14 Pounds
1 Pound = 16 Ounces

COMMON METRIC-IMPERIAL CONVERSIONS

1 kg ≈ 2.2 pounds   1 foot ≈ 30 cm   1 gallon ≈ 4.5 litres   1 mile ≈ 1.6 km

≈ means approximately equal to

You only need to remember the metric conversions, but you should be able to use them all.

3 Step Method for Converting Units

GRADE 1

1. Find the conversion factor (always easy).
2. Multiply AND divide by it.
3. Choose the common sense answer.

Make sure you cross out the incorrect working or you could lose marks.

1 **Convert 7 litres into ml.**

1) Find the conversion factor. — 1 litre = 1000 ml, so conversion factor = 1000
2) Multiply and divide by it. — 7 × 1000 = 7000   ~~7 ÷ 1000 = 0.007~~
3) Choose the sensible answer — there should be more millilitres than litres. — 7 litres = 7000 ml

2 **Neil's jet ski can hold 116 pints of fuel. How many gallons is this? (1 gallon = 8 pints)**

1) Find the conversion factor. — 1 gallon = 8 pints, so conversion factor = 8
2) Multiply and divide by it. — ~~116 × 8 = 928~~   116 ÷ 8 = 14.5
3) Choose the sensible answer — there should be fewer gallons than pints. — 116 pints = 14.5 gallons

3 **Igor unicycles 8 miles to work. How far is this in km? (1 mile ≈ 1.6 km)**

1) Find the conversion factor. — 1 mile ≈ 1.6 km, so conversion factor = 1.6
2) Multiply and divide by it. — 8 × 1.6 = 12.8   ~~8 ÷ 1.6 = 5~~
3) Choose the sensible answer — there should be more km than miles. — 8 miles ≈ 12.8 km

Make sure you've learned all of the metric conversions

Any conversion involving imperial units will be given in the exam. Another thing to remember is the three step method for conversion — and don't forget to cross out any incorrect working.

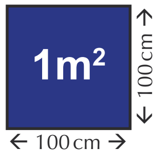
Area and Volume Conversions

Time for some trickier conversions to sink your teeth into. There are a couple of methods for you to remember so that when it comes to the exam you can feel confident converting areas and volumes.

Converting Areas

GRADE 4

You need to be really careful when converting areas — just because 1 m = 100 cm DOES NOT mean 1 m^2 = 100 cm^2.
Follow this method to avoid slipping up:



1 m^2 = 100 cm × 100 cm = 10 000 cm^2
1 cm^2 = 10 mm × 10 mm = 100 mm^2

1) Find the conversion factor — it'll be the same as for converting units (see p.95).
2) Multiply AND divide by the conversion factor TWO TIMES.
3) Choose the common sense answer, and don't forget that the units come with a 2, e.g. mm^2, cm^2

EXAMPLE: **The area of the top of a table is 0.6 m^2. Find its area in cm^2.**

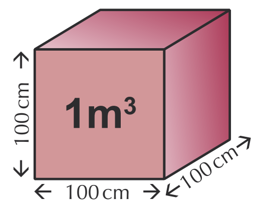
1) Find the conversion factor: 1 m = 100 cm ⟶ Conversion factor = 100
2) It's an area — multiply and divide twice by the conversion factor: 0.6 × 100 × 100 = 6000
~~0.6 ÷ 100 ÷ 100 = 0.00006~~
3) Choose the sensible answer: 0.6 m^2 = 6000 cm^2

1 m = 100 cm so expect more cm than m.

Converting Volumes

GRADE 4

1 m^3 = 100 cm × 100 cm × 100 cm = 1 000 000 cm^3
1 cm^3 = 10 mm × 10 mm × 10 mm = 1000 mm^3



1) Conversion factor — it'll be the same as for converting units (see p.95).
2) Multiply AND divide by the conversion factor THREE TIMES.
3) Choose the common sense answer, and don't forget that the units come with a 3, e.g. mm^3, cm^3.

EXAMPLE: **A glass has a volume of 72 000 mm^3. What is its volume in cm^3?**

1) Find the conversion factor: 1 cm = 10 mm ⟶ Conversion factor = 10
2) It's a volume — multiply and divide 3 times by the conversion factor: ~~72 000 × 10 × 10 × 10 = 72 000 000~~
72 000 ÷ 10 ÷ 10 ÷ 10 = 72
3) Choose the sensible answer: 72 000 mm^3 = 72 cm^3

1 cm = 10 mm so expect fewer cm than mm.

Remember — area comes with a 2 and volume comes with a 3

Learn the rules for converting between units for areas and volumes. Always check your answer to see if it is sensible or not. Common sense will get you a long way.

Time Intervals

Make sure you can convert between time units — it's simple and might grab you a mark or two in the exam.

Converting Time Units

GRADE 1

1) You ought to know the standard time unit conversions by now.
2) Use these standard conversions to change the units of other times.

1 day = 24 hours
1 hour = 60 minutes
1 minute = 60 seconds

EXAMPLES:

1. **How many seconds are there in an hour?**

1) First convert hours into minutes. — 1 hour = 60 minutes
2) Then convert minutes into seconds. — 60 minutes = 60 × 60 = 3600 seconds

2. **Write 186 minutes in hours and minutes.**

1) Work out how many complete hours there are in 186 minutes.
   2 hours = 2 × 60 = 120 minutes
   3 hours = 3 × 60 = 180 minutes ✓
   4 hours = 4 × 60 = 240 minutes — too many
2) Find how many minutes over 3 hours it is.
   186 – 180 = 6 minutes
   So 186 minutes = 3 hours 6 minutes

Be very careful when using calculators — the decimal answers they give are confusing, e.g. 2.5 hours = 2 hours 30 mins, NOT 2 hours 50 mins.

Break Time Calculations into Simple Stages

GRADE 1

EXAMPLE:

Angela watched a film that started at 7.20 pm and finished at 10.05 pm. How long was the film in minutes?

1) Split the time between 7.20 pm and 10.05 pm into simple stages.
   7.20 pm → (+ 2 hours) → 9.20 pm → (+ 40 minutes) → 10.00 pm → (+ 5 minutes) → 10.05 pm
2) Convert the hours to minutes. — 2 hours = 2 × 60 = 120 minutes
3) Add to get the total minutes. — 120 + 40 + 5 = 165 minutes

Timetables

GRADE 2

EXAMPLE:

Use the timetable to answer this question.

Harry wants to get a bus from the bus station to the train station in time for a train that leaves at 19:30. What is the latest bus he can catch?

Bus Timetable

Bus Station	18 45	19 00	19 15	19 30
Market Street	18 52	19 07	19 22	19 37
Long Lane	19 01	19 16	19 31	19 46
Train Station	19 11	19 26	19 41	19 56

1) Read along the train station row. — 19 11   19 26   19 41   19 56
   This is the latest time he could arrive before 19:30.
2) Move up this column to the bus station row and read off the entry.
   The bus that gets to the train station at 19:26 leaves the bus station at 19:00.

This page might look easy, but make sure you learn it all

It's easy to go wrong when you're using your calculator for time questions, so be extra careful. Always try to split questions down into easier stages — that way you'll make fewer mistakes.

Speed, Density and Pressure

Let's see if you can speed through this page. You need to know the formulas and be able to substitute numbers into them — you should also be able to convert between different units (which is a bit tricky).

Speed = Distance ÷ Time

GRADE 3

Speed is the distance travelled per unit time — the number of km per hour or metres per second.

$$\text{SPEED} = \frac{\text{DISTANCE}}{\text{TIME}} \qquad \text{TIME} = \frac{\text{DISTANCE}}{\text{SPEED}} \qquad \text{DISTANCE} = \text{SPEED} \times \text{TIME}$$

A formula triangle is a mighty handy tool for remembering formulas. Here's the one for speed.
To remember the order of the letters (S^DT) we have the words SaD Times.
So if it's a question on speed, distance and time, just say SAD TIMES.

$$\frac{D}{S \times T}$$

How DO YOU USE Formula Triangles?

1) COVER UP the thing you want to find and WRITE DOWN what's left.
2) Now PUT IN THE VALUES for the other two things and WORK IT OUT.

E.g. to get the formula for speed from the triangle, cover up S and you're left with $\frac{D}{T}$.

EXAMPLES:

1. Rob cycles 18 miles in 2 hours. What is his average speed?

1) You want speed so covering S gives: $S = \frac{D}{T}$

2) Put in the numbers — and don't forget the units. $S = 18 \div 2$
$= 9$ mph

CHECK YOUR UNITS MATCH
If the distance is in miles and the time is in hours then you'll get a speed in mph.

2. A cheetah runs at a constant speed of 27 m/s for 20 s. What distance does it cover?

1) You want distance so covering D gives: $D = S \times T$

2) Put in the numbers — and don't forget the units. $D = 27 \times 20$
$= 540$ m

UNITS CHECK: m/s and s go in so m comes out.

Density = Mass ÷ Volume

GRADE 4

Density is the mass per unit volume of a substance. It's usually measured in kg/m³ or g/cm³.

$$\text{DENSITY} = \frac{\text{MASS}}{\text{VOLUME}} \qquad \text{VOLUME} = \frac{\text{MASS}}{\text{DENSITY}} \qquad \text{MASS} = \text{DENSITY} \times \text{VOLUME}$$

Here's the formula triangle for density. To remember the order of the letters in the formula triangle think D^MV or DiMoV (the Russian agent).

$$\frac{M}{D \times V}$$

EXAMPLE:

A giant 'Wunda-Choc' bar has a density of 1.3 g/cm³. If the bar's volume is 1800 cm³, what is the mass of the bar in grams?

1) You want the mass, so covering M gives: $M = D \times V$

2) Put in the numbers — and remember the units. $M = 1.3\ \text{g/cm}^3 \times 1800\ \text{cm}^3$
$= 2340$ g

UNITS CHECK: g/cm³ and cm³ go in so g comes out.

Speed, Density and Pressure

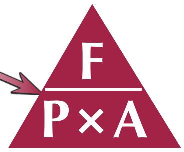
Pressure = Force ÷ Area

'N' stands for 'Newtons'.

Pressure is the amount of force acting per unit area. It's usually measured in N/m^2, or pascals (Pa).

$$\text{PRESSURE} = \frac{\text{FORCE}}{\text{AREA}} \qquad \text{AREA} = \frac{\text{FORCE}}{\text{PRESSURE}} \qquad \text{FORCE} = \text{PRESSURE} \times \text{AREA}$$

Another formula triangle here — this one's for pressure. When you come across a pressure question, a handy trick to remember the order of the letters (P^FA) is to put on your best Spanish accent and say "ohhh PerFectA".



EXAMPLE: **A cuboid box with a weight of 200 N rests on horizontal ground. The side lengths of the face resting on the ground are 0.2 m and 0.8 m. Calculate the pressure exerted by the box on the ground.**

1) Work out the area of the face in contact with the ground: $0.2\text{ m} \times 0.8\text{ m} = 0.16\text{ m}^2$
2) Use the formula triangle — you want pressure so covering P gives: $P = \frac{F}{A}$
3) Put in the numbers. $P = \frac{200\text{ N}}{0.16\text{ m}^2}$
4) Check the units — you put in N and m^2 so you'll get N/m^2. $= 1250\text{ N/m}^2$

Converting Speed, Density and Pressure

1) Units of speed, density and pressure are made up of two measures — a distance and a time, a mass and a volume, or a force and an area.
2) So to convert units of speed, density or pressure, you might need to do two conversions — one for each measure.

EXAMPLE: **A rabbit's top speed is 54 km/h. How fast is this in m/s?**

1) First convert from km/h to m/h:
   1 km = 1000 m, so conversion factor = 1000
   54 × 1000 = 54 000 ~~54 ÷ 1000 = 0.054~~
   54 km/h = 54 000 m/h
2) Now convert from m/h to m/s:
   1 hour = 60 minutes = 60 × 60 = 3600 seconds
   So conversion factor = 3600
   ~~54000 × 3600 = 194400000~~ 54000 ÷ 3600 = 15
   54 km/h = 54 000 m/h = 15 m/s

Formula triangles are dead useful

Cover up the thing you want to find, then write down what's left on show. Put in the values and out pops your answer. Make sure that the units make sense — you won't get a distance in seconds.

Warm-up and Worked Exam Questions

Time to check all that lovely revision has sunk in. Try these first to make sure you've learnt the key stuff:

Warm-up Questions

1) a) How many mm is 6.5 cm? b) How many kg is 250 g?
2) A machine was found to weigh 0.16 tonnes. What is this in kg?
3) A rod is 46 inches long. What is this in feet and inches?
4) a) Roughly how many km is 200 miles? b) Roughly how many feet is 120 cm?
5) Convert these measurements: a) 23 m^2 to cm^2 b) 34 500 cm^2 to m^2.
6) A train leaves at 9.37 am and arrives at 11.16 am. How long is the train journey in minutes?
7) A plane sets off at 10.15 am. The flight lasts 5 hrs 50 mins. What is the arrival time?
8) A cylinder rests with its 8 m^2 circular face on horizontal ground. The pressure it exerts on the ground is 350 N/m^2. What is the force of the cylinder on the ground?
9) A sprinter runs 400 m in 50 seconds. What was his average speed in m/s?
10) A cube has a mass of 25 g and side length 2 cm. What is its density in g/cm^3?

Worked Exam Question

Make sure you really take this stuff in — read the working thoroughly and then have a go yourself to check you've understood. You'll kick yourself if this comes up in the exam and you only gave it a quick glance.

1 Adam has been caught speeding by a pair of cameras measuring average speed.
The speed limit was 30 mph. The cameras are 2.5 km apart.
The time taken for his car to pass between them was 3 minutes.

a) What was Adam's average speed between the cameras?
Give your answer to the nearest mph. (1 mile ≈ 1.6 km) GRADE 4

2.5 km ≈ 2.5 ÷ 1.6 = 1.5625 miles

1 hour = 60 minutes, so 3 minutes = 3 ÷ 60 = 0.05 hours

Speed $= \frac{\text{Distance}}{\text{Time}}$

Speed = 1.5625 miles ÷ 0.05 hours
= 31.25 mph = 31 mph (to nearest mph)

..........31.......... mph
[3 marks]

b) If Adam had been travelling within the speed limit, what is the minimum time it should have taken him to pass between the cameras? GRADE 5
Give your answer to the nearest second.

Time = 1.5625 miles ÷ 30 mph = 0.05208... hours

Time $= \frac{\text{Distance}}{\text{Speed}}$

0.05208... hours × 60 × 60 = 187.5 seconds
= 188 seconds (to nearest second)

..........188.......... s
[2 marks]

Exam Questions

2 Isaac and Ultan spent 13 days building a model robot.
On the first 12 days they built from 4.30 pm till 7.15 pm and on the last day they built for a total of 7 hours 10 minutes.

What is the total amount of time they spent building the robot?
Give your answer in hours and minutes.

.... hours .... minutes
[4 marks]

3 1 gallon = 8 pints. 9 litres ≈ 2 gallons.
Approximately how many litres are there in 64 pints?

.... litres
[3 marks]

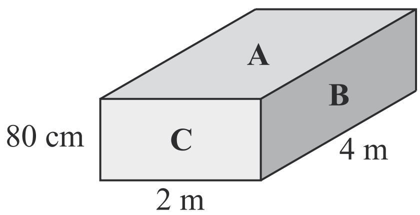
4 Nicole wants to post eight books to a friend in another country.
Each book weighs 0.55 lb and postage is £0.50 per 100 g. (1 kg ≈ 2.2 lb)

How much will it cost her to post all the books to her friend?

£ ....
[4 marks]

5 Look at the cuboid on the right. Three of its faces are labelled A, B and C. The cuboid has a weight of 40 N.

Calculate the pressure, in N/m^2, that the cuboid exerts on horizontal ground when the cuboid is resting on face A.



.... N/m^2
[3 marks]

Revision Questions for Section Four

Lots of things to remember in Section Four — there's only one way to find out what you've taken in...

- Try these questions and tick off each one when you get it right.
- When you've done all the questions for a topic and are completely happy with it, tick off the topic.

Ratios (p78-80)

1) Reduce: a) 18 : 22to its simplest form b) 49 g : 14 g to the form $n:1$
2) Sarah is in charge of ordering stock for a clothes shop. The shop usually sells red scarves and blue scarves in the ratio 5 : 8. Sarah orders 50 red scarves. How many blue scarves should she order?
3) Pencils and rubbers are in the ratio 7 : 2. How many times more pencils are there than rubbers?
4) Ryan, Joel and Sam are sharing 800 lollipops. They split the lollipops in the ratio 5 : 8 : 12.
   a) What fraction of the lollipops does Ryan get? b) How many lollipops does Sam get?
5) The recipe on the right shows CGP's secret pasta sauce recipe. The recipe serves 6 people. How much of each ingredient is needed to make enough for 17 servings?
   - 18 ml olive oil
   - 360 g tomatoes
   - 9 g garlic powder
   - 72 g onions

Proportion Problems (p83-85)

6) 3 gardeners can plant 360 flowers in a day. How many flowers could 8 gardeners plant in a day?
7) 'y is directly proportional to x'. Sketch the graph of this proportion for $x \geq 0$.
8) A DIY shop sells varnish in two different-sized tins. A 500 ml tin costs £8 and a 1800 ml tin costs £30. Which tin represents the better value for money?
9) The amount of time it takes to wash a car is inversely proportional to the number of people washing it. 3 people take 12 minutes to wash a car. How many people are needed to wash a car in 2 minutes?

Percentages (p88-92)

10) If $x = 20$ and $y = 95$: a) Find x% of y. b) Find the new value after y is increased by x%.
    c) Express x as a percentage of y. d) Express y as a percentage of x.
11) What's the formula for finding a change in value as a percentage?
12) An antique wardrobe decreased in value from £800 to £520. What was the percentage decrease?
13) A tree's height has increased by 15% in the last year to 20.24 m. What was its height a year ago?
14) 25% of the items sold by a bakery in one day were pies. 8% of the pies sold were chicken pies. What percentage of the items sold by the bakery were chicken pies?
15) Collectable baseball cards increase in value by 10% each year. A particular card is worth £80.
    a) How much will it be worth in 4 years? b) In how many years will it be worth over £140?

Conversions and Time (p95-97)

16) Convert: a) 5.6 litres to cm^3 b) 8 feet to cm c) 2 weeks into hours
    d) 12 m^3 to cm^3 e) 1280 mm^2 to cm^2 f) 2.75 cm^3 to mm^3
17) A musical production starts at 19:30. The musical is 118 minutes long plus a 20 minute interval. What time does the musical finish? Give your answer in 12-hour time.

Speed, Density and Pressure (p98-99)

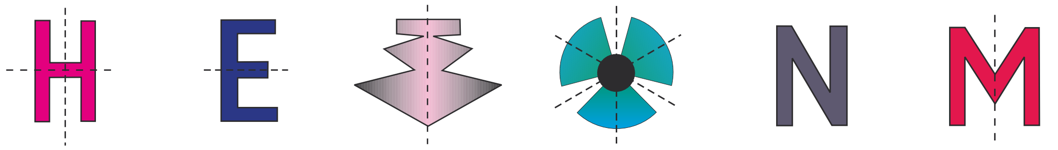
18) Find the average speed, in km/h, of a car if it travels 63 miles in 1.5 hours.
19) Find the volume, in cm^3, of a snowman if its density is 0.4 g/cm^3 and its mass is 5000 g.
20) Find the area of an object in contact with horizontal ground, if the pressure it exerts on the ground is 120 N/m^2 and the force acting on the object is 1320 N.

Properties of 2D Shapes

Here's a nice easy page to get you started on 2D shapes.

Line Symmetry

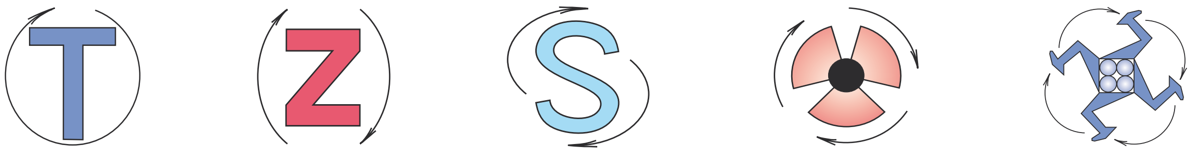
This is where you draw one or more MIRROR LINES across a shape and both sides will fold exactly together.



2 LINES OF SYMMETRY — 1 LINE OF SYMMETRY — 1 LINE OF SYMMETRY — 3 LINES OF SYMMETRY — NO LINES OF SYMMETRY — 1 LINE OF SYMMETRY

Rotational Symmetry

This is where you can rotate the shape into different positions that look exactly the same.



Order 1 — Order 2 — Order 2 — Order 3 — Order 4

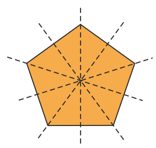
The ORDER OF ROTATIONAL SYMMETRY is the posh way of saying: 'how many different positions look the same'. You should say the Z-shape above has 'rotational symmetry of order 2'.

When a shape has only 1 position you can either say that it has 'rotational symmetry of order 1' or that it has 'NO rotational symmetry'.

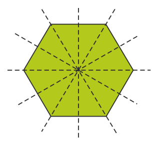
Regular Polygons

In an irregular polygon, the sides and angles aren't all equal.

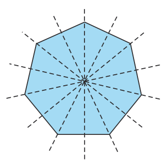
All the sides and angles in a regular polygon are the same.
Learn the names of these regular polygons and how many sides they have.
(An equilateral triangle and a square are both regular polygons — see p.104 and p.105 for their properties.)



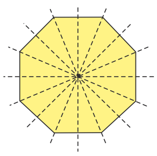
REGULAR PENTAGON
5 sides
5 lines of symmetry
Rotational symmetry of order 5



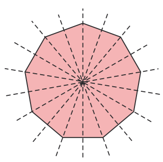
REGULAR HEXAGON
6 sides
6 lines of symmetry
Rotational symmetry of order 6



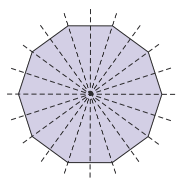
REGULAR HEPTAGON
7 sides
7 lines of symmetry
Rotational symmetry of order 7



REGULAR OCTAGON
8 sides
8 lines of symmetry
Rotational symmetry of order 8



REGULAR NONAGON
9 sides
9 lines of symmetry
Rotational symmetry of order 9



REGULAR DECAGON
10 sides
10 lines of symmetry
Rotational symmetry of order 10

Make sure you learn the two different types of symmetry

You'll then be able to dazzle your friends by spotting symmetry in everyday shapes like road signs and letters — and more importantly, you'll get some nice marks in your exam. Regular polygons have the same number of lines of symmetry and the same order of rotational symmetry as the number of sides.

Properties of 2D Shapes

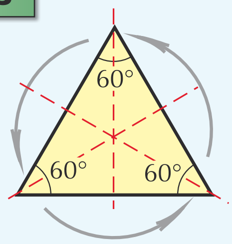
These two pages have a load of details about triangles and quadrilaterals — and you need to learn them all.

Triangles

GRADE 2

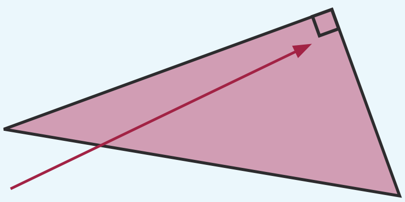
1) Equilateral Triangles

3 equal sides and
3 equal angles of 60°.
3 lines of symmetry,
rotational symmetry order 3.



2) Right-angled Triangles

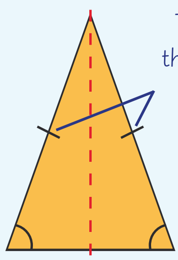
1 right angle (90°).
No lines of symmetry.
No rotational symmetry.



The little square means it's a right angle.

3) Isosceles Triangles

2 sides the same.
2 angles the same.
1 line of symmetry.
No rotational symmetry.

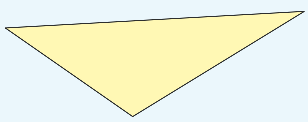


These dashes mean that the two sides are the same length.

An acute-angled triangle has 3 acute angles, and an obtuse-angled triangle has one obtuse angle (see p.126).

4) Scalene Triangles

All three sides different.
All three angles different.
No symmetry (pretty obviously).



Triangles have three sides

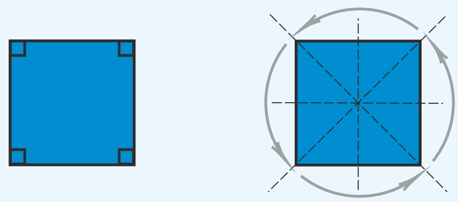
Learn the names (and how to spell them) and the properties of all the triangles on this page.
These are easy marks in the exam — make sure you know them all.

Properties of 2D Shapes

Quadrilaterals

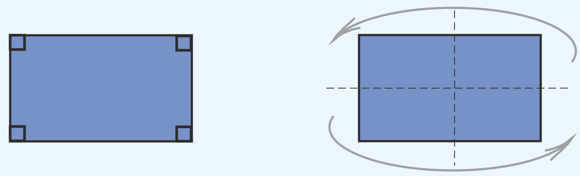
Grade 2

1) Square



4 equal angles of 90° (right angles).
4 lines of symmetry,
rotational symmetry order 4.

2) Rectangle



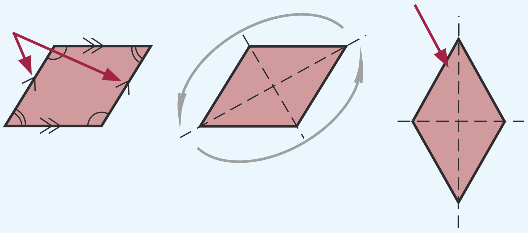
4 equal angles of 90° (right angles).
2 lines of symmetry, rotational symmetry order 2.

3) Rhombus

(A square pushed over)

Matching arrows show parallel sides.

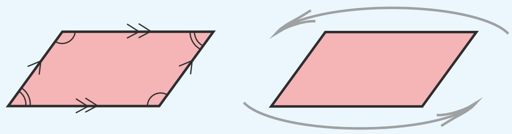
A rhombus is the same as a diamond.



4 equal sides (opposite sides are parallel).
2 pairs of equal angles.
2 lines of symmetry, rotational symmetry order 2.

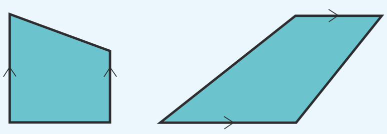
4) Parallelogram

(A rectangle pushed over)



2 pairs of equal sides (each pair are parallel).
2 pairs of equal angles.
NO lines of symmetry,
rotational symmetry order 2.

5) Trapezium

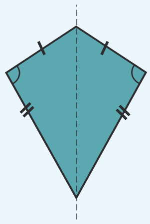


1 pair of parallel sides. NO lines of symmetry*.
No rotational symmetry.

*In an isosceles trapezium, the sloping sides are the same length. An isosceles trapezium has 1 line of symmetry.

6) Kite

2 pairs of equal sides.
1 pair of equal angles.
1 line of symmetry.
No rotational symmetry.



Quadrilaterals have four sides

There are six types of quadrilateral to learn on this page. Make sure you know their features — for example, how many lines of symmetry they have and whether they have rotational symmetry.

Congruent Shapes

Shapes can be congruent, which basically just means 'the same as each other'.
Luckily for you, here's a full page on congruence.

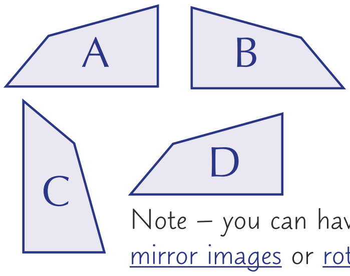
Congruent — Same Shape, Same Size

GRADE 2

Congruence is another ridiculous maths word which sounds really complicated when it's not:

If two shapes are CONGRUENT, they are EXACTLY THE SAME — the SAME SIZE and the SAME SHAPE.

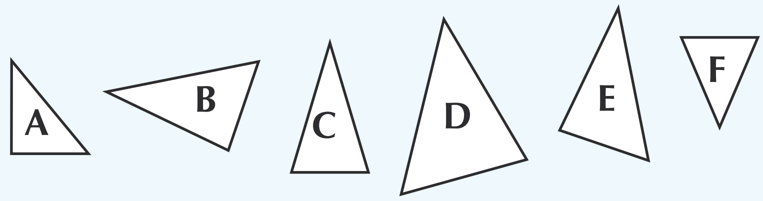
These shapes are all congruent:



Note – you can have mirror images or rotations.

EXAMPLE:

Two of the triangles below are congruent. Write down the letters of the congruent triangles.



Just pick out the two triangles that are exactly the same — remember that the shape might have been rotated or reflected.
By eye, you can see that the congruent triangles are **B and E**.

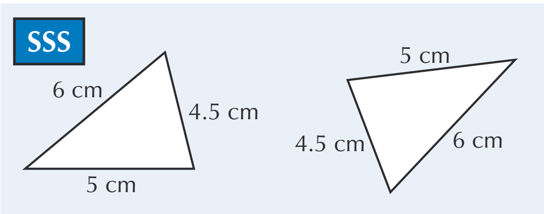
Conditions for Congruent Triangles

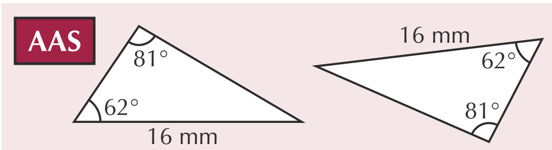
GRADE 5

Two triangles are congruent if one of the four conditions below holds true:

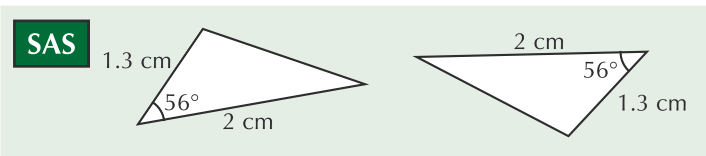
- SSS — three sides are the same
- AAS — two angles and a corresponding side match up
- SAS — two sides and the angle between them match up
- RHS — a right angle, the hypotenuse and one other side all match up

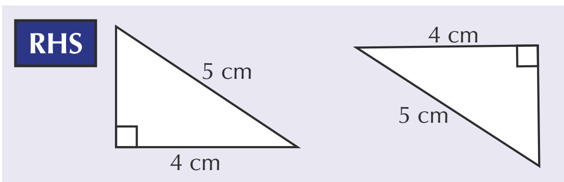
The hypotenuse is the longest side of a right-angled triangle — the one opposite the right angle.





Make sure the sides match up — here, the side is opposite the 81° angle.





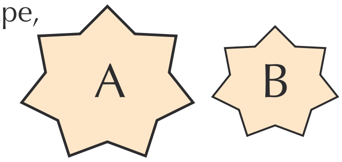
Congruent just means same size, same shape

You need to be able to recognise congruent shapes and learn all 4 conditions for congruent triangles.
Take your time and think carefully — make sure you're using the right sides and angles in each shape.

Similar Shapes

Similar shapes are exactly the same shape, but can be different sizes (they can also be rotated or reflected).

SIMILAR — same shape, different size.

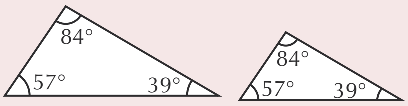


Similar Shapes Have the Same Angles

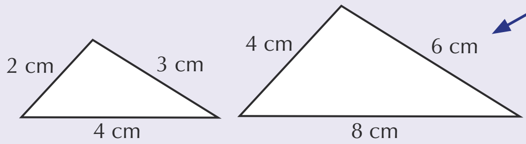
Generally, for two shapes to be similar, all the angles must match and the sides must be proportional. But for triangles, there are three special conditions — if any one of these is true, you know they're similar.

Two triangles are similar if:

1) All the angles match up.

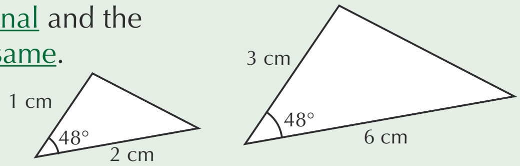


2) All three sides are proportional.



Here, the sides of the bigger triangle are twice as long as the sides of the smaller triangle.

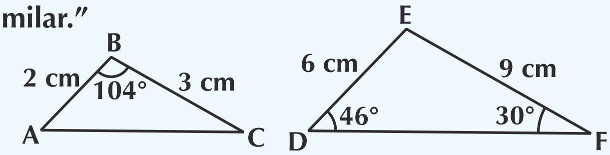
3) Any two sides are proportional and the angle between them is the same.



Watch out — if one of the triangles has been rotated or flipped over, it might look as if they're not similar, but don't be fooled.

EXAMPLE:

Tony says, "Triangles ABC and DEF are similar." Is Tony correct? Explain your answer.



Check condition 3 holds — start by finding the missing angle in triangle DEF:

Angle DEF = 180° – 46° – 30° = 104° so angle ABC = angle DEF

Now check that AB and BC are proportional to DE and EF:

DE ÷ AB = 6 ÷ 2 = 3 and EF ÷ BC = 9 ÷ 3 = 3 so DE and EF are 3 times as long as AB and BC.

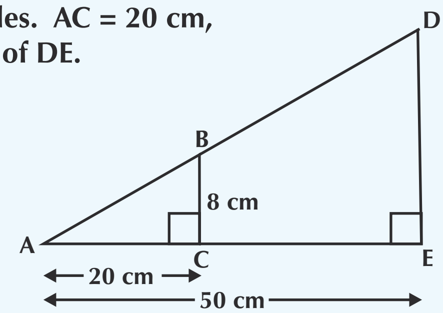
Tony is correct — two sides are proportional and the angle between them is the same so the triangles are similar.

Use Similarity to Find Missing Lengths

You might have to use the properties of similar shapes to find missing distances, lengths etc. — you'll need to use scale factors (see p.109) to find the lengths of missing sides.

EXAMPLE:

ABC and ADE are similar right-angled triangles. AC = 20 cm, AE = 50 cm and BC = 8 cm. Find the length of DE.



The triangles are similar, so work out the scale factor:

scale factor = $\frac{50}{20}$ = 2.5

Now use the scale factor to work out the length of DE:

DE = 8 × 2.5 = 20 cm

Similar means the same shape but a different size

Make sure you know the difference between congruent and similar shapes — to help you remember, think 'similar siblings, congruent clones' — siblings are alike but not the same, clones are identical.

The Four Transformations

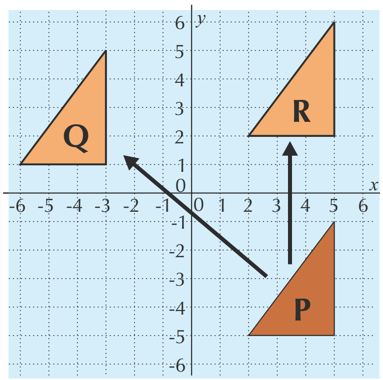
There are four transformations you need to know — translation, rotation, reflection and enlargement.

1) Translations

In a translation, the amount the shape moves by is given as a vector (see p.147-148) written $\begin{pmatrix} x \\ y \end{pmatrix}$ — where x is the horizontal movement (i.e. to the right) and y is the vertical movement (i.e. up). If the shape moves left and down, x and y will be negative.

EXAMPLE:

a) Describe the transformation that maps triangle P onto Q.
b) Describe the transformation that maps triangle P onto R.



a) To get from P to Q, you need to move 8 units left and 6 units up, so...
The transformation from P to Q is a translation by the vector $\begin{pmatrix} -8 \\ 6 \end{pmatrix}$.

b) The transformation from P to R is a translation by the vector $\begin{pmatrix} 0 \\ 7 \end{pmatrix}$.

2) Rotations

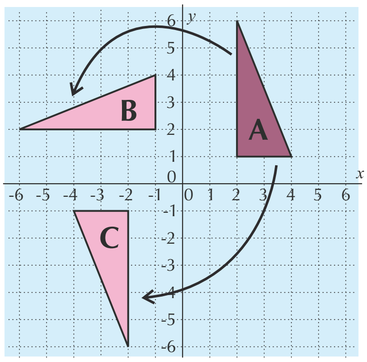
To describe a rotation, you must give 3 details:

1) The angle of rotation (usually 90° or 180°).
2) The direction of rotation (clockwise or anticlockwise).
3) The centre of rotation (often, but not always, the origin).

For a rotation of 180°, it doesn't matter whether you go clockwise or anticlockwise.

EXAMPLE:

a) Describe the transformation that maps triangle A onto B.
b) Describe the transformation that maps triangle A onto C.



a) The transformation from A to B is a rotation of 90° anticlockwise about the origin.

b) The transformation from A to C is a rotation of 180° clockwise (or anticlockwise) about the origin.

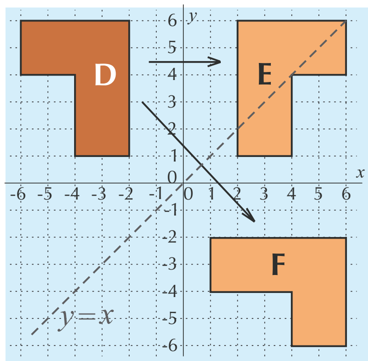
If it helps, you can use tracing paper to help you find the centre of rotation.

3) Reflections

For a reflection, you must give the equation of the mirror line.

EXAMPLE:

a) Describe the transformation that maps shape D onto shape E.
b) Describe the transformation that maps shape D onto shape F.



a) The transformation from D to E is a reflection in the y-axis.

b) The transformation from D to F is a reflection in the line $y = x$.

A rotation is specified by an angle, a direction and a centre

Shapes are congruent under translation, reflection and rotation — this is because their size and shape don't change, just their position and orientation.

The Four Transformations

One more transformation coming up — enlargements. They're the trickiest, but also the most interesting.

4) Enlargements

For an enlargement, you must specify:

1) The scale factor.
2) The centre of enlargement.

$$\text{scale factor} = \frac{\text{new length}}{\text{old length}}$$

1) The scale factor for an enlargement tells you how long the sides of the new shape are compared to the old shape. E.g. a scale factor of 3 means you multiply each side length by 3.
2) If you're given the centre of enlargement, then it's vitally important where your new shape is on the grid.

The scale factor tells you the RELATIVE DISTANCE of the old points and new points from the centre of enlargement.

So, a scale factor of 2 means the corners of the enlarged shape are twice as far from the centre of enlargement as the corners of the original shape.

Describing Enlargements

EXAMPLE: **Describe the transformation that maps Triangle A onto Triangle B.**

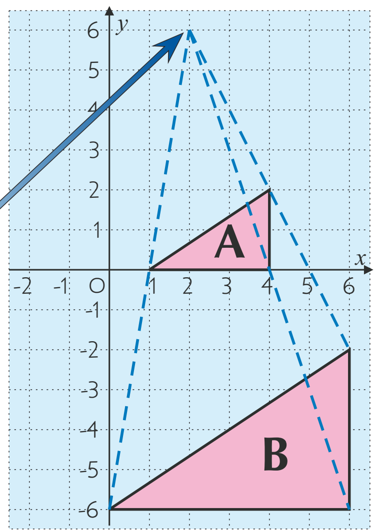
Use the formula to find the scale factor. (Just do this for one pair of sides.)

Old length of triangle base = 3 units
New length of triangle base = 6 units

$$\text{Scale factor} = \frac{\text{new length}}{\text{old length}} = \frac{6}{3} = 2$$

To find the centre of enlargement, draw lines that go through matching corners of both shapes and see where they cross.

So the transformation is an enlargement of scale factor 2, centre (2, 6).

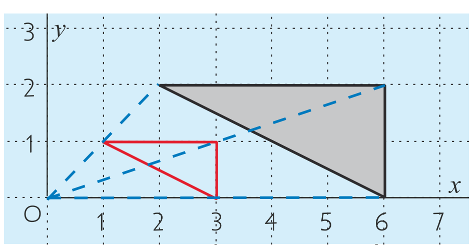


Fractional Scale Factors

1) If the scale factor is bigger than 1 the shape gets bigger.
2) If the scale factor is smaller than 1 (e.g. ½) it gets smaller.

EXAMPLE: **Enlarge the shaded shape by a scale factor of $\frac{1}{2}$, about centre 0.**

1) Draw lines going from the centre to each corner of the original shape. The corners of the new shape will be on these lines.
2) The scale factor is $\frac{1}{2}$, so make each corner of the new shape half as far from 0 as it is in the original shape.



An enlargement is given by a scale factor and a centre of enlargement

Shapes are similar under enlargement — the position and the size change, but the angles and ratios of the sides don't (see p.107). Remember that a scale factor smaller than 1 means the shape gets smaller.

Warm-up and Worked Exam Questions

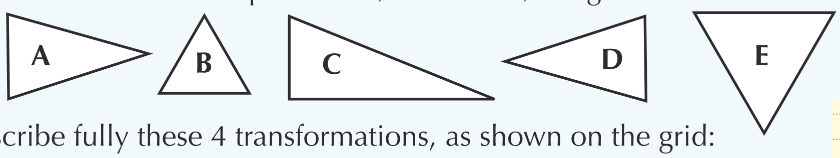
Nothing too tricky so far in this section. Now it's time for some warm-up questions to get your brain ticking — before moving on to the exam-style questions. It's all good practice for the big day.

Warm-up Questions

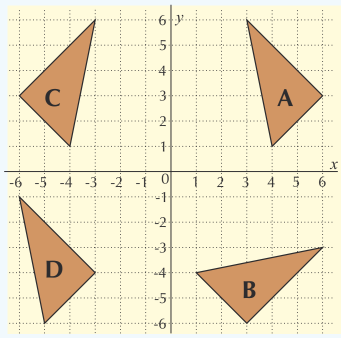
1) Copy the letters, mark in their lines of symmetry and give their order of rotational symmetry:

C W I D Q

2) Give all the properties of an equilateral triangle.
3) How many lines of symmetry does a kite have?
4) Which two of these shapes are: a) similar? b) congruent?



5) Describe fully these 4 transformations, as shown on the grid:
   A→B
   B→C
   C→A
   A→D

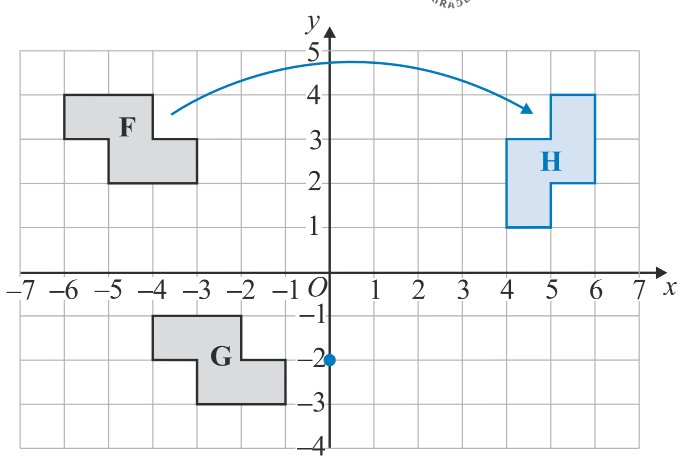


6) On a grid, draw triangle A with vertices (2, 1), (4, 1) and (4, 3), and triangle B with vertices (3, 1), (7, 1) and (7, 5).
   Describe the transformation that maps A to B.

Worked Exam Question

Worked exam questions are the ideal way to get the hang of answering the real exam questions — make sure you understand the answer to this one.

1 Shapes **F** and **G** have been drawn on the grid below. (Grade 3)



a) Write down the vector which describes the translation that maps **F** onto **G**.

To get from F to G, the shape is moved 2 units to the right and 5 units down.

$\begin{pmatrix} 2 \\ -5 \end{pmatrix}$

[2 marks]

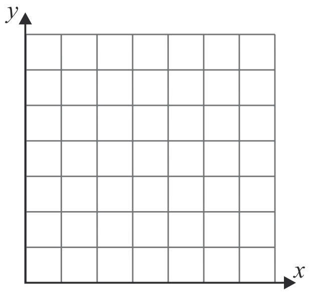
b) Rotate shape **F** by 90° clockwise about the point (0, –2).
Label your image **H**.

See the grid above — you might find it easiest to use tracing paper to draw rotations.

[2 marks]

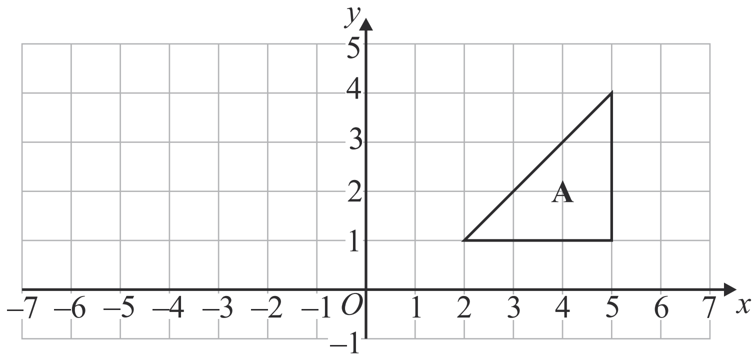
Exam Questions

2 An isosceles triangle has vertices A(1, 1), B(3, 7) and C(5, 1).
Give the equation of its line of symmetry.



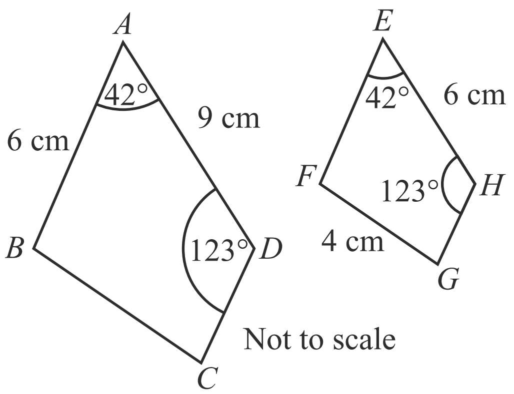
....
[1 mark]

3 Triangle **A** has been drawn on the grid below.
Reflect triangle **A** in the line $x = -1$. Label your image **B**.



[2 marks]

4 The shapes *ABCD* and *EFGH* are mathematically similar.



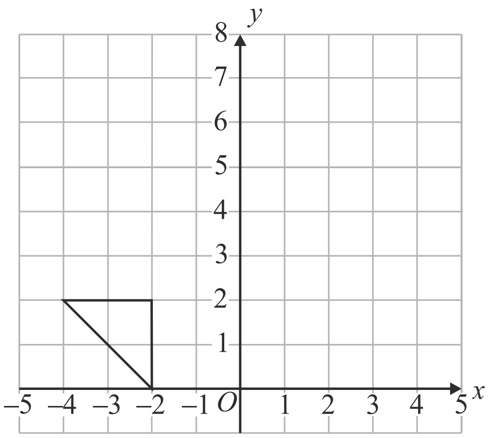
a) Find the length of *EF*.

.... cm
[2 marks]

b) Find the length of *BC*.

.... cm
[1 mark]

5 On the grid enlarge the triangle by a scale factor of 3, centre (–4, 0).



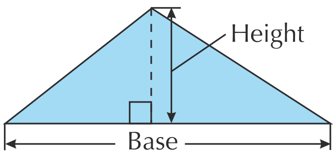
[3 marks]

Perimeter and Area

Perimeter is the distance around the outside of a shape. Area is a bit trickier — you need to learn some formulas. You should already know that the area of a rectangle is A = l × w and the area of a square is A = l².

Area Formulas for Triangles and Quadrilaterals

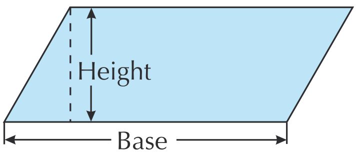
Learn these formulas:



Area of triangle = ½ × base × vertical height

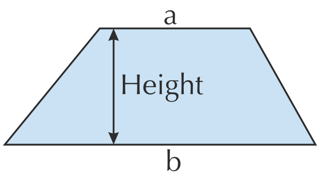
A = ½ × b × h

Note that in each case the height must be the vertical height, not the sloping height.



Area of parallelogram = base × vertical height

A = b × h



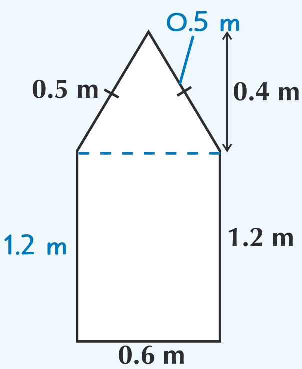
Area of trapezium = average of parallel sides × distance between them (vertical height)

A = ½(a + b) × h

Perimeter and Area Problems

You might have to use the perimeter or area of a shape to answer a slightly more complicated question (e.g. find the area of a wall, then work out how many rolls of wallpaper you need to wallpaper it).

EXAMPLE: **Greg is making a stained-glass window in the shape shown below.**



a) **Find the perimeter of the window.**

Label all the side lengths, then add them up:

0.5 m + 1.2 m + 0.6 m + 1.2 m + 0.5 m = 4 m

When you're adding side lengths it's a good idea to mark them off as you go along to make sure you don't repeat or miss any.

b) **Coloured glass costs £82 per m². Work out the cost of the glass needed for the window.**

Split the shape into a triangle and a rectangle (as shown) to find the area:

Area of rectangle = length × width = 0.6 × 1.2 = 0.72 m²

Area of triangle = $\frac{1}{2}$ × base × height = $\frac{1}{2}$ × 0.6 × 0.4 = 0.12 m²

Total area of shape = 0.72 + 0.12 = 0.84 m²

Then multiply the area by the price to work out the cost:

Cost = area × price per m² = 0.84 × 82 = £68.88

Learn the area formulas

If you have a composite shape (a shape made up of different shapes stuck together), split it into triangles and quadrilaterals, work out the area of each bit and add them together.

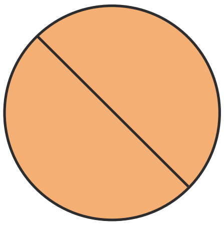
Perimeter and Area — Circles

Another page of formulas — this time on circles. You know the drill...

LEARN these Formulas

Area and Circumference of Circles

GRADE 3



Area of circle = π × (radius)^2

Remember that the radius is half the diameter.

$$A = \pi r^2$$

For these formulas, use the π button on your calculator. For non-calculator questions, use $\pi \approx 3.142$.

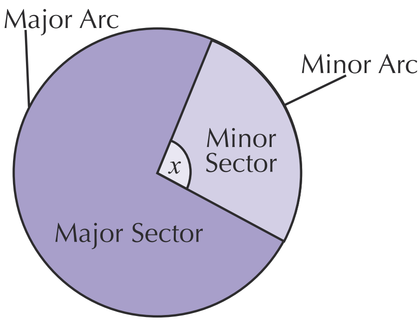
Circumference = π × diameter
= 2 × π × radius

$$C = \pi D = 2\pi r$$

Arc Lengths and Areas of Sectors

GRADE 5

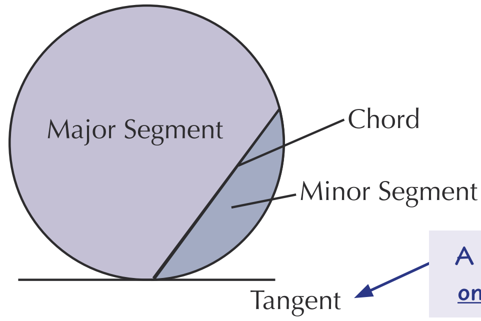
These next ones are a bit more tricky — before you try and learn the formulas, make sure you know what a sector and an arc are (I've helpfully labelled the diagram below — I'm nice like that).



$$\text{Area of Sector} = \frac{x}{360} \times \text{Area of full Circle}$$

$$\text{Length of Arc} = \frac{x}{360} \times \text{Circumference of full Circle}$$

You also need to know what a segment, a chord and a tangent are.



A tangent just touches one point of the circle.

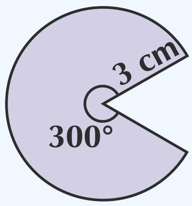
EXAMPLE:

In the diagram on the right, a sector with angle 60° has been cut out of a circle with radius 3 cm. Find the exact area of the shaded shape.

Use the formula to find the area of the shaded sector:

$$\text{area of sector} = \frac{x}{360} \times \pi r^2 = \frac{300}{360} \times \pi \times 3^2$$
$$= \frac{5}{6} \times \pi \times 9 = \frac{15}{2}\pi \text{ cm}^2$$

'Exact area' means leave your answer in terms of π.



Make sure you know all of these circle terms

One more thing — if you're asked to find the perimeter of a semicircle or quarter circle, don't forget to add on the straight edges too. It's an easy mistake to make, and it'll cost you marks.

Warm-up and Worked Exam Questions

There are lots of formulas in this section. The best way to find out what you know is to practise these questions. If you find you keep forgetting the formulas, you need more practice.

Warm-up Questions

1) Find the perimeter of the shape shown on the right.

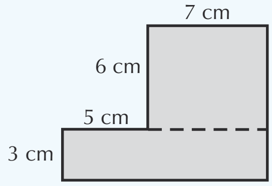
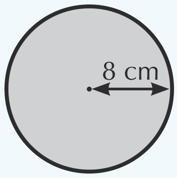


DIAGRAM NOT TO SCALE

2) Give the formulas for:
   a) the area of a rectangle
   b) the circumference of a circle
   c) the area of a parallelogram
3) A triangle has a base of 3 m and a vertical height of 7 m. Calculate its area.
4) A woodworking template has the shape shown:
   Calculate the area of the template.



5) a) What is a tangent?
   b) Draw a circle and label a tangent.

Worked Exam Questions

Here are two juicy worked exam questions for you. Work through each one step by step.

1 The diagram below shows a rectangle and a square. (Grade 4)

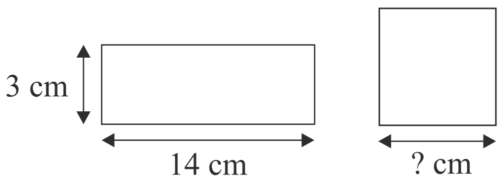


Diagram not accurately drawn

The ratio of the area of the rectangle to the area of the square is 6 : 7.
What is the area of the square?

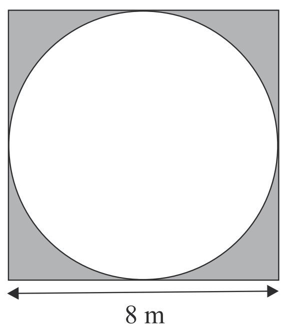
Area of rectangle = $3 \times 14 = 42$ cm^2

area of rectangle : area of square
6 : 7
(×7) 42 : 49 (×7)
So area of the square = 49 cm^2

See p.79 for more on scaling up ratios.

49 cm^2
[2 marks]

2 The diagram below shows a square with a circle inside. The circle touches each of the four sides of the square. Calculate the shaded area. Give your answer to 2 d.p. (Grade 4)

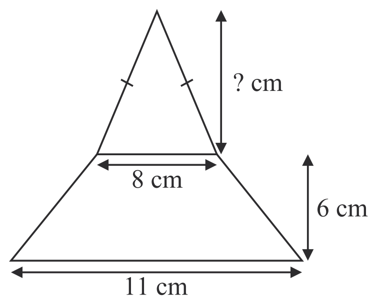


Area of square = $8 \times 8 = 64$ m^2
Area of circle = $\pi \times 4^2 = 50.2654...$ m^2
Shaded area = $64 - 50.2654...$
$= 13.7345...$ m^2

13.73 m^2
[3 marks]

Exam Questions

3 A shape is made up from an isosceles triangle and a trapezium.



The area of the trapezium is 3 times as big as the area of the triangle.

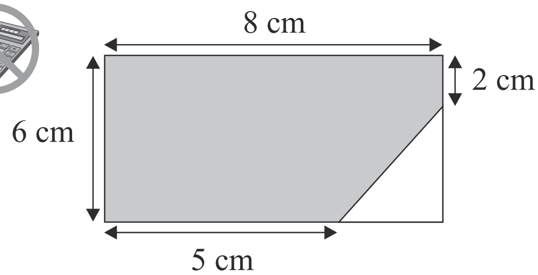
a) Find the total area of the shape.

.... cm^2
[3 marks]

b) Find the height of the triangle.

.... cm
[2 marks]

4 The diagram below shows a rectangle with a right-angled triangle inside.

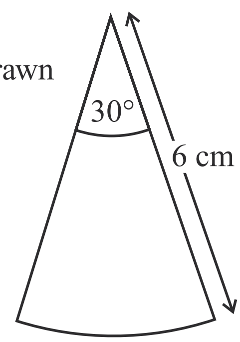


Calculate the area of the shaded part.

.... cm^2
[4 marks]

5 Look at the sector shown in the diagram below.

Diagram not accurately drawn



Find the perimeter and the area of the sector.
Give your answers to 3 significant figures.

Don't forget to add the two radii to the arc length when finding the perimeter.

Perimeter = ... cm

Area = ... cm^2
[5 marks]

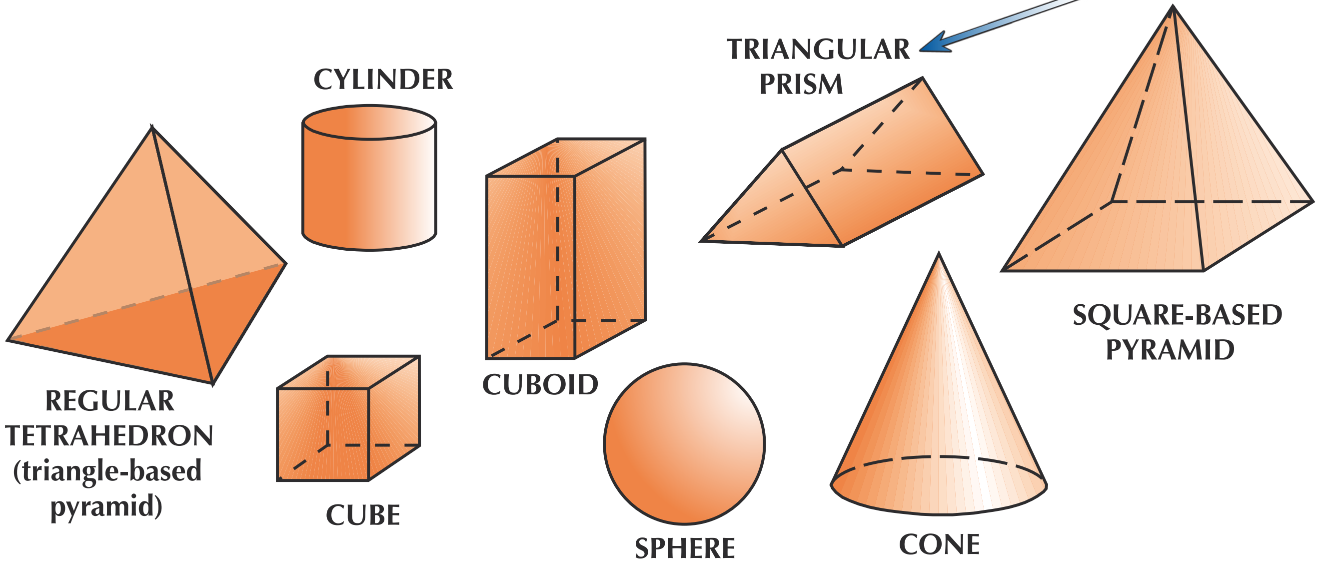
3D Shapes

First up are some 3D shapes for you to learn, closely followed by a look at the different parts of solids.

Eight Solids to Learn

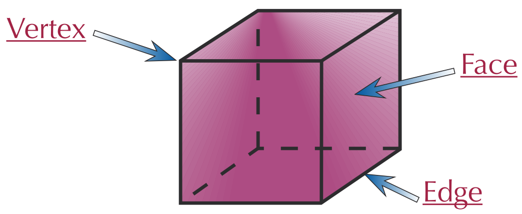
3D shapes are solid shapes. These are the ones you need to know:

There's more about prisms on p.118.



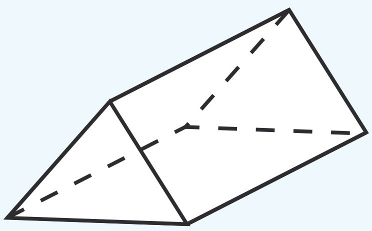
Different Parts of Solids

There are different parts of 3D shapes you need to be able to spot. These are vertices (corners), faces (the flat bits) and edges.
You might be asked for the number of vertices, faces and edges in the exam — just count them up, and don't forget the hidden ones.



EXAMPLE:

For the triangular prism on the right, write down the number of faces, the number of edges and the number of vertices.



A triangular prism has **5 faces** (there are three rectangular faces and two triangular ones).
It has **9 edges** (there are 3 hidden ones — the dotted lines in the diagram).
It has **6 vertices** (there's one hidden at the back).

Faces (especially curved faces) are sometimes called surfaces.

Make sure you can spot the vertices, faces and edges of 3D shapes

Remember — 1 vertex, 2 vertices. They're funny words, designed to confuse you, so don't let them catch you out. You need to know the names of all the solid shapes above too.

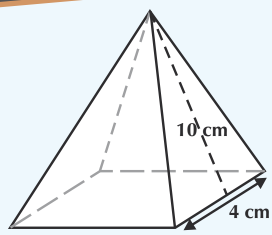
3D Shapes — Surface Area

Surface area is like normal area but for 3D shapes — for some shapes you can use 2D areas and add them up, for others there are special formulas you'll need to use.

Surface Area using Nets

1) SURFACE AREA only applies to 3D objects — it's just the total area of all the faces added together.
2) SURFACE AREA OF A SOLID = AREA OF THE NET (remember that a net is just a 3D shape folded out flat). So if it helps, imagine (or sketch) the net and add up the area of each bit.

EXAMPLE: **Find the surface area of the square-based pyramid below.**

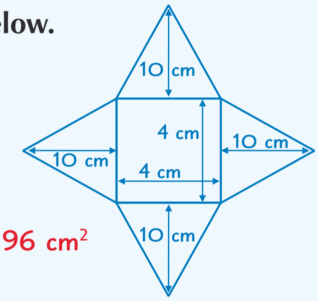


Sketch the net — a square-based pyramid has 1 square face and 4 triangular faces.

Area of square face = $4 \times 4 = 16$ cm^2

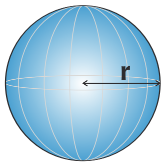
Area of triangular face = $\frac{1}{2} \times 4 \times 10 = 20$ cm^2

Total surface area = $16 + (4 \times 20) = 16 + 80 = 96$ cm^2



Surface Area Formulas

1) SPHERES, CONES AND CYLINDERS have surface area formulas that you need to be able to use.
2) Luckily you don't need to memorise the sphere and cone formulas — you'll be given them in your exam.
3) But you must get lots of practice using them, or you might slip up when it comes to the exam.

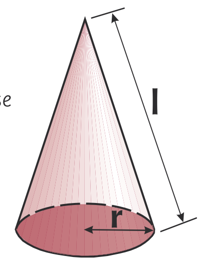


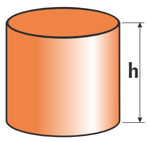
Surface area of a SPHERE = $4\pi r^2$

curved area of cone (l is the slant height)

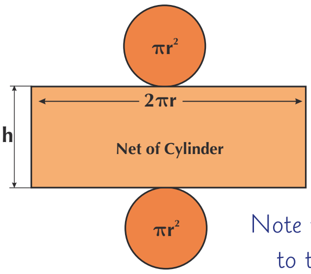
area of circular base

Surface area of a CONE = $\pi r l + \pi r^2$





Cylinder



Surface area of a CYLINDER = $2\pi rh + 2\pi r^2$

Note that the length of the rectangle is equal to the circumference of the circular ends.

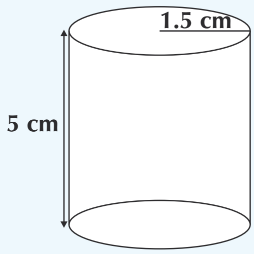
EXAMPLE: **Find the surface area of the cylinder on the right to 1 d.p.**

Just put the measurements into the formula and work it out very carefully in stages:

Surface area of cylinder = $2\pi rh + 2\pi r^2$

$= (2 \times \pi \times 1.5 \times 5) + (2 \times \pi \times 1.5^2)$

$= 47.123... + 14.137... = 61.261... = 61.3$ cm^2



To find the surface area of a solid just add up the area of each face

Nets can help you to find surface area — they make it easier to see each face and work out their areas. Make sure you're familiar with the formulas on this page — get some practice at using them.

3D Shapes — Volume

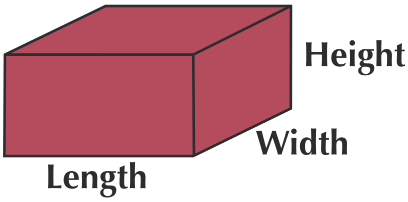
You've already had a couple of pages on 3D shapes — now it's time to work out their volumes.

LEARN these volume formulas...

Volumes of Cuboids

GRADE 2

A cuboid is a rectangular block. Finding its volume is dead easy:



Volume of Cuboid = length × width × height

V = L × W × H

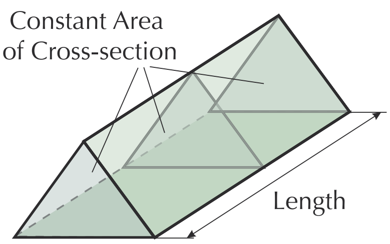
This is the formula for the volume of a cube too — where the length, width and height are all the same.

Volumes of Prisms

GRADE 3

A PRISM is a solid (3D) object which is the same shape all the way through — i.e. it has a CONSTANT AREA OF CROSS-SECTION.

Triangular Prism



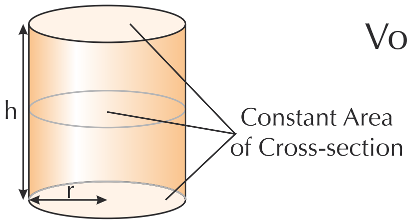
Volume of Prism = cross-sectional area × length

V = A × L

This formula works for any prism.

Cylinder

(circular prism)

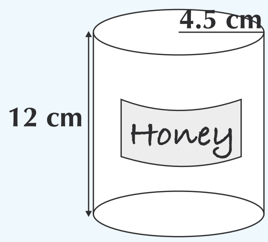


Volume of Cylinder = area of circle × height

$V = \pi r^2 h$

EXAMPLE:

Honey comes in cylindrical jars with radius 4.5 cm and height 12 cm. Dan has a recipe that needs 1 litre of honey. How many jars should he buy?



First, work out the volume of the jar — just use the formula above:

$V = \pi r^2 h = \pi \times 4.5^2 \times 12$

$= 763.4070...\ cm^3$

1 litre = 1000 cm³ (see p.95), so he needs to buy 2 jars of honey.

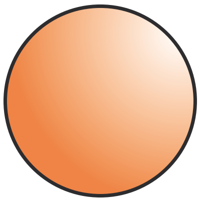
You have to remember what a prism is

You might get a question where you're given a shape made up of 1 cm cubes and asked for its volume. All you have to do here is count up the cubes (not forgetting any hidden ones at the back of the shape).

3D Shapes — Volume

Another page on volumes now — my generosity knows no limits.

Volumes of Spheres

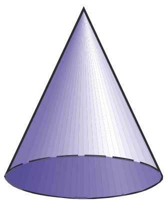


Volume of Sphere = $\frac{4}{3}\pi r^3$

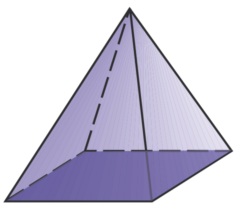
A hemisphere is half a sphere. So the volume of a hemisphere is just half the volume of a full sphere, V = $\frac{2}{3}\pi r^3$.

Volumes of Pyramids and Cones

A pyramid is a shape that goes from a flat base up to a point at the top. Its base can be any shape at all.



Cone



Square-based Pyramid

Volume of Pyramid = $\frac{1}{3}$ × Base Area × Vertical Height

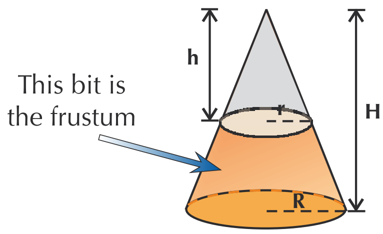
Volume of Cone = $\frac{1}{3}$ × πr^2 × h

Make sure you use the vertical height in these formulas — don't get confused with the slant height, which you used to find the surface area of a cone.

Volumes of Frustums

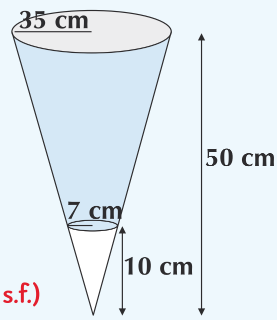
A frustum of a cone is what's left when the top part of a cone is cut off parallel to its circular base. You'll be given the formula for the volume of a cone in your exam, but you'll need to remember:

VOLUME OF FRUSTUM = VOLUME OF ORIGINAL CONE – VOLUME OF REMOVED CONE



EXAMPLE:

A waste paper basket is the shape of a frustum formed by removing the top 10 cm from a cone of height 50 cm and radius 35 cm. Find the volume of the waste paper basket to 3 significant figures.



Use the formula for the volume of a cone above:

Volume of original cone = $\frac{1}{3} \times \pi \times 35^2 \times 50 = 64140.850...\ \text{cm}^3$

Volume of removed cone = $\frac{1}{3} \times \pi \times 7^2 \times 10 = 513.126...\ \text{cm}^3$

Volume of frustum = 64140.850... – 513.126... = 63627.723... = 63600 cm^3 (3 s.f.)

Remember that a frustum is just a cone with the top chopped off

A common mistake is that a frustum is actually called a frustRum. Anyway — learn the formula in the blue box above. It's not too tricky, because it makes sense when you think about it — subtract the volume of the removed cone from the original volume, and you're left with the volume of the bit at the bottom.

3D Shapes — Volume

You might get a question asking you something a little more challenging than just finding the volume — this page will talk you through a couple of trickier types of volume question.

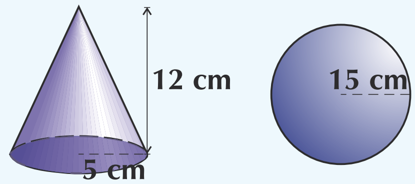
Ratios of Volumes

1) You might need to look at how the volumes of different shapes are linked.
2) So you could be given two shapes and have to show how many times bigger one volume is than the other, or you might need to show the ratio of their volumes:

> 1) Work out each volume separately — make sure the units are the same.
> 2) Write the volumes as a ratio and simplify.

EXAMPLE:

The cone in the diagram has a radius of 5 cm and a height of 12 cm. The sphere has a radius of 15 cm. Find the ratio of their volumes in its simplest form.



Put the numbers into the volume formulas (see p.119):

Volume of cone $= \frac{1}{3} \times \pi r^2 \times h = \frac{1}{3} \times \pi \times 5^2 \times 12 = \frac{1}{3} \times \pi \times 300 = 100\pi \text{ cm}^3$

Volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times 15^3 = \frac{4 \times 3375}{3} \times \pi = 4500\pi \text{ cm}^3$

When comparing volumes, it's usually best to leave your answers in terms of π.

Find the ratio of the volumes and simplify:

volume of cone : volume of sphere

$100\pi : 4500\pi$

$\div 100\pi$ $\quad$ $\div 100\pi$

$1 : 45$

Rates of Flow

You need to be really careful with units in rates of flow questions. You might be given the dimensions of a shape in cm or m but the rate of flow in litres (e.g. litres per minute).

EXAMPLE:

A cube-shaped fish tank with sides of length 30 cm is being filled with water at a rate of 4 litres per minute. How long will it take to fill the fish tank? Give your answer in minutes and seconds. 1 litre = 1000 cm³

Find the volume of the fish tank: $V = (\text{side length})^3 = 30^3 = 27\ 000 \text{ cm}^3$

Then convert the rate of flow into cm³/minute — multiply by 1000:

4 litres per minute $= 4 \times 1000 = 4000 \text{ cm}^3/\text{min}$

So it will take $27\ 000 \div 4000 = 6.75$ minutes

= 6 minutes and 45 seconds to fill the fish tank.

Leave volumes in terms of π when comparing them

Sorry, there's no way round it — this page is a rotten one. When dealing with rates of flow, you might need to convert the rate of flow so its units match the volume (e.g. m³/s), then use this to find your answer.

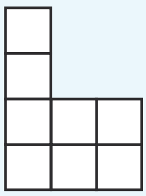
Projections

Projections are just different views of a 3D solid shape — looking at it from the front, the side and the top.

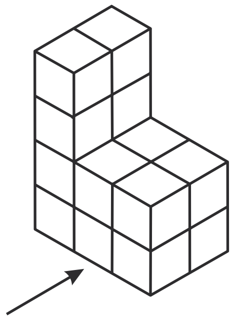
The Three Different Projections

There are three different types of projections — front elevations, side elevations and plans (elevation is just another word for projection).

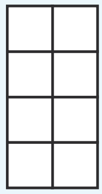
1



FRONT ELEVATION — the view you'd see from directly in front (in the direction of the arrow)

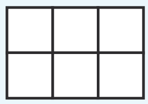


2

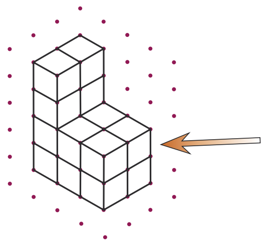


SIDE ELEVATION — the view you'd see from directly to one side

3



PLAN — the view you'd see from directly above

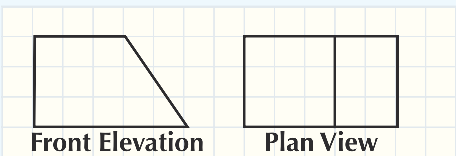


Don't be thrown if you're given a diagram on isometric (dotty) paper like this — it works in just the same way.
If you have to draw shapes on isometric paper, just join the dots.
You should only draw vertical and diagonal lines (no horizontal lines).

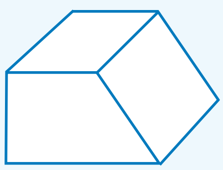
Drawing Projections

EXAMPLES:

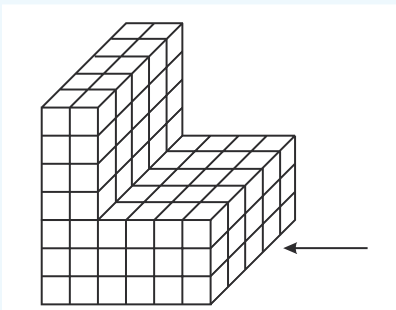
1. **The front elevation and plan view of a shape are shown below. Sketch the solid shape.**



Just piece together the original shape from the information given — here you get a prism in the shape of the front elevation.



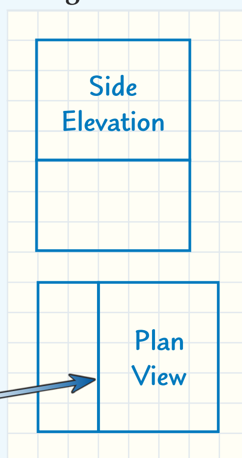
2. a) **On the cm square grid, draw the side elevation of the prism from the direction of the arrow.**
   b) **Draw a plan of the prism on the grid.**



Draw lines like this to show where there's a change in depth.

a)

b)



Check that all your lengths are the same as the shape (count the squares).

You need to know the three different types of projection

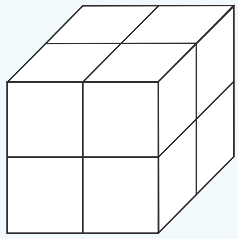
Projection questions aren't too bad — just take your time and sketch the diagrams carefully. Watch out for questions on isometric paper — they may look confusing, but can actually be easier than other questions.

Warm-up and Worked Exam Questions

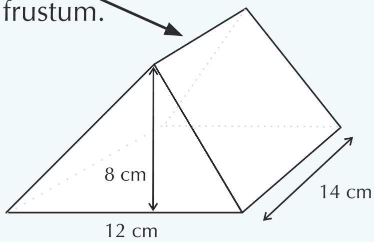
Turns out there's lots to know about 3D shapes. If you have any problems doing these warm-up questions, look back over anything you're unsure of before tackling the exam questions.

Warm-up Questions

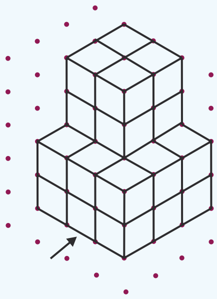
1) Write down the number of faces, vertices and edges for a cuboid.
2) Find the radius of a sphere with surface area of 196π cm^2.
3) The shape on the right is made from 1 cm cubes. Find its volume.



4) Calculate the volume of the triangular prism below on the right.
5) A cone has radius 12 m and vertical height 18 m.
   A cone 3 m high with a radius of 2 m is cut off the cone to make a frustum.
   Find the volume of the frustum, leaving your answer in terms of π.



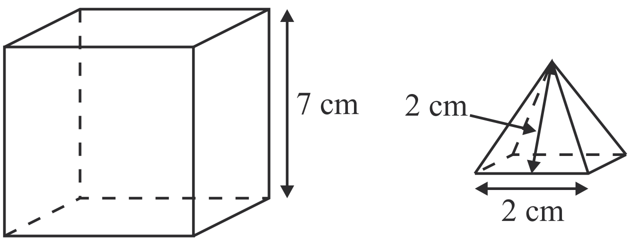
6) A sphere has a radius of 3 cm. A cylinder has a radius of 6 cm and a height of 10 cm. Show that the volume of the cylinder is 10 times as big as the volume of the sphere.
7) For the shape on the right, draw:
   a) the front elevation
   b) the side elevation
   c) the plan view.



Worked Exam Question

Work through this question carefully before having a go at the exam questions.

1 The dimensions of a cube and a square-based pyramid are shown in the diagram below. The side length of the cube is 7 cm. The side length of the pyramid's base is 2 cm and the slant height of the pyramid is 2 cm.



Not drawn accurately

Find the ratio of the surface area of the cube to the surface area of the pyramid in the form $n : 1$.

Surface area of cube = 6 × area of one face = 6 × 7 × 7 = 294 cm^2

Surface area of square-based pyramid = area of base + (4 × area of one triangular face)
= 2 × 2 + (4 × ½ × 2 × 2) = 4 + 8 = 12 cm^2

Surface area of cube : surface area of pyramid
= 294 : 12
= 294 ÷ 12 : 12 ÷ 12 = 24.5 : 1

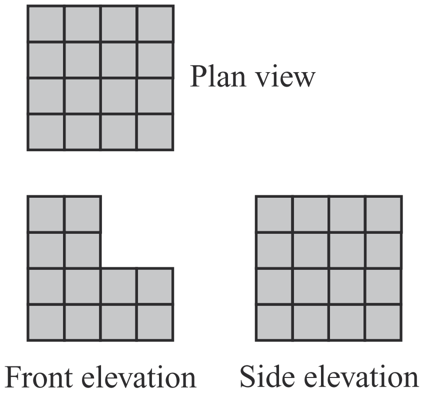
Once you've found the surface area of each, set them as a ratio then divide to get the ratio in the form n:1.

24.5 : 1

[4 marks]

Exam Questions

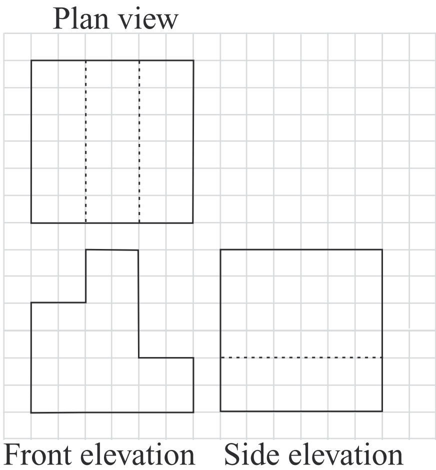
2 The diagram below shows the plan view, and the front and side elevations of a shape made from identical cubes.



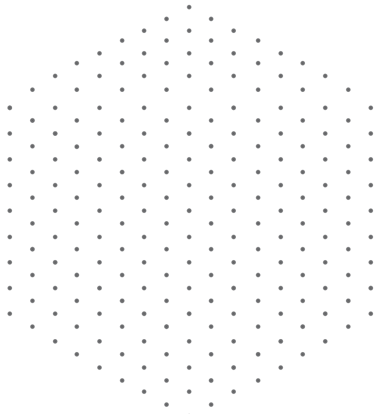
How many cubes make up the shape?

....
[2 marks]

3 The diagram below shows the plan, the front elevation and the side elevation of a prism.

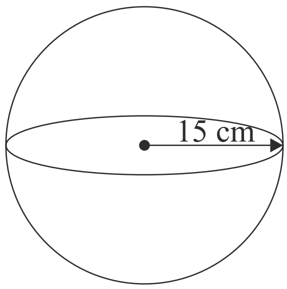


Draw a sketch of the solid prism on the grid below.



[2 marks]

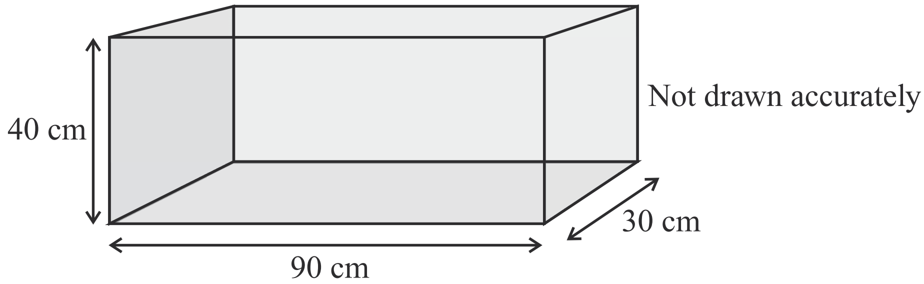
4 Volume of sphere = $\frac{4}{3}\pi r^3$.
Find the volume of the sphere on the right.
Give your answer to 3 significant figures.



.... cm^3
[2 marks]

Exam Questions

5 The tank shown in the diagram below is completely filled with water.



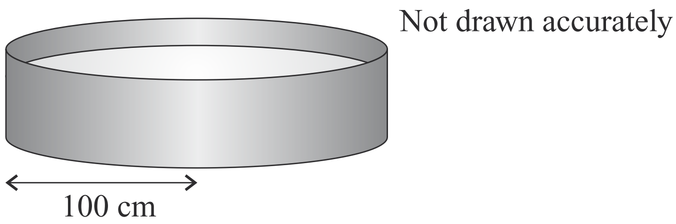
a) Calculate the volume of water in the tank.

.... cm^3
[2 marks]

b) The water from this tank is then poured into a second tank with length 120 cm. The depth of the water is 18 cm. What is the width of the second tank?

.... cm
[2 marks]

6 The diagram below shows a paddling pool with a radius of 100 cm.



a) What is the volume of water in the paddling pool when it is filled to a depth of 40 cm? Give your answer in terms of π.

.... cm^3
[2 marks]

b) The paddling pool is filled at a rate of 300 cm^3 per second. How long does it take to fill the pool to a depth of 40 cm? Give your answer to the nearest minute.

.... minutes
[2 marks]

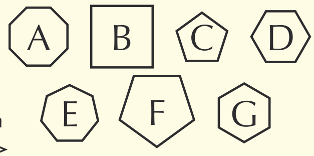
Revision Questions for Section Five

That's right, you made it to the last page of Section Five. Try out your new shape skills on these questions.

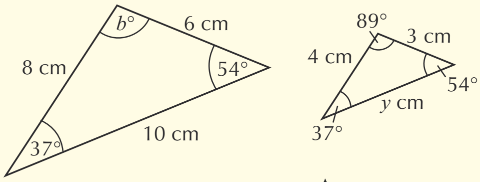
- Try these questions and tick off each one when you get it right.
- When you've done all the questions for a topic and are completely happy with it, tick off the topic.

2D Shapes (p103-107)

1) For each of the letters shown, write down how many lines of symmetry they have and their order of rotational symmetry. **H Z T N E X S**
2) Write down four properties of an isosceles triangle.
3) How many lines of symmetry does a rhombus have? What is its order of rotational symmetry?
4) What are congruent and similar shapes?

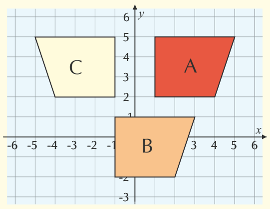


5) Look at the shapes A-G on the right and write down the letters of:
   a) a pair of congruent shapes,
   b) a pair of similar shapes.
6) These two triangles are similar.
   Write down the values of b and y.



Transformations (p108-109)

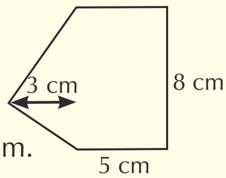
7) Describe the transformation that maps:
   a) Shape A onto shape B
   b) Shape A onto shape C



8) Carry out the following transformations on the triangle X, which has vertices (1, 1), (4, 1) and (2, 3):
   a) a rotation of 90° clockwise about (1, 1)   b) a translation by the vector $\begin{pmatrix} -3 \\ -4 \end{pmatrix}$
   c) an enlargement of scale factor 2, centre (1, 1)

Perimeter and Area (p112-113)

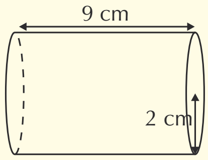
9) Write down the formula for finding the area of a trapezium.
10) Find the area of a parallelogram with base 9 cm and vertical height 4 cm.
11) Find the area of the shape on the right.



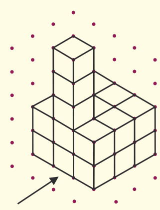
12) Find, to 2 decimal places, the area and circumference of a circle with radius 7 cm.
13) Draw a circle and label an arc, a sector, a chord and a segment.
14) Find, to 2 decimal places, the area and perimeter of a quarter circle with radius 3 cm.

3D Shapes — Surface Area, Volume and Projections (p116-121)

15) Write down the number of faces, edges and vertices for the following 3D shapes:
   a) a square-based pyramid   b) a cone   c) a triangular prism.
16) Find the surface area of a cube with side length 5 cm.
17) Find, to 1 decimal place, the surface area of a cylinder with height 8 cm and radius 2 cm.
18) Write down the formula for the volume of a cylinder with radius r and height h.
19) A pentagonal prism has a cross-sectional area of 24 cm² and a length of 15 cm. Find its volume.
20) a) Find the volume of the cylinder on the right (to 2 d.p.).
   b) How long will it take to fill the cylinder with water if the water is flowing at 1.5 litres per minute? Give your answer in seconds to 1 d.p.



21) On squared paper, draw the front elevation (from the direction of the arrow), side elevation and plan view of the shape on the right.



Angle Basics

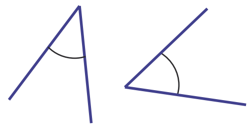
Before we really get going with the thrills and chills of angles and geometry, there are a few things you need to know. Nothing too scary — just some special angles and some fancy notation.

Fancy Angle Names

Some angles have special names. You might have to identify these angles in the exam.

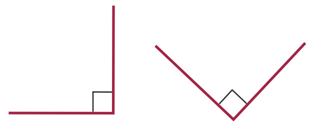
ACUTE angles

Sharp pointy ones (less than 90°)



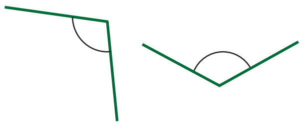
RIGHT angles

Square corners (exactly 90°)



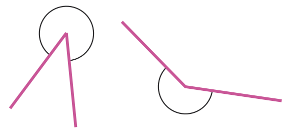
OBTUSE angles

Flatter ones (between 90° and 180°)



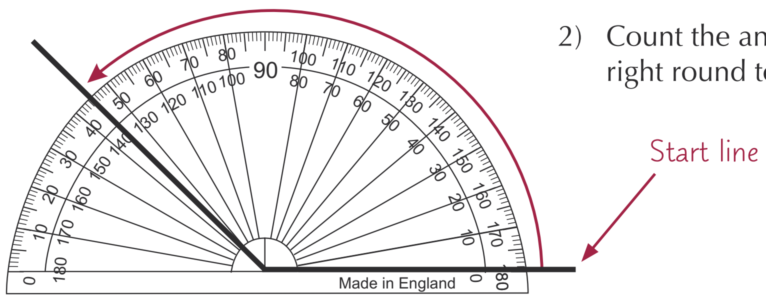
REFLEX angles

Ones that bend back on themselves (more than 180°)



Measuring Angles with a Protractor

1) ALWAYS position the protractor with the base line of it along one of the lines as shown here:



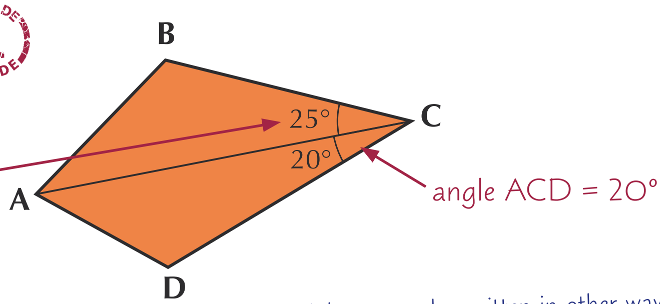
2) Count the angle in 10° STEPS from the start line right round to the other line over there.

Check your measurement by looking at it. If it's between a right angle and a straight line, it's between 90° and 180°.

DON'T JUST READ A NUMBER OFF THE SCALE — chances are it'll be the wrong one because there are TWO scales to choose from.
The answer here is 135° (NOT 45°) which you will only get right if you start counting 10°, 20°, 30°, 40° etc. from the start line until you reach the other line.

Three-Letter Angle Notation

The best way to say which angle you're talking about in a diagram is by using THREE letters.
For example in the diagram, angle ACB = 25°.



1) The middle letter is where the angle is.
2) The other two letters tell you which two lines enclose the angle.

You might see angles written in other ways as well — ∠ABC and $A\hat{B}C$ are both the same as angle ABC.

Four fancy angle names to learn — acute, right, obtuse and reflex

In the exams, it's pretty likely that angles will be referred to using three-letter notation — so make sure you know how to use it. And make sure you've learnt the four special angles too.

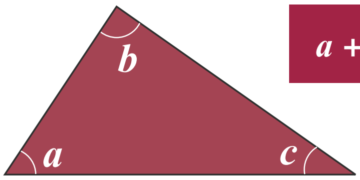
Five Angle Rules

If you know all these rules thoroughly, you'll at least have a fighting chance of working out problems with lines and angles. If you don't — you've no chance. Sorry to break it to you like that.

5 Simple Rules — that's all

GRADE 3 GRADE

1) Angles in a triangle add up to 180°.

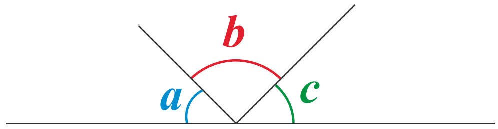


$a + b + c = 180°$

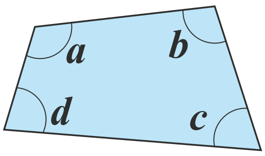
There's a nice proof of this (using parallel lines) on the next page.

2) Angles on a straight line add up to 180°.

$a + b + c = 180°$



3) Angles in a quadrilateral add up to 360°.

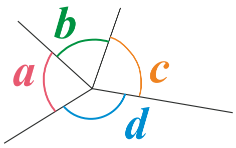


Remember that a quadrilateral is a 4-sided shape.

$a + b + c + d = 360°$

You can see why this is if you split the quadrilateral into two triangles along a diagonal. Each triangle has angles adding up to 180°, so the two together have angles adding up to 180° + 180° = 360°.

4) Angles round a point add up to 360°.

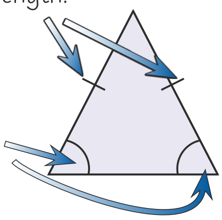


$a + b + c + d = 360°$

5) Isosceles triangles have 2 sides the same and 2 angles the same.

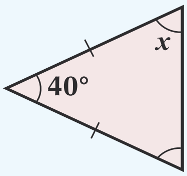
In an isosceles triangle, you only need to know one angle to be able to find the other two.

These dashes indicate two sides the same length.



These angles are the same.

EXAMPLE: **Find the size of angle x.**



180° – 40° = 140°

The two angles on the right are the same (they're both x) and they must add up to 140°, so $2x = 140°$, which means $x = 70°$.

Five simple rules, make sure you LEARN THEM...

Scribble them down again and again until they're ingrained in your brain (or desk).
The basic facts are pretty easy really, but examiners like to combine them in questions to confuse you — if you've learnt them all you've got a much better chance of spotting which ones you need.

Parallel Lines

Parallel lines are always the same distance apart. This page is all about them.

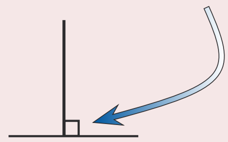
Angles Around **Parallel Lines**

Grade 3

When a line crosses two parallel lines...

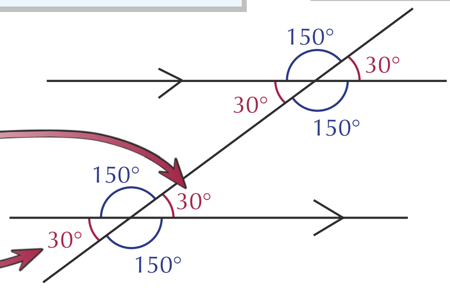
1) The two bunches of angles are the same.
2) There are only two different angles: a small one and a big one.
3) These ALWAYS ADD UP TO 180°. E.g. 30° and 150° below.

You also need to know what perpendicular lines are — they meet at 90°.



The two lines with the arrows on are parallel:

These are vertically opposite angles. They're equal to each other.



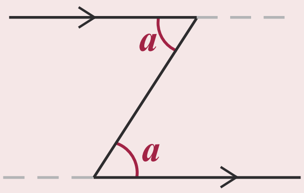
Alternate, Allied and **Corresponding** Angles

Grade 3

Watch out for these 'Z', 'C', 'U' and 'F' shapes popping up.
They're a dead giveaway that you've got a pair of parallel lines.

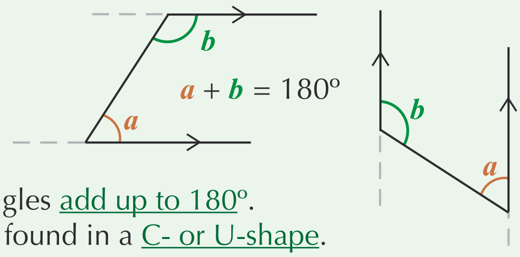
Don't call them Z, C, U and F angles in the exam — you'll need to use their proper names.

ALTERNATE ANGLES



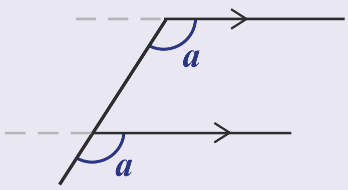
Alternate angles are the same.
They are found in a Z-shape.

ALLIED ANGLES



Allied angles add up to 180°.
They are found in a C- or U-shape.

CORRESPONDING ANGLES



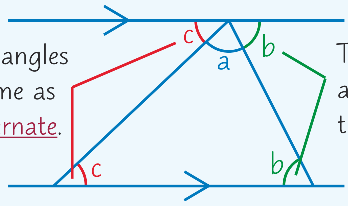
Corresponding angles are the same.
They are found in an F-shape.

EXAMPLE:

Prove that the angles in a triangle add up to 180°.

This is the proof of rule 1 from the previous page.
First, draw a triangle between two parallel lines:

These two angles are the same as they're alternate.



These two angles are the same as they're alternate.

Angles on a straight line add up to 180°, so a + b + c = 180°.

Parallel lines are key things to look out for in geometry

Watch out for parallel lines and Z, C, U and F shapes — extending the lines can make spotting them easier.
Learn the proper names (alternate, allied and corresponding angles) as you'll have to use them in the exam.

Geometry Problems

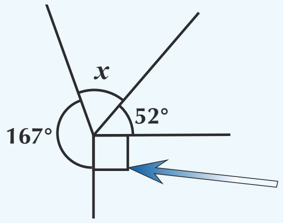
As if geometry wasn't enough of a problem already, here's a page dedicated to geometry problems. Make sure you learn the five angle rules on p.127 — they'll help a lot on these questions.

Using the **Five Angle Rules**

The best method is to find whatever angles you can until you can work out the ones you're looking for. It's a bit trickier when you have to use more than one rule, but writing them all down is a big help.

EXAMPLES:

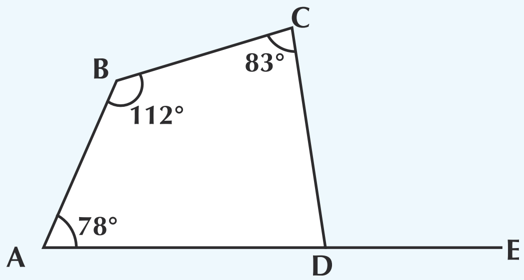
1. **Find the value of x.**



Use rule 4 from p.127:
Angles round a point add up to 360°,
so $x + 52° + 90° + 167° = 360°$
$x = 360° - 52° - 90° - 167° = 51°$

Remember — this little square means that it's a right angle (90°).

2. **Find the size of angle CDE.**



First use rule 3 from p.127:
Angles in a quadrilateral add up to 360°, so the fourth angle in the quadrilateral is
$360° - 78° - 112° - 83° = 87°$

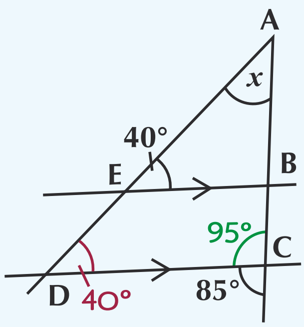
Then use rule 2:
Angles on a straight line add up to 180°.
So $\angle CDE = 180° - 87° = 93°$

Parallel Lines and **Angle Rules**

Sometimes you'll come across questions combining parallel lines and the five angle rules. These look pretty tricky, but like always, just work out all the angles you can find until you get the one you want.

EXAMPLE:

Find the value of angle x on the diagram below.



$\angle AEB$ and $\angle ADC$ are corresponding angles, so they are equal. $\angle ADC = 40°$

Use rule 2 from p.127 to find $\angle ACD$:
Angles on a straight line add up to 180°.
So $\angle ACD = 180° - 85° = 95°$

Use rule 1 to find x:
Angles in a triangle add up to 180°.
So $x = 180° - 95° - 40° = 45°$

It's always a good idea to label your diagram as you work out each angle.

The most important rule of all — don't panic

Geometry problems often look a lot worse than they are — don't panic, just write down everything you can work out. You'll need all the rules from the last two pages, make sure they're clear in your head.

Angles in Polygons

A polygon is a many-sided shape, and can be regular or irregular. A regular polygon (p.103) is one where all the sides and angles are the same. By the end of this page you'll be able to work out the angles in them.

Exterior and Interior Angles

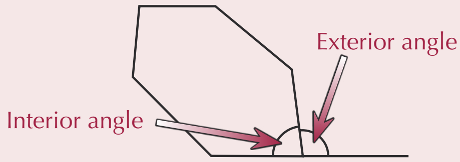
GRADE 3

You need to know what exterior and interior angles are and how to find them.

For ANY POLYGON (regular or irregular):

SUM OF EXTERIOR ANGLES = 360°

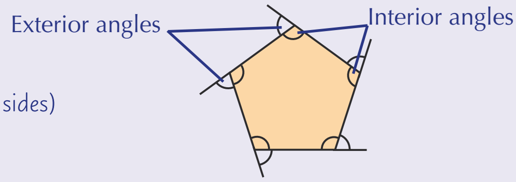
INTERIOR ANGLE = 180° – EXTERIOR ANGLE



For REGULAR POLYGONS only:

$$\text{Exterior angle} = \frac{360°}{n}$$

(n is the number of sides)



EXAMPLE: **Find the exterior and interior angles of a regular octagon.**

Octagons have 8 sides: exterior angle $= \frac{360°}{n} = \frac{360°}{8} = 45°$

Use the exterior angle to find the interior angle: interior angle = 180° – exterior angle

= 180° – 45° = 135°

The Tricky One — Sum of Interior Angles

GRADE 3

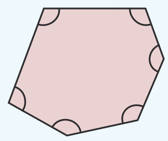
This formula for the sum of the interior angles works for ALL polygons, even irregular ones:

SUM OF INTERIOR ANGLES = (n – 2) × 180°

EXAMPLE: **Find the sum of the interior angles of the polygon on the right.**

The polygon is a hexagon, so n = 6: Sum of interior angles = (n – 2) × 180°

= (6 – 2) × 180° = 720°



Don't panic if the examiners put algebra in an interior angle question. It looks worse than it is.

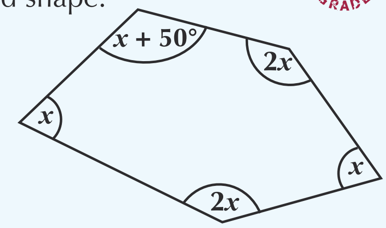
EXAMPLE: **Find the value of x in the diagram on the right.**

GRADE 4

First, find the sum of the interior angles of the 5-sided shape:

Sum of interior angles = (n – 2) × 180°

= (5 – 2) × 180° = 540°



Now write an equation and solve it to find x:

$2x + x + 2x + x + (x + 50°) = 540°$

$7x + 50° = 540° \rightarrow 7x = 490° \rightarrow x = 70°$

Learn these four important formulas

There are all manner of questions they can ask about angles in polygons, but they all come back to these four formulas — so make sure you've learnt them really well.

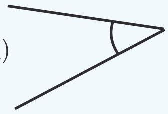
Warm-up and Worked Exam Questions

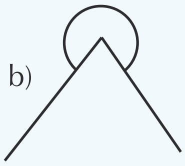
Oh look at all those lovely diagrams. But don't just look at them — you need to work through the questions one by one and polish all those geometry skills.

Warm-up Questions

1) Write down one example of:
   a) an acute angle
   b) an obtuse angle
   c) a reflex angle
   d) a right angle.

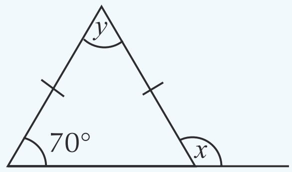
2) Measure these angles accurately with a protractor:

a) 

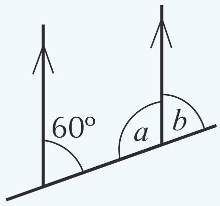
b) 

3) Use a protractor to accurately draw these angles:
   a) 35° b) 150° c) 80°

4) Work out angles x and y in the diagram below.



5) The diagram shown below has one angle given as 60°. Find the other two marked angles.



6) Find the size of the interior angle of a regular decagon.

Worked Exam Questions

There'll probably be a question in the exams that asks you to find angles. That means you have to remember all the different angle rules and practise using them in the right places...

1 The diagram to the right is wrong. Explain why.

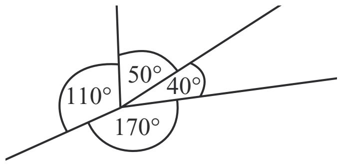


Diagram not accurately drawn

$110° + 170° + 50° + 40° = 370°$

Angles round a point add up to 360°, so these angles do not fit round the point.

[2 marks]

2 *BCD* is a triangle.
AD is a straight line.

a) Find the value of x.

Angles in a triangle add up to 180°

$2x + 3x + 4x = 180$

So $9x = 180$ and $x = 20$

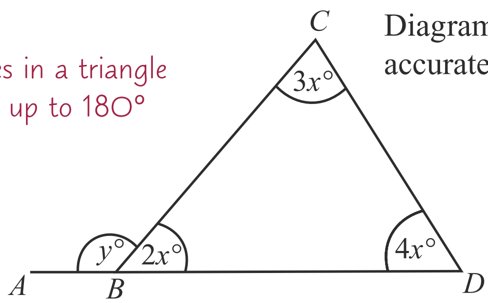


Diagram not accurately drawn

$x = 20$

[2 marks]

b) Find the value of y.

Angles on a straight line add up to 180°

$2x + y = 180$

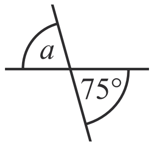
$40 + y = 180$, so $y = 180 - 40 = 140$

$y = 140$

[2 marks]

Exam Questions

3 Find the size of the angle marked a.
Give a reason for your answer.



....

[2 marks]

4 $ABCD$ is a quadrilateral.
Work out the size of angle ADC.

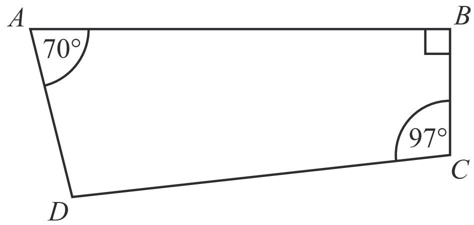


Diagram not accurately drawn

.... °

[2 marks]

5 Part of a regular polygon is on the right. Each interior angle is 150°.

Calculate the number of sides of the polygon.

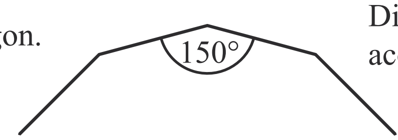


Diagram not accurately drawn

....

[3 marks]

6 BD and EF are parallel straight lines.
AH is a straight line.
Work out the size of angle x.

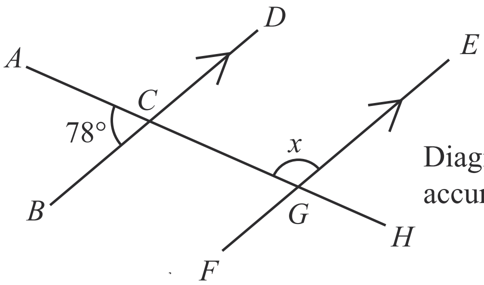
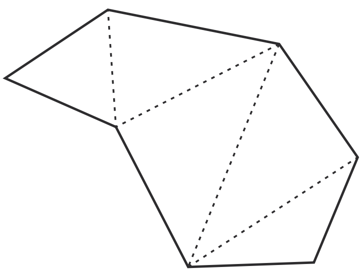


Diagram not accurately drawn

.... °

[2 marks]

7 The irregular polygon below has been divided into triangles as shown.



Use the triangles to show that the sum of the interior angles of the polygon is 900°.

[3 marks]

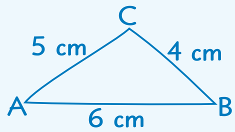
Triangle Construction

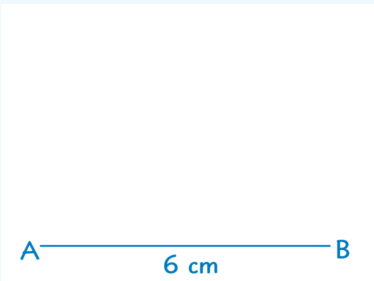
How you construct a triangle depends on what info you're given about the triangle...

Three sides — Use a Ruler and Compasses

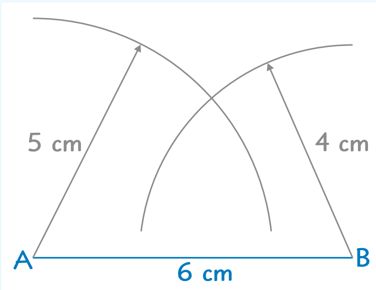
EXAMPLE: **Construct the triangle ABC where AB = 6 cm, BC = 4 cm, AC = 5 cm.**

1. First, sketch and label a triangle so you know roughly what's needed. It doesn't matter which line you make the base line.

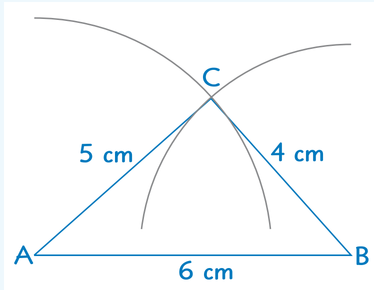




2. Draw the base line. Label the ends A and B.



3. For AC, set the compasses to 5 cm, put the point at A and draw an arc. For BC, set the compasses to 4 cm, put the point at B and draw an arc.

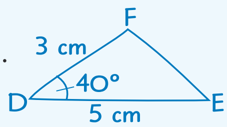


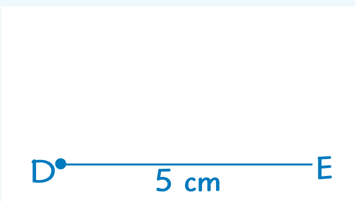
4. Where the arcs cross is point C. Now you can finish your triangle.

Sides and Angles — use a Ruler and Protractor

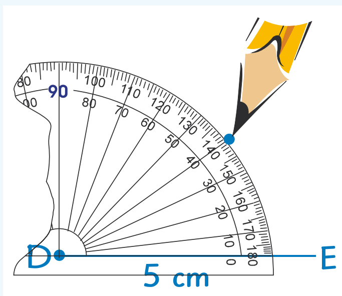
EXAMPLE: **Construct triangle DEF. DE = 5 cm, DF = 3 cm, and angle EDF = 40°.**

1. Roughly sketch and label the triangle.

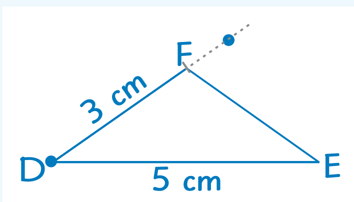




2. Draw the base line.



3. Draw angle EDF (the angle at D) — place the centre of the protractor over D, measure 40° and put a dot.



4. Measure 3 cm towards the dot and label it F. Join up D and F. Now you've drawn the two sides and the angle. Just join up F and E to complete the triangle.

Don't forget your compasses and protractor for the exam

Constructing a triangle isn't difficult, so long as you learn the methods on this page — and remember to take your ruler, compasses and protractor with you into the exam. You won't get far without them.

Loci and Construction

A LOCUS (another ridiculous maths word) is simply: **A LINE or REGION that shows all the points which fit a given rule.**

Make sure you learn how to do these PROPERLY using a ruler and compasses as shown on the next few pages.

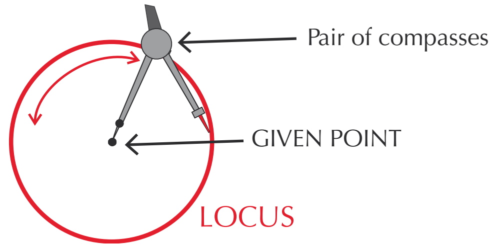
The **Four** Different Types of **Loci**

GRADE 4 GRADE

Loci is just the plural of locus.

1) The locus of points which are 'A FIXED DISTANCE from a given POINT'.

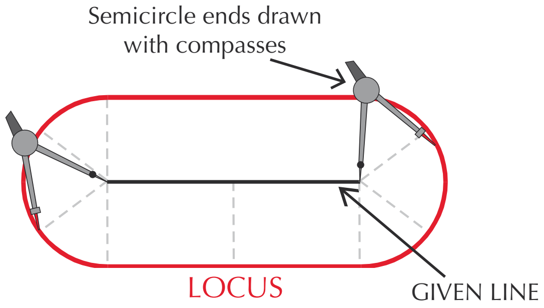
This locus is simply a CIRCLE.



2) The locus of points which are 'A FIXED DISTANCE from a given LINE'.

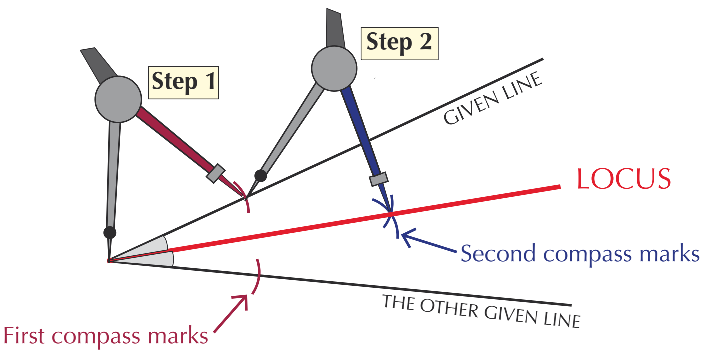
This locus is a SAUSAGE SHAPE.

It has straight sides (drawn with a ruler) and ends which are perfect semicircles (drawn with compasses).



3) The locus of points which are 'EQUIDISTANT from TWO GIVEN LINES'.

1) Keep the compass setting THE SAME while you make all four marks.
2) Make sure you leave your compass marks showing.
3) You get two equal angles — i.e. this LOCUS is actually an ANGLE BISECTOR.



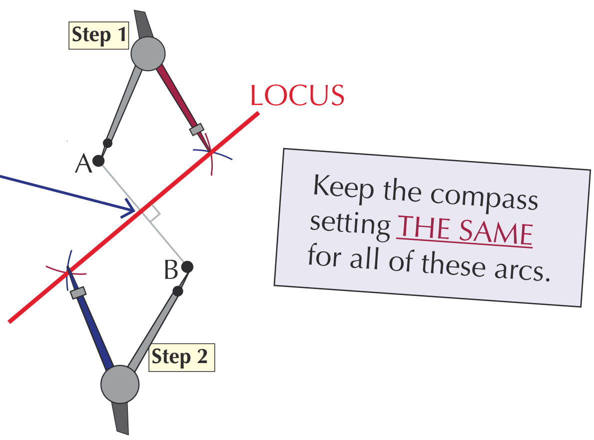
4) The locus of points which are 'EQUIDISTANT from TWO GIVEN POINTS'.

This LOCUS is all points which are the same distance from A as they are from B.

This time the locus is actually the PERPENDICULAR BISECTOR of the line joining the two points.

The perpendicular bisector of line segment AB is a line at right angles to AB, passing through the midpoint of AB. This is the method to use if you're asked to draw it.

(In the diagram below, A and B are the two given points.)

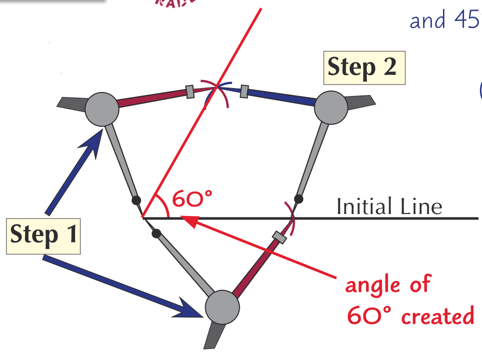


Loci and Construction

Don't just read the page through once and hope you'll remember it — work through each example and see if you can do each one. It's the only way of testing whether you really know this stuff.

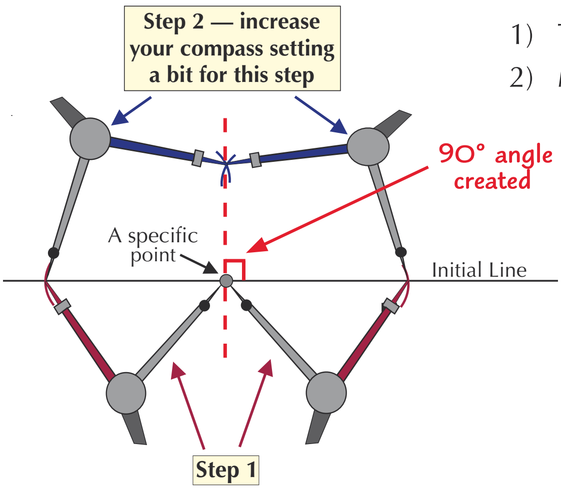
Constructing Accurate 60° Angles

1) They may well ask you to draw an accurate 60° angle without a protractor.
2) Follow the method shown in this diagram (make sure you leave the compass settings the same for each step).



You can construct 30° angles and 45° angles by bisecting 60° and 90° angles (see previous page).

Constructing Accurate 90° Angles

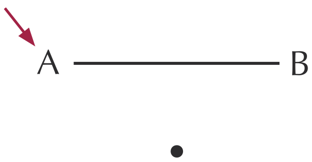


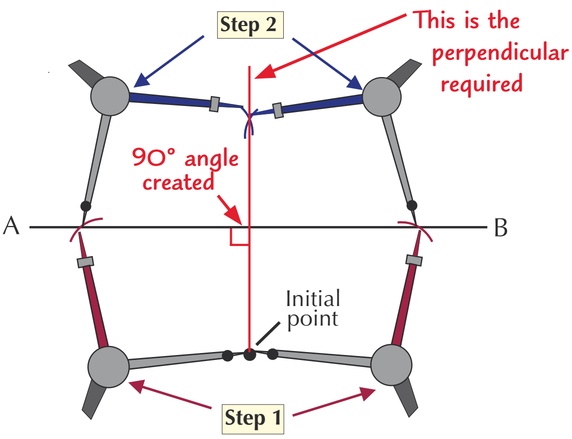
1) They might want you to construct an accurate 90° angle.
2) Make sure you can follow the method shown in this diagram.

The examiners WON'T accept any of these constructions done 'by eye' or with a protractor. You've got to do them the PROPER WAY, with compasses. DON'T rub out your compass marks, or the examiner won't know you used the proper method.

Drawing the Perpendicular from a Point to a Line

1) This is similar to the one above but not quite the same — make sure you can do both.
2) You'll be given a line and a point, like this:





Get out your ruler, pencil and compasses...

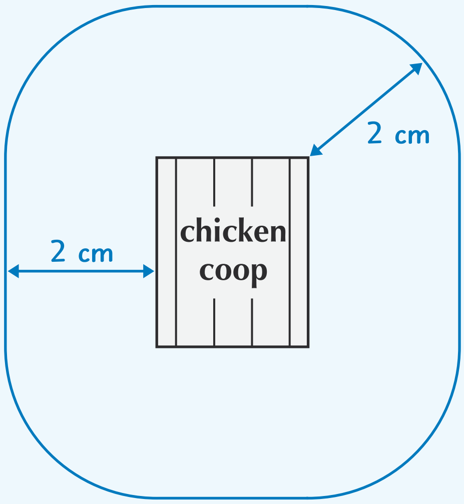
...because you'll need them if you're asked to do a construction or locus question in the exam. Practise drawing the constructions by following the instructions first, and then try to do them from memory.

Loci and Construction — Worked Examples

It's not break time just yet — first you should learn how to put those constructions to good use.

EXAMPLES:

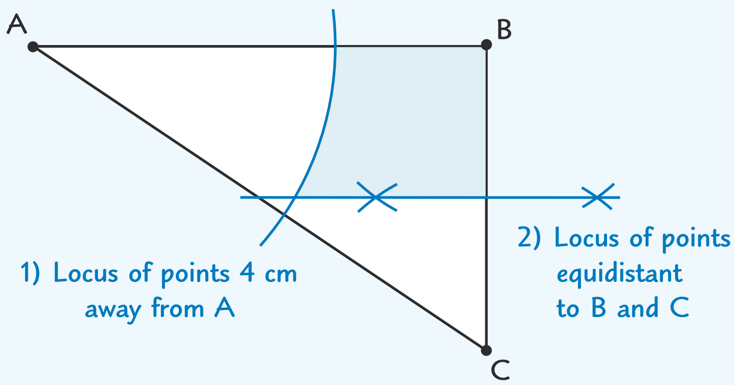
1. A farmer is drawing a diagram he can use to place a fence around his chicken coop. On the diagram the fence should be exactly 2 cm from the edge of the coop on all sides. Complete the diagram below using a ruler and compass:



1) Set your compasses to 2 cm.
2) Place the point on each corner and draw a quarter-circle at each one.
3) Draw lines parallel to each edge of the chicken coop 2 cm away, joining up the ends of the quarter-circles.

2. Point P lies somewhere in triangle ABC. Using the following information, a ruler and a compass, shade the area that P could lie in:

P is at least 4 cm away from A.
P is closer to B than C.



1) Set your compasses to 4 cm. Place the point on A and draw an arc.
2) Draw the perpendicular bisector of line BC using the method given on page 134.
3) Shade the area that's at least 4 cm away from A and closer to B than C.

Always leave your construction lines showing.
They show the examiner that you used the proper method.

If there are several rules, draw each locus then find the bit you want

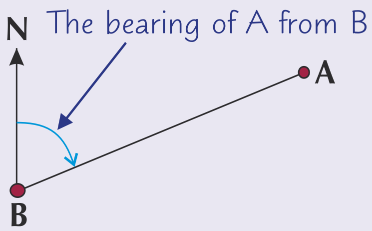
Don't panic if you get a wordy loci question — just deal with one condition at a time and work out which bit you need to shade at the end. Make sure you draw your loci accurately — it's really important. Use a ruler and a pair of compasses, otherwise you won't pick up all the marks.

Bearings

Bearings. They'll be useful next time you're off sailing. Or in your Maths exam.

Bearings

GRADE 3



1) A bearing is just a direction given as an angle in degrees.
2) All bearings are measured clockwise from the North line.
3) All bearings are given as 3 figures:
   e.g. 060° rather than just 60°, 020° rather than 20° etc.

The 3 Key Words

To find or draw a bearing you must remember three key words:

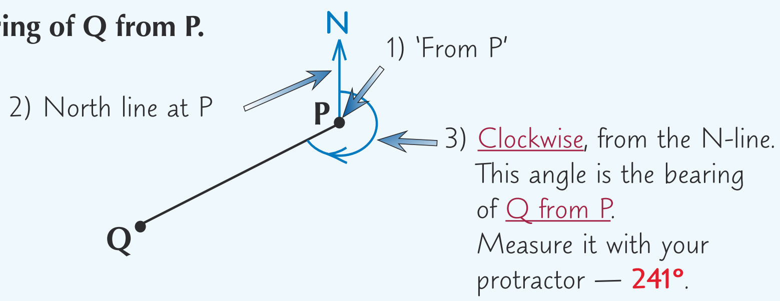
1) 'FROM' — Find the word 'FROM' in the question, and put your pencil on the diagram at the point you are going 'from'.

2) NORTH LINE — At the point you are going FROM, draw in a NORTH LINE.

3) CLOCKWISE — Now draw in the angle CLOCKWISE from the NORTH LINE to the line joining the two points — this angle is the bearing.

EXAMPLES:

1. **Find the bearing of Q from P.**



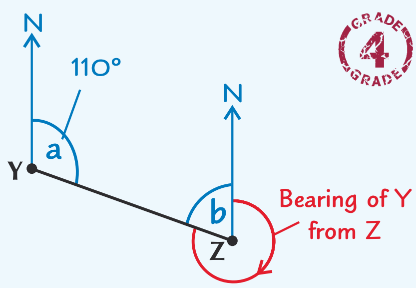
2. **The bearing of Z from Y is 110°.**
   Find the bearing of Y from Z.

GRADE 4

See page 128 for allied angles.

First sketch a diagram so you can see what's going on.
Angles a and b are allied, so they add up to 180°.

Angle b = 180° – 110° = 70°
So bearing of Y from Z = 360° – 70° = 290°.



From a point draw a North line then draw the angle clockwise

Make sure you've learnt the three key words above and the method for using them — scribble them out from memory to check you know them. You might need to use some of the geometry rules here too.

Maps and Scale Drawings

Scales tell you what a distance on a map or drawing represents in real life. They can be written in various ways, but they all boil down to something like "1 cm represents 5 km".

Map Scales

GRADE 2

1 cm = 3 km — "1 cm represents 3 km"

1 : 2000 — 1 cm on the map means 2000 cm in real life.
Converting to m gives "1 cm represents 20 m".

Use a ruler — the line's 2 cm long, so 2 cm means 1 km.
Dividing by 2 gives "1 cm represents 0.5 km".

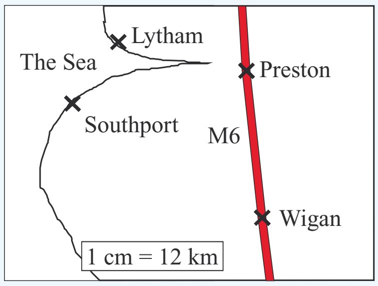
To convert between maps and real life, learn these rules:

- Make sure your map scale is of the form "1 cm = ..."
- To find REAL-LIFE distances, MULTIPLY by the MAP SCALE.
- To find MAP distances, DIVIDE by the MAP SCALE.
- Always check your answer looks sensible.

Converting from Map Distance to real life — Multiply

EXAMPLE:

This map shows the original Roman M6 motorway built by the Emperor Hadrian in the year AD 120. Work out the length of the section of the M6 between Wigan and Preston in km.



1) Measure with a ruler: Distance on map = 2 cm
2) Read off the scale: Scale is 1 cm = 12 km
3) For real life, multiply: Real distance is: 2 × 12 = 24 km

This looks sensible ✓

Converting from Real Life to Map Distance — Divide

EXAMPLE:

Helmsley is 18 km west of Pickering.

a) How far apart would they be on this map?

Real-life distance = 18 km

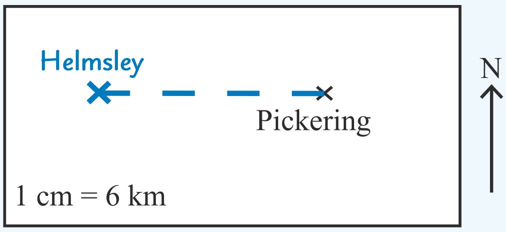
Scale is 1 cm = 6 km

Divide for a map distance. — Distance on map = 18 ÷ 6 = 3 cm

This looks sensible ✓

b) Mark Helmsley on the map.

Measure 3 cm to the west (left) of Pickering:



Practise reading scales on any maps you can find

Map scales can be a bit confusing at first — but as long as you work through the examples on this page and make sure you understand them, you shouldn't have a problem in the exam.

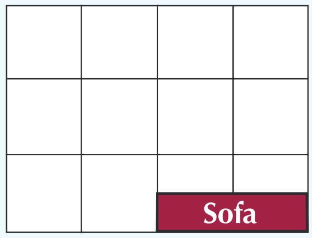
Maps and Scale Drawings

Scale Drawings

Scale drawings work just like maps. To convert between real life and scale drawings, just replace the word 'map' with 'drawing' in the rules on the previous page.

EXAMPLE:

This is a scale drawing of a room in Clare's house. 1 cm represents 1.5 m.



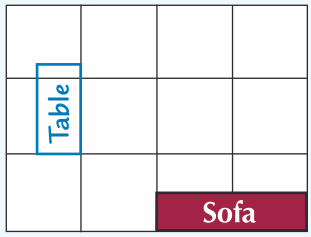
Scale drawings will often be shown on a grid.

a) Find the real length and width of the sofa in m.

1. Measure with a ruler. — Length on drawing = 2 cm; Width on drawing = 0.5 cm
2. Multiply to get real-life length. — Real length = 2 × 1.5 = 3 m; Real width = 0.5 × 1.5 = 0.75 m

b) Clare's dining table is 90 cm wide and 180 cm long. Draw the table on the scale drawing.

1. Scale uses m, so convert cm to m. — Width = 90 cm = 0.9 m; Length = 180 cm = 1.8 m
2. Divide to get scale drawing length. — Width on drawing = 0.9 ÷ 1.5 = 0.6 cm; Length on drawing = 1.8 ÷ 1.5 = 1.2 cm
3. Draw with a ruler in any sensible position and label.



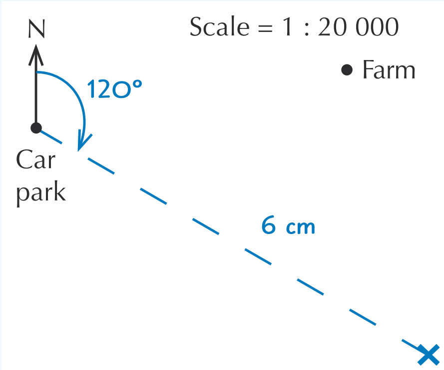
Map Questions Using Bearings

EXAMPLE:

Liam walks 1.2 km from the car park on a bearing of 120°.

a) Mark his position on the map.

1. Work out how many km 1 cm represents. — 1 cm = 20 000 cm = 200 m = 0.2 km. So 1 cm = 0.2 km
2. Divide to get distance on map. — Distance walked on map = 1.2 ÷ 0.2 = 6 cm
3. Mark a point 6 cm away, 120° clockwise from the North line.



b) How far is he from the farm in km?

1. Measure distance between Liam and farm. — Distance between Liam and farm = 4 cm
2. For real life, multiply: — Real distance = 4 × 0.2 = 0.8 km

Bearings can crop up in map questions too

You could come across questions which ask you to use maps and bearings. Just remember to measure your angle from the North line. Take a look back at page 137 if you get a bit stuck.

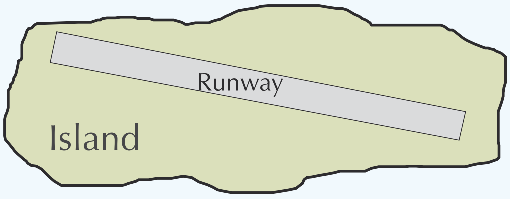
Warm-up and Worked Exam Questions

Have a go at these warm-up questions to prepare yourself for the exam-style questions coming up.

Warm-up Questions

1) a) Construct a triangle ABC with side AB = 3 cm, side BC = 4 cm and angle ABC = 90°.
   b) Measure the length of side AC.
2) Using a ruler and compasses construct an equilateral triangle with length of side 4 cm.
3) Draw the locus of the point P that moves around a 3 cm vertical line at a constant distance of 1 cm.
4) a) Work out the length in m of the runway shown on the right:
   b) How many cm on the map would a 500 m runway be?

Scale: 1 cm to 200 m

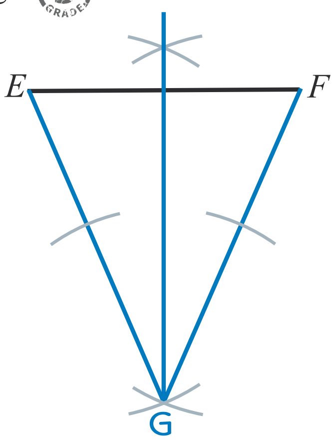


5) Draw a dot on a piece of paper to represent home, and then draw 2 lines, one going out in a south-westerly direction and the other on a bearing of 080°.

Worked Exam Questions

Some lovely worked exam questions here before you get into doing the questions yourself.

1 *EFG* is an isosceles triangle. Sides *EG* and *FG* are both 4.5 cm long. Side *EF* has been drawn on the right.



a) Complete the construction of triangle *EFG* by drawing sides *EG* and *FG*.

Set the compasses to 4.5 cm, put the point at E and draw an arc, then do another with the point at F. Point G is where they intersect.

[2 marks]

b) Construct the bisector of angle *EGF*.

Put the point at G, then draw arcs intersecting EG and FG. Keeping the compasses the same, use these intersections to draw another two arcs, then join their intersection to G with a straight line.

[2 marks]

2 Find the bearing of *B* from *A*.

Figure out which angle you need to find before starting any calculations.

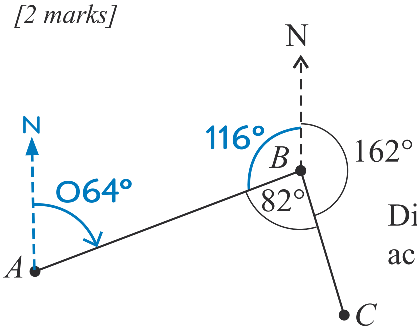


Diagram not accurately drawn

360° – 162° – 82° = 116° (angles round a point)

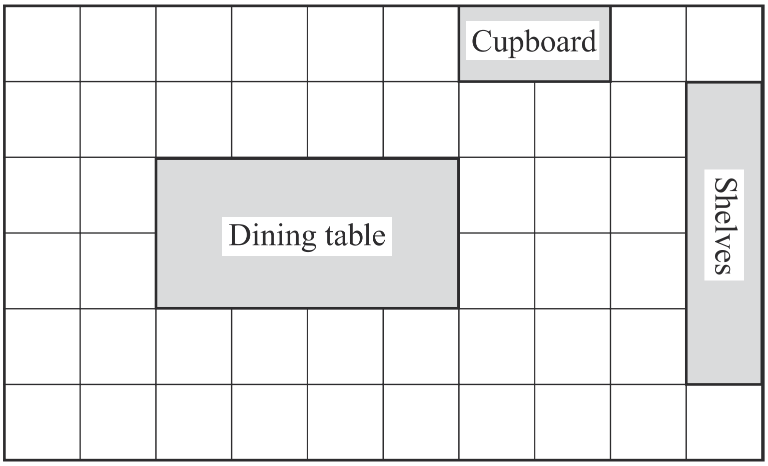
180° – 116° = 64° (allied angles)

Bearing of B from A is 064°.

............064............°
[2 marks]

Exam Questions

3 Douglas drew a scale drawing of one of the rooms in his house. (Grade 2)



a) His dining table is 2 m long. What is the scale of this drawing?

1 cm to .... m
[1 mark]

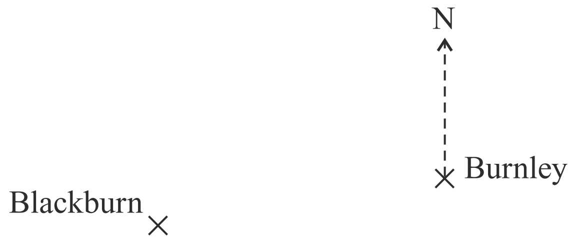
b) Work out the real distance from the dining table to the shelves.

.... m
[1 mark]

c) Douglas wants to put a chair measuring 1 m × 1.5 m in the room so that there is a space of at least 0.5 m around it. Is this possible? Give a reason for your answer.

....
[2 marks]

4 Using the diagram below, find the three-figure bearing of Blackburn from Burnley. (Grade 3)



......................°
[1 mark]

5 Ruth cycles in a straight line from V to U. (Grade 3)

Find the bearing on which she travels to get from V to U.

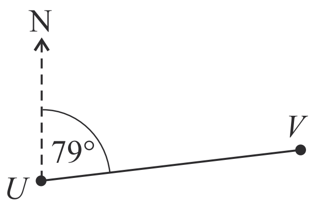


Diagram not accurately drawn

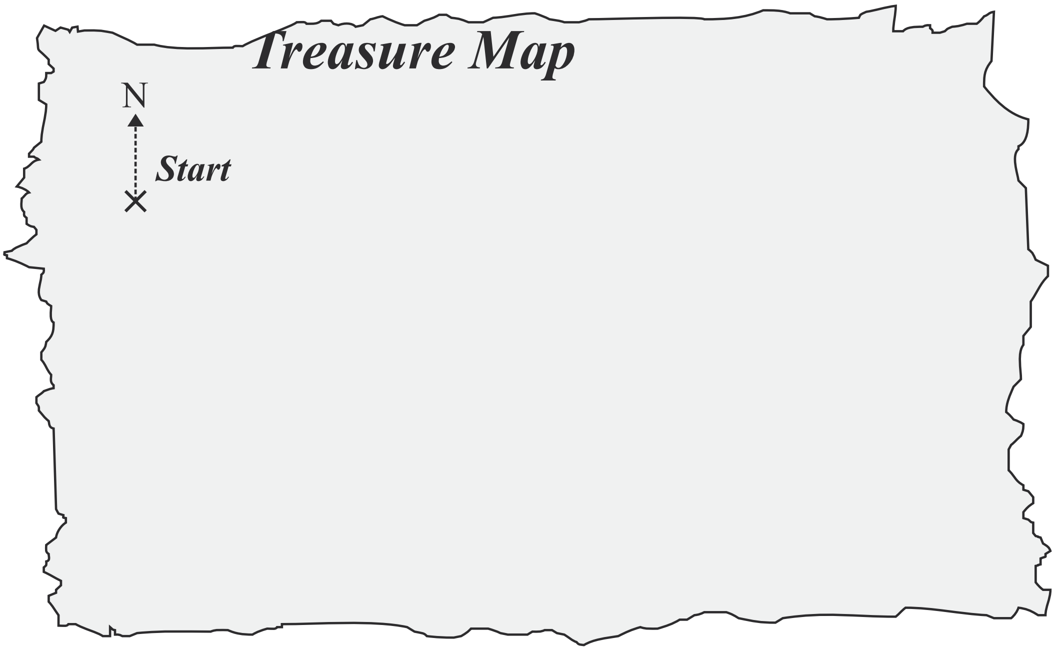
......................°
[2 marks]

Exam Questions

6 The instructions on a treasure map say "start at the cross and walk 400 metres on a bearing of 150°. Then walk 500 metres on a bearing of 090° to find the treasure."

Using a scale of 1 cm = 100 m, accurately draw the path that must be taken to find the treasure on the map below.

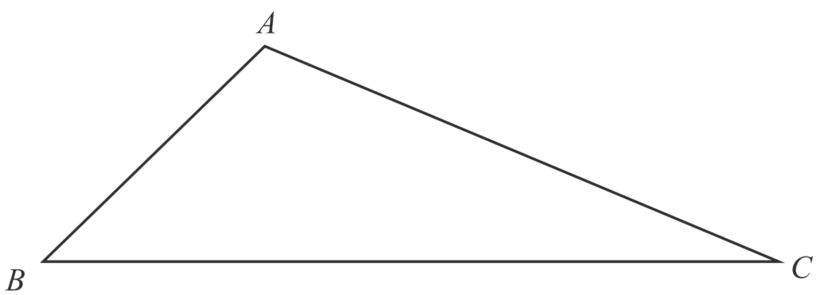
Make sure you draw the North line accurately for the second bearing.



[3 marks]

7 *ABC* is a triangle.

Find and shade the region inside the triangle which is **both** closer to the line *AB* than the line *BC*, **and** more than 6.5 cm from the point *C*.



[4 marks]

Pythagoras' Theorem

Pythagoras' Theorem is dead important — make sure you learn how to use it.

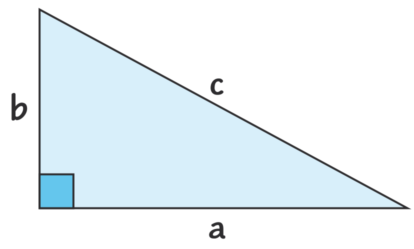
Pythagoras' Theorem is Used on **Right-Angled Triangles**

Pythagoras' theorem only works for RIGHT-ANGLED TRIANGLES.
It uses two sides to find the third side.

The formula for Pythagoras' theorem is:

$$a^2 + b^2 = c^2$$

short sides — long side

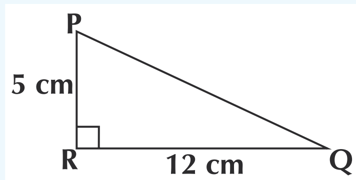


The trouble is, the formula can be quite difficult to use. Instead, it's a lot better to just remember these three simple steps, which work every time:

1) SQUARE THEM	SQUARE THE TWO NUMBERS that you are given, (use the x^2 button if you've got your calculator.)	
2) ADD or SUBTRACT	To find the longest side, ADD the two squared numbers. To find a shorter side, SUBTRACT the smaller from the larger.	$a^2 + b^2 = c^2$ $c^2 - b^2 = a^2$
3) SQUARE ROOT	Once you've got your answer, take the SQUARE ROOT (use the $\sqrt{\ }$ button on your calculator).	$c = \sqrt{a^2 + b^2}$ $a = \sqrt{c^2 - b^2}$

EXAMPLES:

1. **Find the length of side PQ in this triangle.**



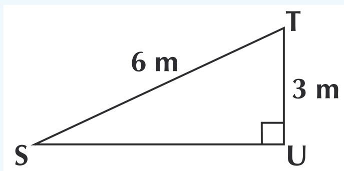
1) Square them: $a^2 = 5^2 = 25$, $b^2 = 12^2 = 144$

2) You want to find the longest side, so ADD: $a^2 + b^2 = c^2$
$25 + 144 = 169$

3) Square root: $c = \sqrt{169} = 13$ cm

Always check the answer's sensible — 13 cm is longer than the other two sides, but not too much longer, so it seems OK.

2. **Find the length of SU to 1 decimal place.**



1) Square them: $b^2 = 3^2 = 9$, $c^2 = 6^2 = 36$

2) You want to find a shorter side, so SUBTRACT: $c^2 - b^2 = a^2$
$36 - 9 = 27$

3) Square root: $a = \sqrt{27} = 5.196...$
$= 5.2$ m (to 1 d.p.)

Check the answer is sensible — yes, it's a bit shorter than the longest side.

Use Pythagoras' Theorem to find lengths in right-angled triangles

This is one of the most famous of all maths theorems and it'll probably be in your exam at some point. You really need to learn the method and make sure you've practised plenty of questions.

Trigonometry — Sin, Cos, Tan

Trigonometry — it's clever stuff. The three trig formulas are used on right-angled triangles to:
a) find an unknown side if you know a side and an angle, or b) find an angle if you know two lengths.

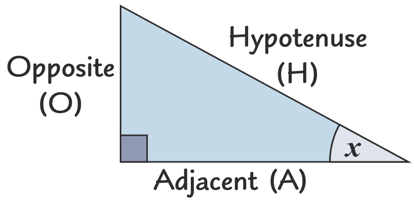
The 3 Trigonometry Formulas

GRADE 5

There are three basic trig formulas — each one links two sides and an angle of a right-angled triangle.

$$\text{Sin } x = \frac{\text{Opposite}}{\text{Hypotenuse}} \qquad \text{Cos } x = \frac{\text{Adjacent}}{\text{Hypotenuse}} \qquad \text{Tan } x = \frac{\text{Opposite}}{\text{Adjacent}}$$

- The Hypotenuse is the LONGEST SIDE.
- The Opposite is the side OPPOSITE the angle being used (x).
- The Adjacent is the (other) side NEXT TO the angle being used.



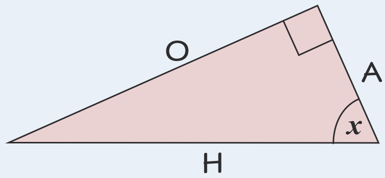
Formula Triangles Make Things Easier

GRADE 5

There's more about formula triangles on p.98 if you need to jog your memory.

A great way to tackle trig questions is to convert the formulas into formula triangles.
Then you can use the same method every time, no matter which side or angle is being asked for.

1) Label the three sides O, A and H (Opposite, Adjacent and Hypotenuse).
2) Write down 'SOH CAH TOA'.
3) Decide which two sides are involved: O,H A,H or O,A and choose SOH, CAH or TOA accordingly.



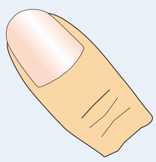
4) Turn the one you choose into a FORMULA TRIANGLE:

SOH: O over S × H

CAH: A over C × H

TOA: O over T × A

In the formula triangles, S represents sin x, C is cos x, and T is tan x.



5) Cover up the thing you want to find with your finger, and write down whatever is left showing.
6) Stick in the numbers and work it out using the sin, cos and tan buttons on your calculator.

If you're finding an angle, you'll need to add an extra step to find the inverse. See next page.

H = longest, O = opposite, A = next to, and remember SOH CAH TOA

You need to know this stuff off by heart — so go over this page a few times until you've got those formulas firmly lodged and all ready to reel off in the exam.

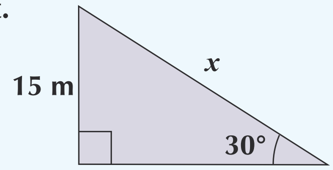
Trigonometry — Examples

Here are some lovely examples using the method from the previous page to help you through the trials of trig.

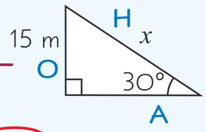
Examples:

GRADE 5

1 **Find the length of x in the triangle to the right.**



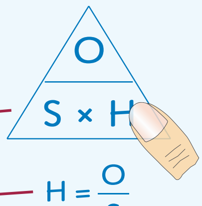
1) Label the sides



2) Write down — SOH CAH TOA

3) O and H involved

4) Write down the formula triangle

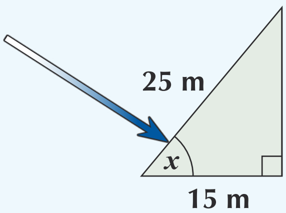


5) You want H so cover it up to give — $H = \frac{O}{S}$

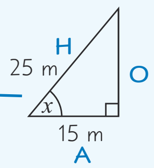
6) Put in the numbers — $x = \frac{15}{\sin 30°} = \frac{15}{0.5} = 30\text{ m}$

15 ÷ sin 30 =

2 **Find the angle x in the triangle to the right.**



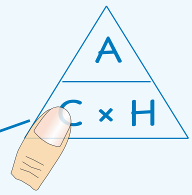
1) Label the sides



2) Write down — SOH CAH TOA

3) A and H involved

4) Write down the formula triangle



5) You want the angle so cover up C to give — $C = \frac{A}{H}$

6) Put in the numbers

15 ÷ 25 =

$\cos x = \frac{15}{25} = 0.6$

7) Find the inverse.

shift cos 0.6 =

$\Rightarrow x = \cos^{-1}(0.6) = 53.1301...°$

$= 53.1° \text{ (1 d.p.)}$

When you're finding an angle you'll have to find the INVERSE at the end. Press SHIFT or 2ndF, followed by sin, cos or tan — your calculator will display $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$.

You need to have learnt all seven steps

In example 2 you can see the six steps from the previous page, plus an extra one put into action. Remember — you only need to do step 7 if you're finding an angle.

Trigonometry — Common Values

Now that you're in the swing of trigonometry questions it's time to put those calculators away. Sorry.

Learn these Common Trig Values

The tables below contain a load of useful trig values. You might get asked to work out some exact trig answers in your non-calculator exam, so having these in your brain will come in handy.

$\sin 30° = \frac{1}{2}$	$\sin 60° = \frac{\sqrt{3}}{2}$	$\sin 45° = \frac{1}{\sqrt{2}}$
$\cos 30° = \frac{\sqrt{3}}{2}$	$\cos 60° = \frac{1}{2}$	$\cos 45° = \frac{1}{\sqrt{2}}$
$\tan 30° = \frac{1}{\sqrt{3}}$	$\tan 60° = \sqrt{3}$	$\tan 45° = 1$

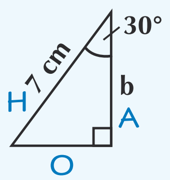
If you're asked for exact answers, don't convert them to decimals at the end.

	$\tan 0° = 0$
$\cos 90° = 0$	$\cos 0° = 1$
$\sin 90° = 1$	$\sin 0° = 0$

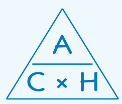
Have a look at the examples below — they might help cement a few values into your head.

EXAMPLES:

1. **Without using a calculator, find the exact length of side b in the right-angled triangle shown.**



1) It's a right-angled triangle so use SOH CAH TOA to pick the correct trig formula to use.



$A = C \times H$

2) Put in the numbers from the diagram in the question.

$b = \cos 30° \times 7$

3) You know the value of cos 30°, so substitute this in.

$b = \frac{\sqrt{3}}{2} \times 7 = \frac{7\sqrt{3}}{2}$ cm

2. **Without using a calculator, show that $\cos 60° + \sin 30° = 1$**

Put in the right values for cos 60° and sin 30°, then do the sum.

$\cos 60° = \frac{1}{2}$ $\sin 30° = \frac{1}{2}$

$\cos 60° + \sin 30° = \frac{1}{2} + \frac{1}{2} = 1$

Learn these common trig values off by heart

There are a lot of angles to learn on this page but you need to know them all.
If a trigonometry question asks for an exact answer, you'll need to use them.

Vectors — Theory

Vectors represent a movement of a certain size in a certain direction.
They might seem a bit weird at first, but there are really just a few facts to get to grips with...

The Vector Notations

GRADE 5

There are several ways to write vectors...

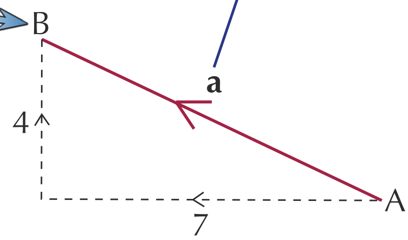
1) Column vectors: $\begin{pmatrix}2\\-5\end{pmatrix}$ — 2 units right, 5 units down; $\begin{pmatrix}-7\\4\end{pmatrix}$ — 7 units left, 4 units up

2) **a** — exam questions use bold like this

3) $\underset{\sim}{a}$ — you should always underline them

4) $\overrightarrow{AB}$ — this means the vector from point A to point B

They're shown on a diagram by an arrow.



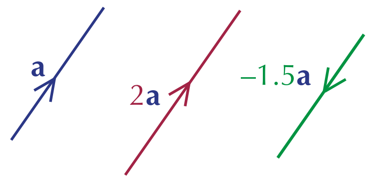
Multiplying a Vector by a Number

GRADE 5

1) Multiplying a vector by a positive number changes the vector's size but not its direction.

2) If the number's negative then the direction gets switched.

Vectors that are multiples of each other are parallel.

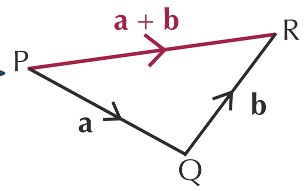


Adding and Subtracting Vectors

GRADE 5

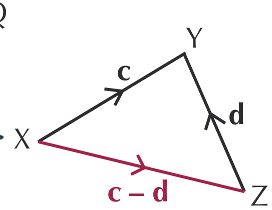
You can describe movements between points by adding and subtracting known vectors.

"$\underset{\sim}{a} + \underset{\sim}{b}$" means 'go along $\underset{\sim}{a}$ then $\underset{\sim}{b}$'.



In the diagrams, $\overrightarrow{PR} = \underset{\sim}{a} + \underset{\sim}{b}$ and $\overrightarrow{XZ} = \underset{\sim}{c} - \underset{\sim}{d}$.

"$\underset{\sim}{c} - \underset{\sim}{d}$" means 'go along c then backwards along d'
(the minus sign means go the opposite way).



When adding column vectors, add the top to the top and the bottom to the bottom. The same goes when subtracting. $\begin{pmatrix}3\\-1\end{pmatrix}+\begin{pmatrix}5\\3\end{pmatrix}=\begin{pmatrix}3+5\\-1+3\end{pmatrix}=\begin{pmatrix}8\\2\end{pmatrix}$

That's three vital vector facts done

But they're only really 'done' if you've learnt them. So be sure you know how vectors can be written, what multiplying a vector by a number does and how to add and subtract vectors — then you're done.

Vectors — Examples

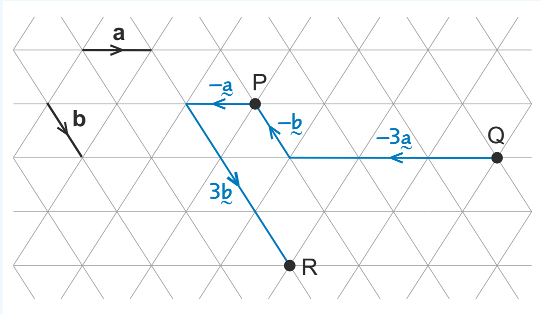
Here's a full page of worked examples to get you in the mood for some tasty vector questions of your own.

Examples

GRADE 5 GRADE

When you're going in the opposite direction to the vector, reverse the sign.

1. The diagram to the right shows an isometric grid. Vectors **a** and **b** are shown on the grid.
Find the following vectors in terms of **a** and **b**:



a) $\overrightarrow{PR}$

Use multiples of $\underset{\sim}{a}$ and $\underset{\sim}{b}$ to get from P to R.

$\overrightarrow{PR} = -\underset{\sim}{a} + 3\underset{\sim}{b}$

b) $\overrightarrow{QP}$

Use multiples of $\underset{\sim}{a}$ and $\underset{\sim}{b}$ to get from Q to P.

$\overrightarrow{QP} = -3\underset{\sim}{a} - \underset{\sim}{b}$

2. Given that $\mathbf{p} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ find:

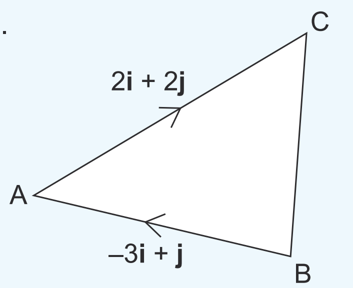
a) $4\mathbf{p}$

$$4 \times \begin{pmatrix} 4 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \times 4 \\ 4 \times -1 \end{pmatrix} = \begin{pmatrix} 16 \\ -4 \end{pmatrix}$$

b) $-2\mathbf{p} + 4\mathbf{r}$

$$-2 \times \begin{pmatrix} 4 \\ -1 \end{pmatrix} + 4 \times \begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \times 4 \\ -2 \times -1 \end{pmatrix} + \begin{pmatrix} 4 \times -3 \\ 4 \times 2 \end{pmatrix}$$
$$= \begin{pmatrix} -8 \\ 2 \end{pmatrix} + \begin{pmatrix} -12 \\ 8 \end{pmatrix} = \begin{pmatrix} -20 \\ 10 \end{pmatrix}$$

3. The diagram to the right shows the vectors $\overrightarrow{BA}$ and $\overrightarrow{AC}$.



a) Find $\overrightarrow{BC}$ in terms of **i** and **j**.

You need to go from B to A, then from A to C.

$\overrightarrow{BA} = -3\underset{\sim}{i} + \underset{\sim}{j}$

$\overrightarrow{AC} = 2\underset{\sim}{i} + 2\underset{\sim}{j}$

$\overrightarrow{BC} = -3\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{i} + 2\underset{\sim}{j}$

$= -\underset{\sim}{i} + 3\underset{\sim}{j}$

b) Alison adds a new line, BD, to the diagram. Given that $\overrightarrow{BD}$ is in the opposite direction to $\overrightarrow{BC}$ and is twice the length of $\overrightarrow{BC}$, find $\overrightarrow{BD}$ in terms of **i** and **j**.

$\overrightarrow{BD}$ is in the opposite direction to $\overrightarrow{BC}$, so reverse the sign. $\Longrightarrow$ $\underset{\sim}{i} - 3\underset{\sim}{j}$

$\overrightarrow{BD}$ is also twice the length of $\overrightarrow{BC}$, so multiply by 2. $\Longrightarrow$ $2\underset{\sim}{i} - 6\underset{\sim}{j}$

So $\overrightarrow{BD} = 2\underset{\sim}{i} - 6\underset{\sim}{j}$

Cover up the working, then try the questions on your own

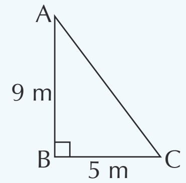
In questions like these, just use the vectors you're given to find what you're asked for. It's worth learning how to do these questions, because you could get very similar ones on the exam.

Warm-up and Worked Exam Questions

There are a lot of different ideas to take in from this mini-section, so here's a bit of a warm-up to get you into the swing of things before the exam questions on the next page.

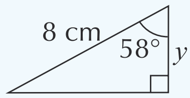
Warm-up Questions

1) Find the length of AC correct to 1 decimal place.

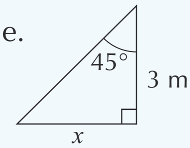


2) A 4 m long ladder leans against a wall. Its base is 1.2 m from the wall. How far up the wall does the ladder reach? Give your answer to 1 decimal place.

3) Find the length of y and give your answer to 2 decimal places.

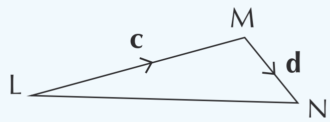


4) Using common trigonometry values, find the exact length of side x in this triangle.



5) Calculate $\begin{pmatrix}4\\7\end{pmatrix} - \begin{pmatrix}3\\2\end{pmatrix}$.

6) Find $\overrightarrow{LN}$ on the diagram to the right.



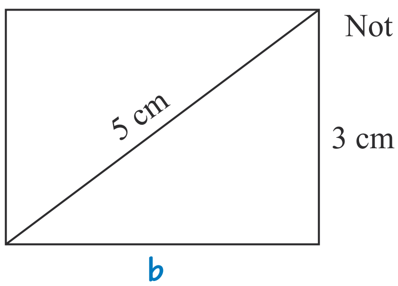
Worked Exam Questions

Some more exam questions with the answers written in — just what the doctor ordered.

1 A rectangle has a height of 3 cm and a diagonal length of 5 cm.

Calculate the area of the rectangle.

Not to scale



$5^2 = 3^2 + b^2$ — Use Pythagoras' Theorem

$b^2 = 25 - 9 = 16$

$b = \sqrt{16}$

$b = 4$ cm

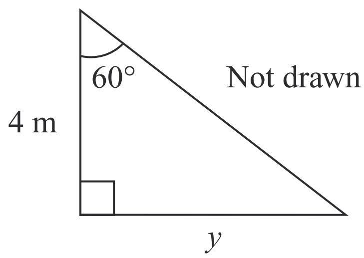
Area of rectangle = 3 cm × 4 cm = 12 cm^2

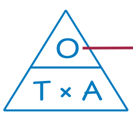
..........12.......... cm^2
[4 marks]

2 The diagram shows a right-angled triangle.

Work out the exact length of the side marked y.

Not drawn accurately





You want to find the opposite length, so cover up O.

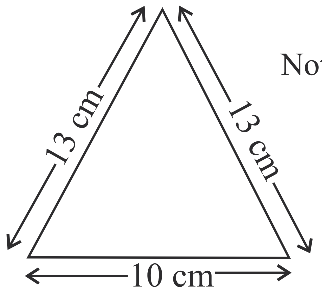
$y = \tan 60° \times 4$

$y = \sqrt{3} \times 4 = 4\sqrt{3}$ m

..........$4\sqrt{3}$.......... m
[3 marks]

Exam Questions

3 An isosceles triangle has a base of 10 cm. Its other two sides are both 13 cm long. GRADE 4
Calculate the height of the triangle.



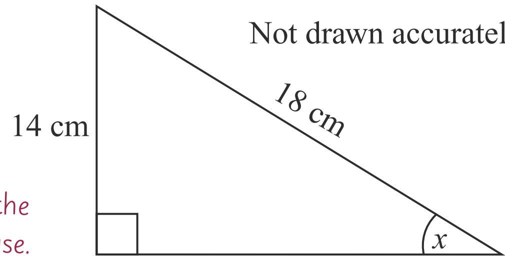
Not to scale

Think about whether you're trying to find the hypotenuse or one of the shorter sides before using Pythagoras' Theorem.

.... cm
[3 marks]

4 The diagram shows a right-angled triangle. GRADE 5
Work out the size of the angle marked x.
Give your answer to 1 decimal place.

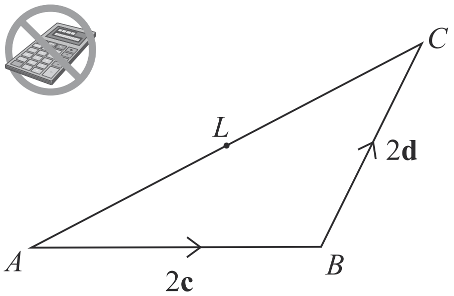
The sides involved here are the Opposite and the Hypotenuse.



Not drawn accurately

..........................°
[3 marks]

5 ABC is a triangle. $\overrightarrow{AB} = 2\mathbf{c}$ and $\overrightarrow{BC} = 2\mathbf{d}$. L is the midpoint of AC. GRADE 5



Not drawn accurately

Write in terms of **c** and **d**:

a) $\overrightarrow{AC}$

....
[2 marks]

b) $\overrightarrow{AL}$

The midpoint is found halfway along the line.

....
[2 marks]

c) $\overrightarrow{BL}$

....
[2 marks]

Revision Questions for Section Six

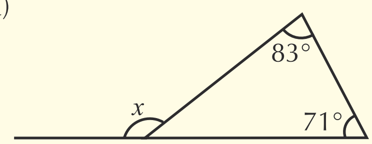
There are lots of opportunities to show off your artistic skills here (as long as you use them to answer the questions).

- Try these questions and tick off each one when you get it right.
- When you've done all the questions for a topic and are completely happy with it, tick off the topic.

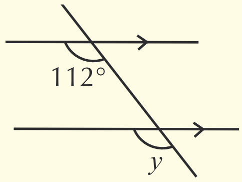
Angles and Geometry Problems (p126-129)

1) What is the name for an angle larger than 90° but smaller than 180°?
2) What do angles in a quadrilateral add up to?
3) Find the missing angles in the diagrams below.

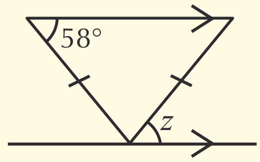
a)



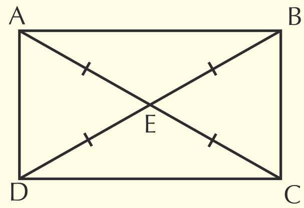
b)



c)



4) Given that angle DAC = 70°, work out angle CED.



Angles in Polygons (p130)

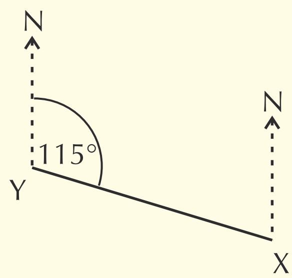
5) Find the exterior angle of a regular hexagon.
6) Find the sum of interior angles in a regular octagon.
7) Find the interior angle of a regular 20-sided polygon.

Constructions and Loci (p133-136)

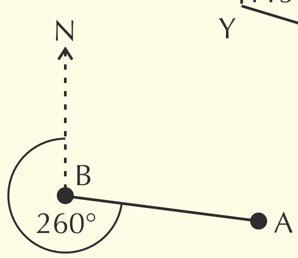
8) Construct triangle XYZ, where XY = 5.6 cm, XZ = 7.2 cm and angle YXZ = 55°.
9) What shape does the locus of points that are a fixed distance from a given point make?
10) Draw a horizontal line with a length of 8 cm.
    Draw the locus of points exactly 2 cm away from the line.
11) Construct an accurate 90° angle.
12) Draw a square with sides of length 6 cm and label it ABCD. Shade the region that is nearer to AB than CD and less than 4 cm from vertex A.

Bearings (p137)

13) Using the diagram on the right, find the bearing of Y from X.



14) Look at the diagram to the right.
    Tom wants to travel from point A to point B.
    Find the bearing he should travel on.

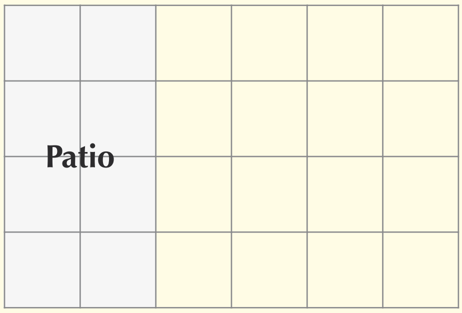


Revision Questions for Sections Six

Maps and Scale Drawings (p138-139) ☐

15) The scale on a map is 1 cm = 4 km. On the map, Leadz is 6.5 cm away from Horrowgate. How far is this in real life? ☐

16) The garden plan on the right has a scale of 1 : 200. Allen wants to build a square shed in the top right corner of the garden and a rectangular pond in the bottom right. The shed will measure 4 m × 4 m and the pond will measure 6 m × 2 m. Draw these accurately on the plan. ☐



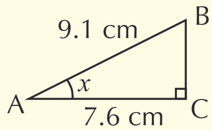
17) John travels on a bearing of 180° for 3 km. He travels on a bearing of 145° from his new position for 6 km. Using a scale of 1 cm = 1.5 km, draw an accurate diagram to represent this. ☐

Pythagoras and Trigonometry (p143-146) ☐

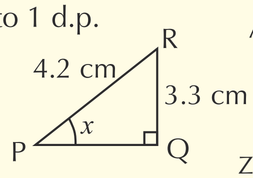
18) A rectangle has a diagonal of 15 cm. Its short side is 4 cm. Calculate the length of the rectangle's long side to 1 d.p. ☐

19) Write down the three trigonometry formula triangles. ☐

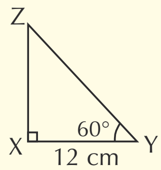
20) Find the size of angle x in triangle ABC to 1 d.p. ☐



21) Work out the value of x in triangle PQR. ☐



22) Find the exact length of side XZ in triangle XYZ without using your calculator. ☐

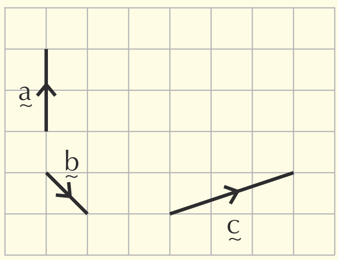


23) Without using your calculator, show that $\tan 45° + \sin 60° = \dfrac{2+\sqrt{3}}{2}$ ☐

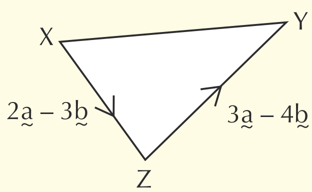
Vectors (p147-148) ☐

24) $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are column vectors, where $\underset{\sim}{a} = \begin{pmatrix}4\\-2\end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix}7\\6\end{pmatrix}$.
   a) Find $\underset{\sim}{a} - \underset{\sim}{b}$
   b) Find $5\underset{\sim}{a}$ ☐

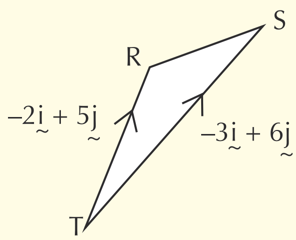
25) The diagram to the right shows a grid of unit squares. Find the vector $\underset{\sim}{c}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. ☐



26) Find the vector that describes $\overrightarrow{XY}$ in the form $m\underset{\sim}{a} + n\underset{\sim}{b}$. ☐



27) Find the vector that describes $\overrightarrow{RS}$ in the form $m\underset{\sim}{i} + n\underset{\sim}{j}$. ☐



Probability Basics

A lot of people think probability is tough. But learn the basics well, and it'll all make sense.

All Probabilities are Between 0 and 1

GRADE 1

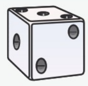
1) Probabilities are always between 0 and 1.
2) The higher the probability of something, the more likely it is.
3) A probability of ZERO means it will NEVER HAPPEN.
4) A probability of ONE means it DEFINITELY WILL HAPPEN.

You can show the probability of something happening on a scale from 0 to 1.
Probabilities can be given as fractions, decimals or percentages.

Impossible	Unlikely	Evens	Likely	Certain
0	¼	½	¾	1
0	0.25	0.5	0.75	1
0%	25%	50%	75%	100%


Every day next year in the UK will be hot and sunny.


A coin tossed twice will come up "heads" both times.


An odd number will be rolled on a dice.


A randomly selected playing card won't be a heart.


It will rain in the UK this year.

Use This Formula When All Outcomes are Equally Likely

GRADE 2

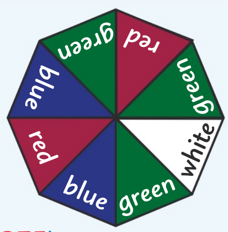
Use this formula to find probabilities for a fair spinner, coin or dice.
A spinner/coin/dice is 'fair' when it's equally likely to land on any of its sides.

$$\text{Probability} = \frac{\text{Number of ways for something to happen}}{\text{Total number of possible outcomes}}$$

Outcomes are just 'things that could happen'.

EXAMPLE:

The picture on the right shows a fair, 8-sided spinner.



a) Work out the probability of this spinner landing on green.

There are 8 sides so there are 8 possible outcomes.
There are 3 ways for it to land on green.

P(green) means 'The probability of the spinner landing on green'.

$$P(\text{green}) = \frac{\text{number of ways for 'green' to happen}}{\text{total number of possible outcomes}} = \frac{3}{8} \text{ (or 0.375)}$$

b) Which of its four colours is the spinner least likely to land on?

It's least likely to land on the colour that 'can happen in the fewest ways' — this is the one on the fewest sections.

White

A probability of 1 means it's certain to happen

A probability of 0 means it definitely won't happen — the higher the probability, the more likely it is.

More Probability

Probabilities Add Up To 1

1) If only one possible result can happen at a time, then the probabilities of all the results add up to 1.

Probabilities always ADD UP to 1

2) So since something must either happen or not happen (i.e. only one of these can happen at a time):

P(event happens) + P(event doesn't happen) = 1

EXAMPLE:

A spinner has different numbers of red, blue, yellow and green sections.

Colour	red	blue	yellow	green
Probability	0.1	0.4	0.3	

a) What is the probability of spinning green?

All the probabilities must add up to 1. P(green) = 1 – (0.1 + 0.4 + 0.3) = 0.2

b) What is the probability of not spinning green?

P(green) + P(not green) = 1 P(not green) = 1 – P(green) = 1 – 0.2 = 0.8

Listing All Outcomes

A sample space diagram shows all the possible outcomes.

1) It can just be a simple list...

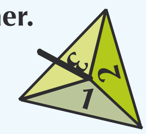
Try to order your lists — here there are 3 choices for the first digit, then the other 2 digits can swap round.

E.g. Find all the 3-digit numbers that include the digits 1, 2 and 3. 123, 132, 213, 231, 321, 312

2) Or you can draw a two-way table if there are two activities going on (e.g. two coins being tossed, or a dice being thrown and a spinner being spun).

EXAMPLE:

The spinner on the right is spun twice, and the scores added together.



a) Complete this sample space diagram showing all the possible outcomes.

+	1	2	3
1	2	3	4
2	3	4	5
3	4	5	6

There are 9 possible outcomes here — even though some of the actual scores are repeated.

b) Find the probability of spinning a total of 3.

$$P(\text{total of } 3) = \frac{\text{ways to score 3}}{\text{total number of possible outcomes}} = \frac{2}{9}$$

There are 2 ways to score 3.

c) Find the probability of spinning a total of 4 or more.

$$P(\text{total of 4 or more}) = \frac{\text{ways to score 4 or more}}{\text{total number of possible outcomes}} = \frac{6}{9} = \frac{2}{3}$$

There are 6 ways to score either 4, 5 or 6.

List all possible outcomes in a methodical way

Be extra careful when you're listing outcomes, you don't want to lose an easy mark by missing one out. Remember, if there are two activities it's usually better to list your outcomes in a two-way table.

Probability Experiments

The formula on page 153 only works when the outcomes are equally likely. If they're not equally likely, you can use the results from experiments to estimate the probability of each outcome.

Do the Experiment Again and Again...

You need to do an experiment over and over again and count how many times each outcome happens (its frequency). Then you can calculate the relative frequency using this formula:

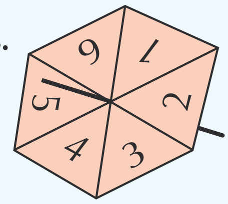
$$\text{Relative frequency} = \frac{\text{Frequency}}{\text{Number of times you tried the experiment}}$$

An experiment could just mean rolling a dice.

You can use the relative frequency of a result as an estimate of its probability.

EXAMPLE: **The spinner on the right was spun 100 times. Use the results in the table below to estimate the probability of getting each of the scores.**

Score	1	2	3	4	5	6
Frequency	3	14	41	20	18	4



Divide each of the frequencies by 100 to find the relative frequencies.

Score	1	2	3	4	5	6
Relative Frequency	$\frac{3}{100} = 0.03$	$\frac{14}{100} = 0.14$	$\frac{41}{100} = 0.41$	$\frac{20}{100} = 0.2$	$\frac{18}{100} = 0.18$	$\frac{4}{100} = 0.04$

The MORE TIMES you do the experiment, the MORE ACCURATE your estimate of the probability should be.

E.g. if you spun the above spinner 1000 times, you'd get a better estimate of the probability for each score.

Fair or Biased?

'Fair' means all the outcomes are equally likely. If something is unfair, it's called biased.

1) If the dice/spinner/coin/etc. is fair, then the relative frequencies of the results should roughly match the probabilities you'd get using the formula on p.153.
2) If the relative frequencies are far away from those probabilities, you can say it's probably biased.

EXAMPLE: **Do the above results suggest that the spinner is biased?**

Yes, because the relative frequency of 3 is much higher than you'd expect, while the relative frequencies of 1 and 6 are much lower.

For a fair 6-sided spinner, you'd expect all the relative frequencies to be about 1 ÷ 6 = 0.17(ish).

More experiments mean a more accurate probability estimate

Learn the formula for calculating relative frequency — you can then use the relative frequency of a result to estimate its probability. If something is biased, this just means it isn't fair. Remember that even with a fair dice you're unlikely to get exactly the expected result.

Probability Experiments

OK, I'll admit it, probability experiments aren't as fun as science experiments but they are useful.

Record Results in **Frequency Trees**

Grade 4

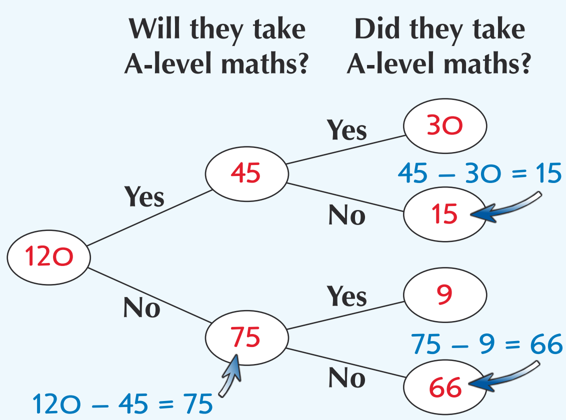
When an experiment has two or more steps, you can record the results using a frequency tree.

EXAMPLE:

120 GCSE maths students were asked if they would go on to do A-level maths.

- **45 of them said they would go on to do A-level maths.**
- **30 of the students who said they would do A-level maths actually did.**
- **9 of the students who said they wouldn't do A-level maths actually did.**

a) Complete the frequency tree below.



b) A student who said they wouldn't do A-level maths is chosen at random. What is the probability that they did do A-level maths?

9 out of the 75 students who said they wouldn't do A-level maths actually did.

So the probability is $\frac{9}{75} = 0.12$

Use Probability to Find an "**Expected Frequency**"

Grade 4

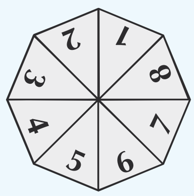
1) You can estimate how many times you'd expect something to happen if you do an experiment *n* times.
2) This expected frequency is based on the probability of the result happening.

Expected frequency of a result = probability × number of trials

A trial is a single experiment.

EXAMPLE:

A person spins the fair spinner on the right 200 times.
Estimate how many times it will land on 5.



1) First calculate the probability of the spinner landing on 5.

$$\text{P(lands on 5)} = \frac{\text{ways to land on 5}}{\text{number of possible outcomes}} = \frac{1}{8}$$

2) Then multiply by 200 spins.

$$\text{Expected number of 5's} = \text{P(lands on 5)} \times \text{number of trials}$$
$$= \frac{1}{8} \times 200 = 25 \text{ times}$$

If you don't know the probability of a result, fear not...
... you can estimate the probability using the relative frequency of the result in past experiments.

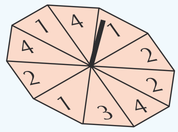
Expected frequency is how many times you'd expect something to happen

Make sure you can remember the formula for expected frequency in the box above. Try to get your head around where you can use frequency trees — they're really useful for experiments with two or more steps.

Warm-up and Worked Exam Questions

These probability basics aren't that difficult, but it's easy to throw away marks by being a little slap-dash with your calculations. So it's important to get loads of practice. Try these questions.

Warm-up Questions



1) Calculate the probability of the fair spinner on the right landing on 4.
2) The probability of rolling a double from two dice rolls is $\frac{1}{6}$.
   What is the probability of not rolling a double?
3) Two fair coins are tossed: a) List all the possible outcomes.
   b) Find the probability of getting exactly 1 head.
4) Sandro rolled a dice 1000 times and got the results shown in the table below.

Score	1	2	3	4	5	6
Frequency	140	137	138	259	161	165

   Find the relative frequencies for each of the scores 1-6.
5) Using the frequency tree on page 156, find the probability that a randomly chosen student said they were going to do A-level maths but didn't actually do it.
6) The spinner on page 156 is spun 300 times.
   Estimate how many times it will land on an even number.

Worked Exam Questions

Take a look at these worked exam questions. They should give you a good idea of how to answer similar questions. You'll usually get at least one probability question in the exam.

1 Steven records the positions of all the members of his football team. The table on the right shows his results.

GRADE 2

Position	Frequency
Attacker	6
Midfielder	9
Defender	4
Goalkeeper	1

A member of the team is chosen at random.
What is the probability they're a midfielder?
Give your answer as a decimal.

There are 6 + 9 + 4 + 1 = 20 people on the team

P(midfielder) = $\frac{9}{20}$ = 0.45

9 out of the 20 people on the team are midfielders.

0.45
[2 marks]

2 Georgie has a biased 5-sided spinner numbered 1-5.
The table below shows the probabilities of the spinner landing on numbers 1-4.

GRADE 3

Number	1	2	3	4	5
Probability	0.3	0.15	0.2	0.25	

She spins the spinner 100 times. Estimate the number of times it will land on 5.

P(landing on a 5) = 1 – (0.3 + 0.15 + 0.2 + 0.25) = 0.1

In 100 spins, you'd estimate: 0.1 × 100 = 10 fives

Expected frequency = probability × no. of trials

10
[2 marks]

Exam Questions

3 There are 10 counters in a bag. 4 of the counters are blue and the rest are red.
One counter is picked out at random. On the scale below,
mark with an arrow (↓) the probability that a red counter is picked.

[2 marks]

4 Katie decides to attend two new after-school activities. She can do one on Monday and one on Thursday. Below are lists of the activities she could do on these days.

Monday	**Thursday**
Hockey	Netball
Orchestra	Choir
Drama	Orienteering

a) List all nine possible combinations of two activities Katie could try in one week.

[2 marks]

Katie randomly picks an activity to do on each day.
Use your answer to part a) to find:

b) the probability that she does hockey on Monday and netball on Thursday,

....
[1 mark]

c) the probability that she does drama on Monday.

....
[1 mark]

5 Sammi has 3 pieces of homework, English (E), History (H) and Maths (M).
She has to do all 3 pieces tonight but she can do them in any order.

a) List all the different orders in which she could do her pieces of homework.

....

....
[2 marks]

Sammi randomly chooses the order in which to do her pieces of homework.

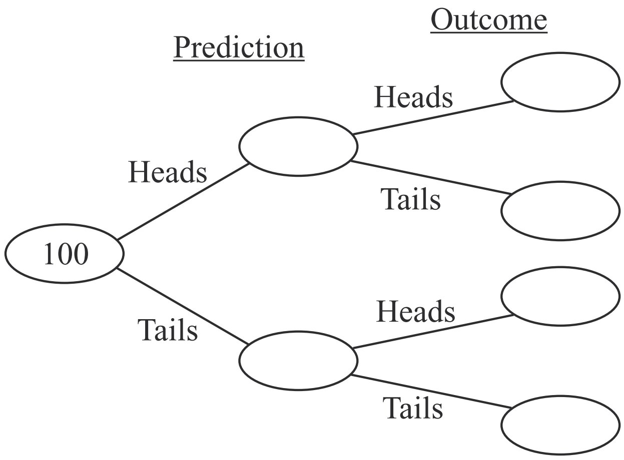
b) Use your answer to part a) to find the probability that she does her Maths homework before her English homework. Give your answer as a fraction in its simplest form.

....
[1 mark]

Exam Questions

6 Danielle flipped a coin 100 times and predicted the outcome before each flip.
She predicted heads 47 times and got 25 correct.
Of the times she predicted tails, she got 26 correct.

a) Complete the frequency tree below to show these results.



[2 marks]

b) Work out the relative frequency of Danielle predicting the outcome correctly.

....
[2 marks]

7 John throws a ball at a target using his left and right hands.
His results are shown in the table on the right.

	Left Hand	Right Hand
Throws	20	100
Hit target	12	30

a) Estimate the probability that John will hit the target with his next throw if he uses his left hand.

....
[2 marks]

b) John uses his results to estimate the probabilities of him hitting the target using each hand. Explain which of his estimated probabilities will be more reliable.

....

....
[1 mark]

The AND / OR Rules

This page will show you two really important probability rules.

Combined Probability — Two or More Events

GRADE 5

1) Always start by working out what different SINGLE EVENTS you're interested in.
2) Find the probability of EACH of these SINGLE EVENTS.
3) Apply the AND/OR rule.

And now for the rules. Say you have two events — call them A and B...

The AND Rule

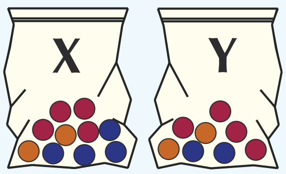
GRADE 5

This only works when the two events are independent — one event happening does not affect the chances of the other happening.

The probability of event A AND event B happening is equal to the probability of event A MULTIPLIED BY the probability of event B.

EXAMPLE:

Dave picks one ball at random from each of bags X and Y.
Find the probability that he picks a blue ball from both bags.



1) The single events you're interested in are 'picks a blue ball from bag X' and 'picks a blue ball from bag Y'.
2) Find the probabilities of the events.
   P(Dave picks a blue ball from bag X) = $\frac{4}{10}$ = 0.4
   P(Dave picks a blue ball from bag Y) = $\frac{2}{8}$ = 0.25
3) Use the AND rule.
   P(Dave picks a blue ball from bag X AND bag Y) = 0.4 × 0.25 = 0.1

The OR Rule

GRADE 5

This rule only works when the two events can't both happen at the same time.

The probability of EITHER event A OR event B happening is equal to the probability of event A ADDED TO the probability of event B.

EXAMPLE:

A spinner with red, blue, green and yellow sections is spun — the probability of it landing on each colour is shown in the table.
Find the probability of spinning either red or green.

Colour	red	blue	yellow	green
Probability	0.25	0.3	0.35	0.1

1) The single events you're interested in are 'lands on red' and 'lands on green'.
2) Write down the probabilities of the events.
   P(lands on red) = 0.25
   P(lands on green) = 0.1
3) Use the OR rule.
   P(lands on either red OR green) = 0.25 + 0.1 = 0.35

Two rules to learn here

You won't go far if you don't learn the AND/OR rules. The way to remember them is that it's the wrong way round — you'd want AND to go with '+' but it doesn't. It's 'AND with ×' and 'OR with +'.

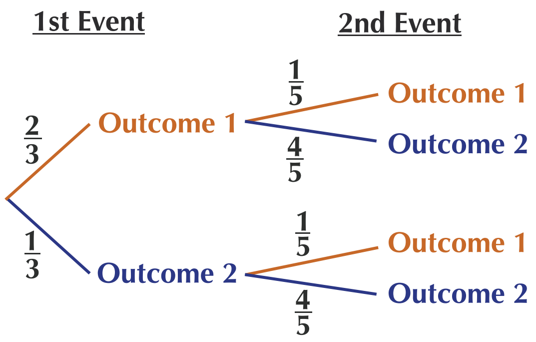
Tree Diagrams

Tree diagrams can really help you work out probabilities when you have a combination of events.

Remember These **Four Key Tree Diagram Facts**

GRADE 5

1) On any set of branches which meet at a point, the probabilities must add up to 1.



$\frac{2}{3} \times \frac{1}{5} = \frac{2}{15}$

$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$

$\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$

$\frac{1}{3} \times \frac{4}{5} = \frac{4}{15}$

Total = 1

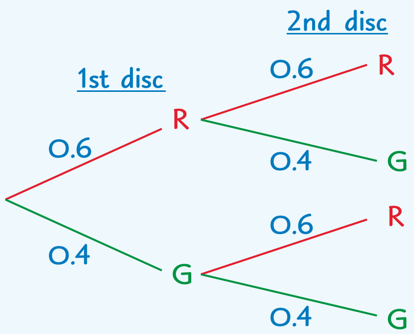
2) Multiply along the branches to get the end probabilities.

3) If you work out all the end probabilities they should add up to 1.

4) Pick the probability you need to answer the question.

EXAMPLE:

A box contains only red and green discs. A disc is taken at random and replaced. A second disc is then taken. The tree diagram below shows the probabilities of picking each colour.



a) **What is the probability that both discs are red?**

Multiply along the branches to find the probability you want:

P(both discs are red) = P(red, red) = 0.6 × 0.6 = 0.36

b) **What is the probability that you pick a green disc then a red disc?**

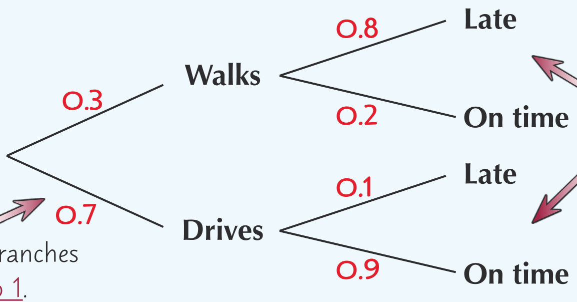
Multiply along the branches to find the probability you want:

P(green, red) = 0.4 × 0.6 = 0.24

Watch out for events that are affected by what else has happened — you'll get different probabilities on different sets of branches.

EXAMPLE:

Florence either walks or drives to work. The probability that she walks is 0.3. If she walks, the probability that she is late is 0.8. If she drives, the probability that she is late is 0.1. Complete the tree diagram below.



Fill in the empty branches so they add to 1.

The probabilities of being late or on time depend on whether she walks or drives — they are dependent events.

See how useful tree diagrams are

With probability questions that seem hard, drawing a tree diagram can be a good place to start. It takes some thinking to decide how to draw it and which bits you need, but after that it's plain sailing.

Sets and Venn Diagrams

Venn diagrams are a way of displaying sets in intersecting circles.

A Set is a Collection of Numbers or Objects

1) Sets are just collections of things — we call these 'things' elements.
2) Sets can be written in different ways but they'll always be in a pair of curly brackets {}. You can:

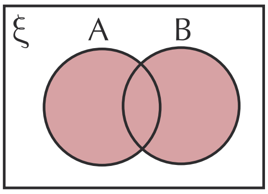
Each of these describes the same set.

- list the elements in the set, e.g. {2, 3, 5, 7}.
- give a description of the elements in the set, e.g. {prime numbers less than 10}.
- use formal notation, e.g. $\{x : x$ is a prime number less than $10\}$

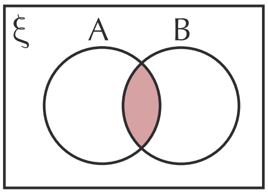
3) The symbol $\in$ means 'is a member of'. So $x \in A$ means 'x is a member of A'.
4) The universal set (ξ) is the group of things that the elements of the set are selected from.

Show Sets on Venn Diagrams

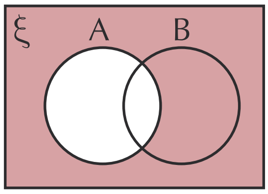
1) On a Venn diagram, each set is represented by a circle.
   The universal set is everything inside the rectangle.
2) The diagram can show either the actual elements of each set, or the number of elements in each set.



The union of sets A and B (written $A \cup B$) contains all the elements in either set A or set B — it's everything inside the circles.



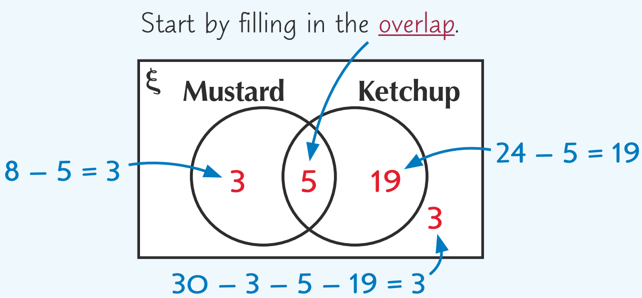
The intersection of sets A and B (written $A \cap B$) contains all the elements in both set A and set B — it's where the circles overlap.



The complement of set A (written A') contains all members of the universal set that aren't in set A — it's everything outside circle A.

EXAMPLE: **In a class of 30 pupils, 8 of them like mustard, 24 of them like ketchup and 5 of them like both mustard and ketchup.**

a) **Complete the Venn diagram below showing this information.**



b) **How many pupils like mustard or ketchup?**

This is the number of pupils in the union of the two sets. $3 + 5 + 19 = 27$

c) **What is the probability that a randomly selected pupil will like mustard and ketchup?**

5 out of 30 pupils are in the intersection. $\frac{5}{30} = \frac{1}{6}$

This is $P(M \cap K)$.

Learn what each bit of a Venn diagram represents

It's tricky to find the probability of an event by using a Venn diagram — you'll need to know what each section of the Venn diagram represents to give yourself the best chance of answering these questions.

Warm-up and Worked Exam Questions

Probability can be tricky to get your head around so it's important to get loads of practice.
Try these warm-up questions and take a look back at anything you're unsure about.

Warm-up Questions

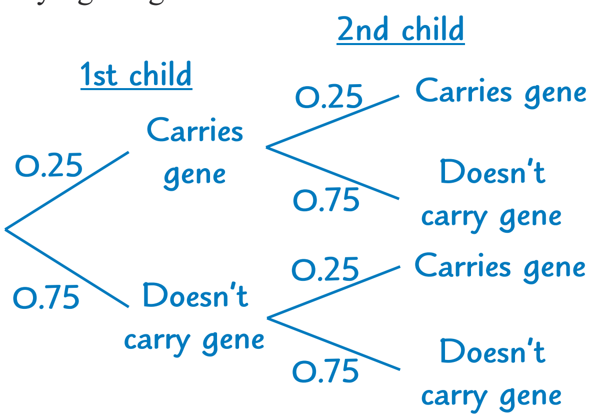
1) What is the probability of rolling a six three times in a row with a six-sided dice?
2) Dimitri is a car salesman. The probability that he sells a car on a Monday is 0.8.
   The probability that he sells a car on a Tuesday is 0.9.
   What is the probability that he sells a car on both Monday and Tuesday?
3) For the spinner on page 160, find the probability of spinning:
   a) blue OR yellow   b) blue THEN green
4) A bag contains 6 red balls and 4 black ones. If one ball is picked at random, placed back into the bag, then another ball is drawn at random, find the probability that they're both red.
5) 50 birdwatchers were looking for pigeons and seagulls. 28 of them saw a pigeon, 15 of them saw both birds and 10 of them didn't see either bird.
   a) Show this information on a Venn diagram.
   b) Find the probability that a randomly selected birdwatcher saw a seagull.

Worked Exam Question

Look through this worked exam question and make sure you understand it — don't leave it to chance.

1 A couple are both carriers of a gene that causes a disease.
If they have a child, the probability that the child will carry the gene is 0.25.

a) The couple have two children. Draw a tree diagram to show the probabilities of each child carrying or not carrying the gene.



[1 mark]

b) Find the probability that both children will carry the gene.

P(both children carry the gene) = $0.25 \times 0.25 = 0.0625$

0.0625

[2 marks]

Exam Questions

2 A biased 5-sided spinner is numbered 1-5.

The probability that the spinner will land on each of the numbers 1 to 5 is given in this table.

Number	1	2	3	4	5
Probability	0.3	0.15	0.2	0.25	0.1

a) What is the probability of the spinner landing on a 4 or a 5?

....
[2 marks]

b) The spinner is spun twice. What is the probability that it will land on a 1 on the first spin and a 3 on the second spin?

....
[2 marks]

3 Shaun is playing the game ‘hook-a-duck’. The probability that he wins a prize is 0.3.

a) What is the probability that he does not win a prize?

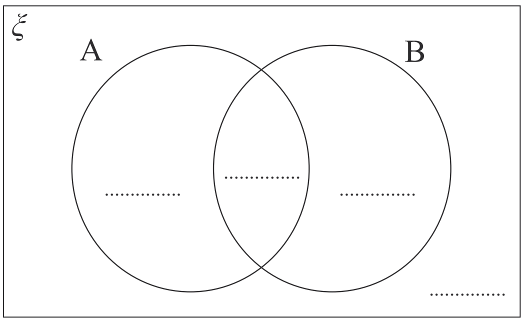
....
[1 mark]

b) If he plays two games, what is the probability that he doesn’t win a prize in either game?

....
[2 marks]

4 100 Year 7 students were asked if they like apples (A) or bananas (B).
70 like apples, 40 like bananas and 20 like apples and bananas.

a) Complete the Venn diagram below to show this information.



It’s a good idea to start by filling in the intersection.

[3 marks]

b) One of the students is selected at random.
Find P(A ∪ B).

....
[2 marks]

Sampling and Bias

Sampling is about using what you know about smaller groups to tell you about bigger groups.

Use a Sample to Find Out About a Population

GRADE 3

1) For any statistical project, you need to find out about a group of people or things. E.g. all the pupils in a school, or all the trees in a forest. This whole group is called the POPULATION.
2) Often you can't collect information about every member of the population because there are too many. So you select a smaller group from the population, called a SAMPLE, instead.
3) It's really important that your sample fairly represents the WHOLE population. This allows you to apply any conclusions about your sample to the whole population.

E.g. if you find that ¾ of the people in your sample like cheese, you can estimate that ¾ of the people in the whole population like cheese.

You Need to Spot Problems with Sampling Methods

GRADE 3

A BIASED sample (or survey) is one that doesn't properly represent the whole population.

To SPOT BIAS, you need to think about:

1) WHEN, WHERE and HOW the sample is taken.
2) HOW MANY members are in it.

If certain groups are left out of the sample, there can be BIAS in things like age, gender, or different interests.
If the sample is too small, it's also likely to be biased.

Bigger populations need bigger samples to represent them.

EXAMPLE:

Samir's school has 800 pupils. Samir is interested in whether these pupils would like to have more music lessons. For his sample he selects 5 members of the school orchestra to ask.
Explain why the opinions Samir collects from his sample might not represent the whole school.

Only members of the orchestra are included, so the opinions are likely to be biased in favour of more music lessons. And a sample of 5 is too small to represent the whole school.

If possible, the best way to AVOID BIAS is to select a large sample at random from the whole population.

Simple Random Sampling — choosing at Random

GRADE 3

One way to get a random sample is to use 'simple random sampling'.

To SELECT a SIMPLE RANDOM SAMPLE...

1) Assign a number to every member of the population.
2) Create a list of random numbers.
3) Match the random numbers to members of the population

E.g. by using a computer, calculator or picking numbers out of a bag.

Before you begin collecting data, think about your sampling method

You want to make sure that you choose a sample that represents your population.
Make sure you know how to spot poor sampling methods too.

Collecting Data

Data you collect yourself is called primary data. If you use data that someone else has collected, e.g. you get it from a website, it's called secondary data. You need to record primary data in a way that's easy to analyse and suitable for the type of data you've got.

There are **Different Types** of **Data**

GRADE 2

QUALITATIVE DATA is descriptive. It uses words, not numbers. — E.g. pets' names — Smudge, Snowy, Dave, etc. Favourite flavours of ice cream — 'vanilla', 'caramel-marshmallow-ripple', etc.

QUANTITATIVE DATA measures quantities using numbers. — E.g. heights of people, times taken to finish a race, numbers of goals scored in football matches, and so on.

There are two types of quantitative data.

DISCRETE DATA

1) It's discrete if the numbers can only take certain exact values.
2) E.g. the number of customers in a shop each day has to be a whole number — you can't have half a person.

CONTINUOUS DATA

1) If the numbers can take any value in a range, it's called continuous data.
2) E.g. heights and weights are continuous measurements.

You can **Organise** your **Data** into **Classes**

GRADE 2

1) To record data in a table, you often need to group it into classes to make it more manageable. Discrete data classes should have 'gaps' between them, e.g. '0-1 goals', '2-3 goals' (it jumps from 1 to 2 because there are no values in between). Continuous data classes should have no 'gaps', so are often written using inequalities (see p.177).
2) Whatever the data you have, make sure none of the classes overlap and that they cover all the possible values.

When you group data you lose some accuracy because you don't know the exact values any more.

EXAMPLE:

Jonty wants to find out about the ages (in whole years) of people who use his local library. Design a table he could use to collect his data.

Include columns for: the data values, 'Tally' to count the data and 'Frequency' to show the totals.

Use non-overlapping classes — with gaps because the data's discrete.

Include classes like '...or over', '...or less' or 'other' to cover all options in a sensible number of classes.

Age (whole years)	Tally	Frequency
0-19		
20-39		
40-59		
60-79		
80 or over		

Questionnaires should be **Designed Carefully**

GRADE 2

Another way to record data is to ask people to fill in a questionnaire. Your questions should be:

1) **Clear and easy to understand**
2) **Easy to answer**
3) **Fair — not leading or biased**

Watch out for response boxes that could be interpreted in different ways, that overlap, or that don't allow for all possible answers.

A question is 'leading' if it guides you to picking a particular answer.

Tables are a really good way to record data

You need to know what type of data you've got so you can record and display it in a suitable way — if you're using a table, make sure you clearly label each of the columns.

Mean, Median, Mode and Range

Mean, median, mode and range pop up all the time in exams — make sure you know what they are.

MODE = MOST common (GRADE 2)

MEDIAN = MIDDLE value (when values are in order of size)

MEAN = TOTAL of items ÷ NUMBER of items

RANGE = Difference between highest and lowest

REMEMBER:
Mode = most (emphasise the 'mo' in each when you say them)
Median = mid (emphasise the m*d in each when you say them)
Mean is just the average, but it's mean 'cos you have to work it out.

*The **Golden** Rule*

There's one vital step for finding the median that lots of people forget:

Always REARRANGE the data in ASCENDING ORDER (and check you have the same number of entries!)

You must do this when finding the median, but it's also really useful for working out the mode too.

EXAMPLE: **Find the median, mode, mean, and range of these numbers: 2, 5, 3, 2, 0, 1, 3, 3**

1. Rearrange the numbers into ascending order. The MEDIAN is the middle value.
   When there are two middle numbers, the median is halfway between the two.

   0, 1, 2, 2, 3, 3, 3, 5
   ← 4 numbers either side →
   Median = 2.5

   To find the position of the median of n values, you can use the formula (n + 1) ÷ 2. Here, (8 + 1) ÷ 2 = position 4.5 — that's halfway between the 4th and 5th values.

2. MODE (or modal value) is the most common value = 3

3. MEAN = $\frac{\text{total of items}}{\text{number of items}} = \frac{0+1+2+2+3+3+3+5}{8} = \frac{19}{8} = 2.375$

4. RANGE = difference between highest and lowest values = 5 – 0 = 5

A Trickier Example (GRADE 4)

EXAMPLE: **The heights (to the nearest cm) of 8 penguins at a zoo are 41, 43, 44, 44, 47, 48, 50 and 51. Two of the penguins are moved to a different zoo. If the mean height of the remaining penguins is 44.5 cm, find the heights of the two penguins that moved.**

Mean = $\frac{\text{total height}}{\text{no. of penguins}}$

So total height = no. of penguins × mean

Total height of 8 penguins = 368 cm.
Total height of remaining 6 penguins = 6 × 44.5 = 267 cm.
Combined height of penguins that moved = 368 – 267 = 101 cm.
So the heights must be 50 cm and 51 cm.

Mean, median, mode, range — easy marks for learning 4 definitions

The maths involved in working these out is simple, so you'd be mad not to learn the definitions.

Warm-up and Worked Exam Questions

By the time the big day comes you need to know all the facts in these warm-up questions — and how to use them to answer exam questions. It's not easy, but it's the only way to get good marks.

Warm-up Questions

1) An investigation into the average number of people in households in Britain was done by surveying 100 households in one city centre.
   Give two reasons why this is a poor sampling technique.
2) James asks some students how many times they went to the cinema in the last year.
   Say whether this data is discrete or continuous and design a table to record it in.
3) Give one criticism of each of these questions:
   a) Do you watch a lot of television?
   b) Do you agree that maths is the most important subject taught in schools?
   c) What is your favourite drink? Answer A, B or C. A) Tea B) Milk C) Coffee
4) Find the mean, median, mode and range for this set of data:
   1, 3, 14, –5, 6, –12, 18, 7, 23, 10, –5, –14, 0, 25, 8
5) Three different numbers have a mean of 4 and a range of 4. What are the numbers?

Worked Exam Question

There's no better preparation for exam questions than doing, err... practice exam questions.
Hang on, what's this I see...

1 Lee has 6 pygmy goats. Their weights, in kg, are listed below.

32 23 31 28 36 26

a) Which three weights from the list above would have a range which is half the value of the median of the three weights? Also write the range and median of the three weights. GRADE 3

Use trial and error to find the three weights that work — 23, 26 and 36 have a range of 36 – 23 = 13 and a median of 26.

........23........, ...26........, ...36........

range = ...13........, median = ...26........
[2 marks]

b) Two of the goats wander off and don't return. The mean weight of the herd is now 27.25 kg. Find the weights of the two goats who wandered off. GRADE 4

Total weight of all 6 goats = 32 + 23 + 31 + 28 + 36 + 26 = 176 kg

Weight of 4 remaining goats = 4 × 27.25 = 109 kg

The goats that wondered off weigh: 176 kg – 109 kg = 67 kg

Find the total weight of the goats that wandered off.

so, goats weighing 31 kg and 36 kg

Use trial and error to find which two weights add up to 67 kg.

........31........ kg and ....36........ kg
[3 marks]

Exam Questions

2 Faye is investigating how many chocolate bars teenagers buy each week.
She is going to collect data by asking her teenage friends how many they buy.

a) Design a table that Faye could use to record her data.

[2 marks]

b) Comment on whether she can use her results to draw conclusions about teenagers in the UK.

....

....
[2 marks]

3 A bakery records the number of cookies it sells each day for ten days.
The mean number is 17 and the median number is 15.

The next day the bakery sells 18 cookies.

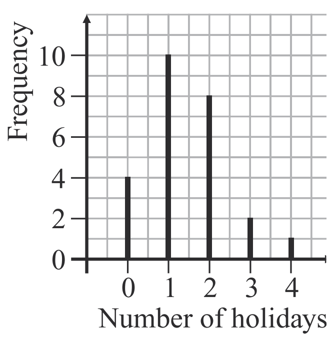
a) Is the mean number sold over all eleven days higher than 17? Explain your answer.

....
[1 mark]

b) Is the median number sold over all eleven days higher than 15? Explain your answer.

....
[1 mark]

4 25 people were asked how many holidays they went on last year.
The vertical line graph below shows the results.



a) Write down the modal number of holidays.

....
[1 mark]

b) Find the median number of holidays.

Imagine the data in a list — 0, 0, 0, 0, 1...
Find the position of the median and count
up through the bars till you get there.

....
[2 marks]

Simple Charts and Graphs

Pictograms and bar charts both show frequencies. (Remember... frequency = 'how many of something'.)

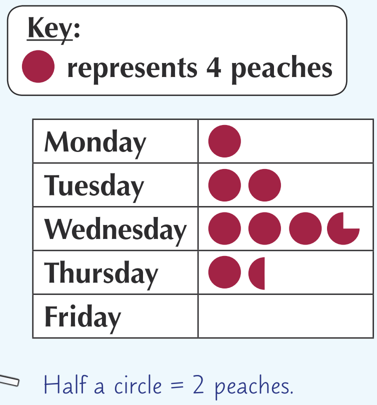
Pictograms Show Frequencies Using Symbols

GRADE 1

Every pictogram has a key telling you what one symbol represents.

With pictograms, you MUST use the KEY.

EXAMPLE: **This pictogram shows how many peaches were sold by a greengrocer on different days.**



a) **How many peaches were sold on Tuesday?**
Each circle represents 4 peaches. $2 \times 4 = 8$ peaches

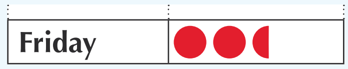
b) **How many peaches were sold on Wednesday?**
Three-quarters of a circle = 3 peaches. $(3 \times 4) + 3 = 15$ peaches

c) **43 peaches were sold altogether. Work out how many peaches were sold on Friday and show this information on the diagram.**

1) Add up the peaches for Mon-Thurs. $4 + 8 + 15 + (4 + 2) = 33$

2) Subtract this from the total. Friday $= 43 - 33 = 10$

3) You need 2 whole circles (= 8), plus another half a circle (= 2).

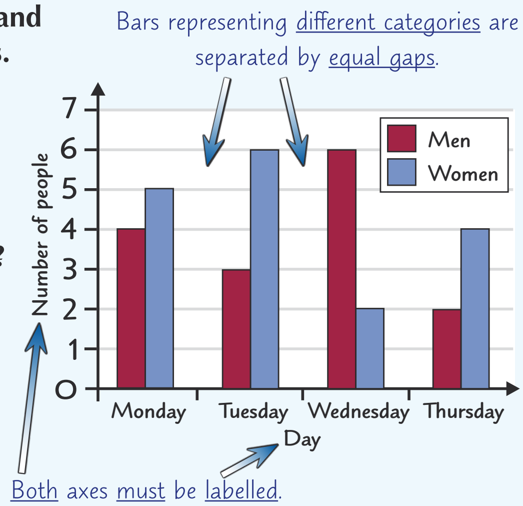


Bar Charts Show Frequencies Using Bars

GRADE 1

1) Frequencies on bar charts are shown by the heights of the different bars.
2) Dual bar charts show two things at once — they're good for comparing different sets of data.

EXAMPLE: **This dual bar chart shows the number of men and women visiting a coffee shop on different days.**



a) **How many men visited the coffee shop altogether?**
Add up the numbers shown by the heights of the dark red bars. $4 + 3 + 6 + 2 = 15$ men

b) **On which day did the most women visit the coffee shop?**
Find the tallest purple bar. Tuesday

c) **On which day was there the biggest difference between the numbers of men and women?**
Count up from the smaller to the larger bar for each day to find the differences. Wednesday

Pictograms and bar charts are good for comparing data

The most important part of a pictogram is the key — without it you won't be able to tell what the symbols represent. When you're drawing a bar chart, make sure both axes are clearly labelled.

Simple Charts and Graphs

Here are two-way tables, stem and leaf diagrams and line graphs. Make sure you can draw and interpret them.

Two-Way Tables Show How Many in each Category

GRADE 2

EXAMPLE: **This table shows the number of cakes and pies a bakery sold on Friday and Saturday.**

	Cakes	Pies	Total
Friday	12	10	
Saturday	4	14	18
Total	16	24	40

a) **How many pies were sold on Saturday?**

Read across to 'Pies' and down to 'Saturday'. 14 pies

b) **How many items were sold in total on Friday?**

Add the number of cakes for Friday to the number of pies. 12 + 10 = 22

Or you could subtract the total for Saturday from the overall total: 40 – 18 = 22.

Stem and Leaf Diagrams put data in Order

GRADE 2

An ordered stem and leaf diagram shows a set of data in order of size.
This makes it easy to find things like the median and range (see p.167).

EXAMPLE: **This stem and leaf diagram shows the ages of some school teachers.**

3	3	5		
4	0	5	7	8
5	1	4	9	
6	1	3		

Key: 5 | 4 = 54 years

a) **How old is the oldest teacher?**

Use the key to help you read off the diagram. 6 | 3 = 63 years old

b) **What is the median age?**

The median is the middle value.
Find its position, then read off the value.

There are 11 values, so the median is the 6th value.
So median age is 4 | 8 = 48 years

Line Graphs can show Time Series

GRADE 3

1) A time series is when you measure the same thing at different times. A line graph of the data has 'time' along the bottom and the thing being measured down the side.
2) A basic pattern often repeats itself — this is called seasonality (but it doesn't have to match the seasons).
3) You can also see the overall trend by looking at the peaks and troughs.

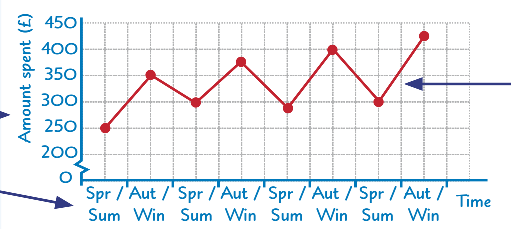
EXAMPLE: **The table shows how much Jamie spent on heating during Spring/Summer and Autumn/Winter over 4 years.**

6-month period	Spr / Sum	Aut / Win	Spr / Sum	Aut / Win	Spr / Sum	Aut / Win	Spr / Sum	Aut / Win
Amount spent (£)	250	350	300	375	290	400	300	425

a) **Draw a line graph of this data on the grid opposite.**

Put 'Time' along the bottom and 'Amount spent' down the side.

Label your axes at equal intervals.



Plot the points and join them up with straight lines.
The pattern repeats itself every 2 points.

b) **Describe the trend in the data.**

Look at the trend in the peaks. The data shows an upward trend in the amount Jamie spent.

Time series often show a repeating pattern

To see the overall trend of a time series graph you have to ignore any repeating pattern and just look at the peaks and troughs of the graph. A trend can be upward, downward or there can be no trend at all.

Pie Charts

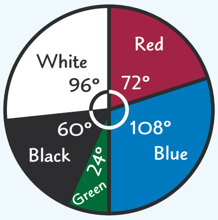
Unlike other charts, pie charts DON'T tell you numbers of things, they show the proportion in each category. Remember that. And here's another thing to remember... the Golden Pie Chart Rule...

The TOTAL of Everything = 360°

1) Fraction of the Total = Angle ÷ 360°

GRADE 3

EXAMPLE:



This pie chart shows the colour of all the cars sold by a dealer. What fraction of the cars were red?

Fraction of red cars $= \dfrac{\text{angle of red cars}}{\text{angle of everything}} = \dfrac{72^\circ}{360^\circ} = \dfrac{1}{5}$

2) Find a Multiplier to Calculate Your Angles

GRADE 3

EXAMPLE:

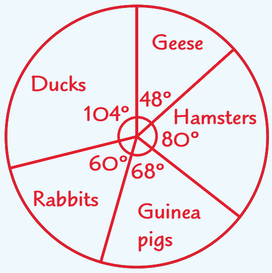
Draw a pie chart to show this information about the types of animal in a petting zoo.

Animal	Geese	Hamsters	Guinea pigs	Rabbits	Ducks
Number	12	20	17	15	26

1) Find the total by adding. $12 + 20 + 17 + 15 + 26 = 90$
2) 'Everything = 360°' — so find the multiplier that turns your total into 360°. Multiplier $= 360 \div 90 = 4$
3) Multiply every number by 4 to get the angle for each sector.

Angle	$12 \times 4 = 48°$	$20 \times 4 = 80°$	$17 \times 4 = 68°$	$15 \times 4 = 60°$	$26 \times 4 = 104°$

4) Draw your pie chart accurately using a protractor.

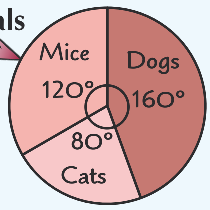


3) Find How Many by Using the Angle for 1 Thing

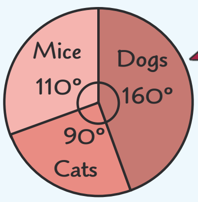
GRADE 3

EXAMPLE:

The pie chart on the right shows information about the types of animals liked most by different students. There were 90 students altogether.



a) **Work out the number of students who liked dogs most.**
   1) 'Everything = 360°', so... → 90 students = 360°
   2) Divide by 90 to find... → 1 student = 4°
   3) Divide the angle for dogs by the angle for 1 student to get: → $160° \div 4° = 40$ — 40 students liked dogs most



b) **The pie chart on the left shows information about the types of animals liked most by a different group of students. Dave says, "This means that 40 students in this group like dogs most." Explain why Dave is not correct.**
   We don't know how many students in total the pie chart represents, so we can't work out how many students liked dogs most.

Pie charts are all about angles

The thing to remember is that the angles of each sector in a pie chart will add up to 360°.
That's the key to all pie-chart questions — from working out numbers to drawing your own chart.

Scatter Graphs

A scatter graph tells you how closely two things are related — the fancy word is CORRELATION.

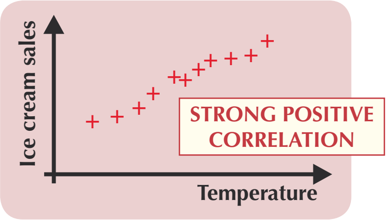
Scatter Graphs Show Correlation

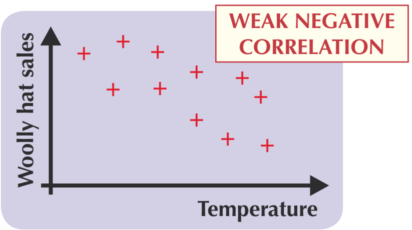
GRADE 3

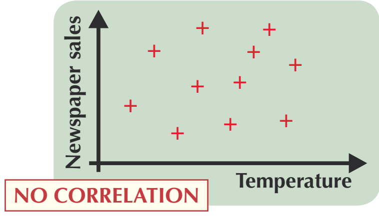
1) If you can draw a line of best fit pretty close to most of your data points, the two things are correlated. If the points are randomly scattered, and you can't draw a line of best fit, then there's no correlation.

2) Strong correlation is when your points make a fairly straight line — the two things are closely related. Weak correlation is when your points don't line up so nicely, but you can still draw a line of best fit.

3) If the points form a line sloping uphill from left to right, then there is positive correlation. If the line slopes downhill from left to right, then there is negative correlation.







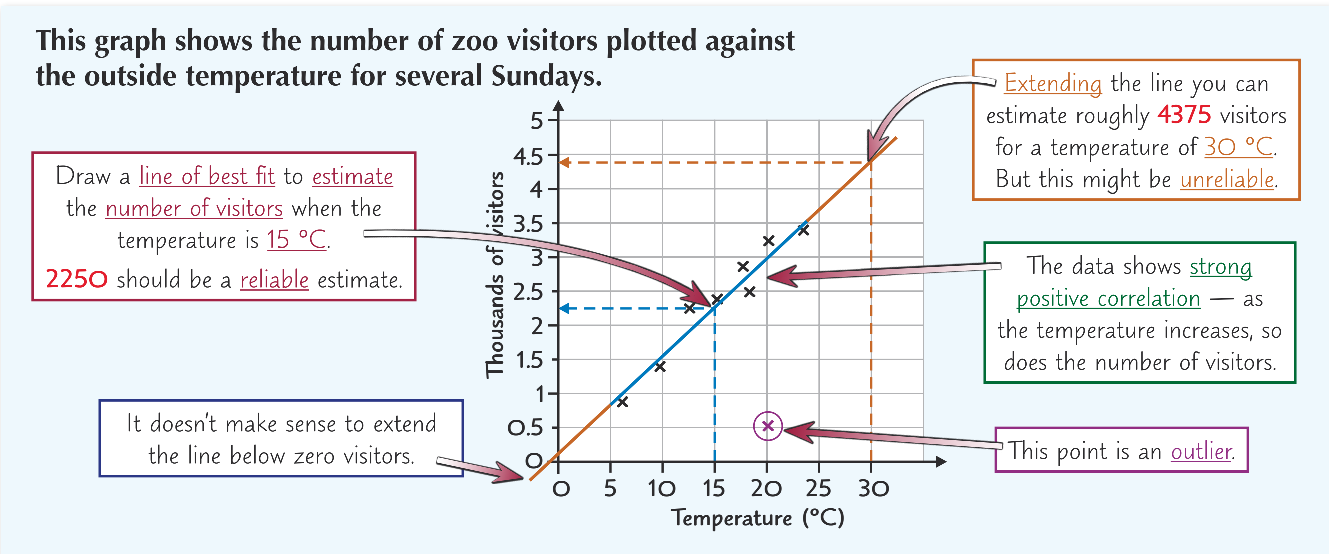
Use a Line of Best Fit to Make Predictions

GRADE 4

1) Predicting a value within the range of data you have should be fairly reliable. But if you extend your line outside the range of data your prediction might be unreliable, because the pattern might not continue.

2) Also watch out for outliers — data points that don't fit the general pattern. Outliers can drag your line of best fit away from the other values, so it's best to ignore them when you're drawing the line.

This graph shows the number of zoo visitors plotted against the outside temperature for several Sundays.



BE CAREFUL with correlation — if two things are correlated it doesn't mean that one causes the other. There could be a third factor affecting both, or it could just be a coincidence.

Correlation means that there's a relationship between two things

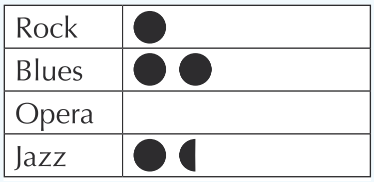
Make sure you know all the terms in case you have to describe a correlation — positive correlation, negative correlation, and how to say how strong the correlation is.

Warm-up and Worked Exam Questions

There's a nice selection of warm-up questions here covering tables, charts and graphs. Now's the time to go back over any bits you're not sure of — in the exam it'll be too late.

Warm-up Questions

1) This pictogram shows the different types of CDs Javier owns, but the key is missing. Javier owns 20 blues CDs.
   a) How many jazz CDs does Javier own?
   b) He owns 5 opera CDs. Complete the pictogram.



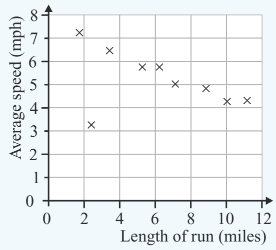
2) Complete the two-way table below showing how a group of students get to school.

	Walk	Car	Bus	Total
Male	15	21		
Female			22	51
Total	33	32		100

3) Work out the angles of the sectors for each type of DVD if a pie chart were to be created from the table on the right.

Type of DVD	Number
Rom Com	23
Western	25
Action	12

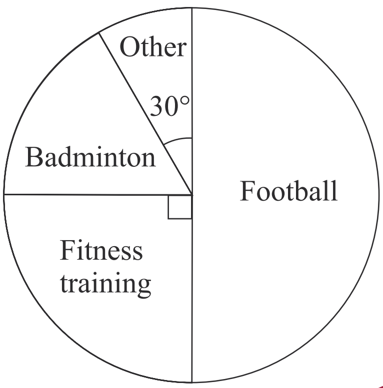
4) This graph shows Sam's average speed on runs of different lengths.
   a) Describe the relationship between length of run and average speed.
   b) Circle the point that doesn't follow the trend.
   c) Estimate Sam's average speed for an 8-mile run.
   d) Comment on the reliability of your estimate in part c).



Worked Exam Question

It's no good learning all the facts in the world if you go to pieces or write nonsense in the exam. This exam question shows how to turn those facts into good answers — and earn yourself marks.

1 A survey was carried out at a leisure centre to find out which sport people prefer to do. The results are shown in the pie chart. GRADE 3



a) What fraction of people prefer to do fitness training?

$90° = \frac{1}{4}$ of a circle

$\frac{1}{4}$

[1 mark]

60 people said they prefer to play football.

b) How many people prefer to play badminton?

Badminton = 360° – 180° – 90° – 30° = 60°

Football = 180°, so 60 people = 180°

1 person = 180° ÷ 60 = 3°

Find the angle that represents 1 person.

Number of people who prefer badminton = 60° ÷ 3°
= 20

Divide the angle of 'badminton' by the angle for '1 person'.

20

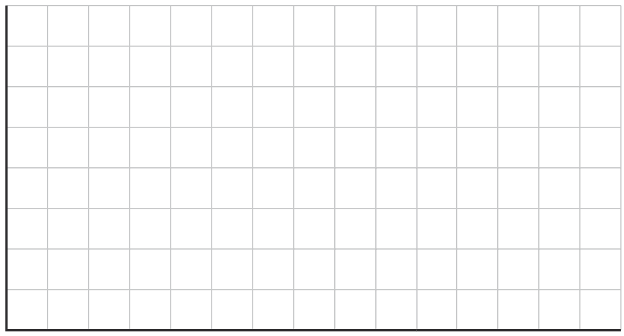
[2 marks]

Exam Questions

2 This table shows some information about the favourite sports of some students.

Sport	Students
Football	14
Swimming	5
Athletics	9
Netball	1
Hockey	6

Show this information as a bar chart on the grid below.



[4 marks]

3 50 people were asked if they've ever been skiing. The table on the right shows the results.

	Have been skiing	Have not been skiing
Male	15	20
Female	5	10

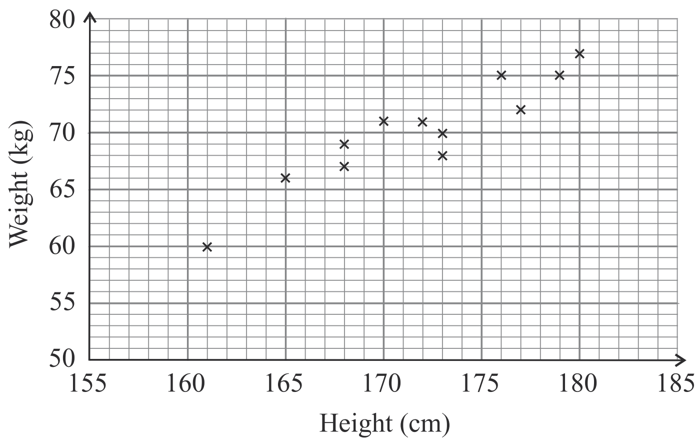
a) Write down the number of males who have been skiing to the number of males who have not been skiing as a ratio in its simplest form.

....
[1 mark]

b) What percentage of all the people asked have been skiing?

.... %
[2 marks]

4 The scatter graph below shows the heights and weights of boys playing in a rugby team.



Two more boys join the team. Their heights and weights are shown in this table.

Player	Height (cm)	Weight (kg)
13	169	70
14	183	76

a) Add the information in the table to the scatter graph.

[1 mark]

b) What fraction of the players have a height of less than 170 cm?

....
[1 mark]

c) Describe the relationship between the height and weight of the players.

....
[1 mark]

Frequency Tables — Finding Averages

You saw how to find averages and range on p.167 — the same applies here, but with the data in a table.

Find Averages from Frequency Tables

GRADE 4

1) The MODE is just the CATEGORY with the MOST ENTRIES.
2) The RANGE is found from the extremes of the first column.
3) The MEDIAN is the CATEGORY of the middle value.
4) To find the MEAN, you have to WORK OUT A THIRD COLUMN yourself.

The MEAN is then: **3rd Column Total ÷ 2nd Column Total**

Categories — How many

Number of cats	Frequency	
0	17	
1	22	
2	15	
3	7	

Mysterious 3rd column...

EXAMPLE:

Some people were asked how many sisters they have. The table opposite shows the results. Find the mode, the range, the mean and the median of the data.

Number of sisters	Frequency
0	7
1	15
2	12
3	8
4	4
5	0

1. The MODE is the category with the most entries — i.e. the one with the highest frequency:

The highest frequency is 15 for '1 sister', so MODE = 1

2. The RANGE is the difference between the highest and lowest numbers of sisters — that's 4 sisters (no one has 5 sisters) and no sisters, so:

RANGE = 4 – 0 = 4

3. To find the MEAN, add a 3rd column to the table showing 'number of sisters × frequency'. Add up these values to find the total number of sisters of all the people asked.

Number of sisters	Frequency	No. of sisters × Frequency
0	7	0
1	15	15
2	12	24
3	8	24
4	4	16
5	0	0
Total	46	79

$$\text{MEAN} = \frac{\text{total number of sisters}}{\text{total number of people asked}} = \frac{79}{46} = 1.72 \text{ (3 s.f.)}$$

3rd column total

2nd column total

4. The MEDIAN is the category of the middle value. Work out its position, then count through the 2nd column to find it.

It helps to imagine the data set out in an ordered list:

0000000111111111111111222222222222333333334444

median

The median is in position $(n + 1) \div 2 = (46 + 1) \div 2 = 23.5$ — halfway between the 23rd and 24th values. There are a total of $(7 + 15) = 22$ values in the first two categories, and another 12 in the third category takes you to 34. So the 23rd and 24th values must both be in the category '2 sisters', which means the MEDIAN is 2.

Remember — mode is most, median is middle and mean is average

When you're finding the mean, add a third column to the frequency table showing number × frequency. Find the total of this column, then divide it by the total frequency to get the mean.

Grouped Frequency Tables

Grouped frequency tables are like ordinary frequency tables, but they group the data into classes.

GRADE 4

NO GAPS between classes for CONTINUOUS data.

Use inequality symbols to cover all possible values.

Height (h millimetres)	Frequency
$5 < h \leq 10$	12
$10 < h \leq 15$	15

See p.166 for grouped discrete data.

To find MID-INTERVAL VALUES:

- Add together the end values of the class and divide by 2.
- E.g. $(5 + 10) \div 2 = 7.5$

Find Averages from Grouped Frequency Tables

GRADE 5

Unlike with ordinary frequency tables, you don't know the actual data values, only the classes they're in. So you have to ESTIMATE THE MEAN, rather than calculate it exactly — you do this by adding columns:

1) **Add a 3RD COLUMN and enter the MID-INTERVAL VALUE for each class.**
2) **Add a 4TH COLUMN to show 'FREQUENCY × MID-INTERVAL VALUE' for each class.**

You'll be asked to find the MODAL CLASS and the CLASS CONTAINING THE MEDIAN, not exact values. And the RANGE can only be estimated too — using the class boundaries.

EXAMPLE:

This table shows information about the weights, in kilograms, of 60 school children.

a) Write down the modal class.
b) Write down the class containing the median.
c) Calculate an estimate for the mean weight.
d) Estimate the range of weights.

Weight (w kg)	Frequency
$30 < w \leq 40$	8
$40 < w \leq 50$	16
$50 < w \leq 60$	18
$60 < w \leq 70$	12
$70 < w \leq 80$	6

a) The modal class is the one with the highest frequency.

Modal class is $50 < w \leq 60$

b) Work out the position of the median, then count through the 2nd column.

The median is in position $(n + 1) \div 2 = (60 + 1) \div 2 = 30.5$, halfway between the 30th and 31st values. Both these values are in the third class, so the class containing the median is $50 < w \leq 60$.

c) Add extra columns for 'mid-interval value' and 'frequency × mid-interval value'. Add up the values in the 4th column to estimate the total weight of the 60 children.

Weight (w kg)	Frequency	Mid-interval value	Frequency × mid-interval value
$30 < w \leq 40$	8	35	280
$40 < w \leq 50$	16	45	720
$50 < w \leq 60$	18	55	990
$60 < w \leq 70$	12	65	780
$70 < w \leq 80$	6	75	450
Total	60	—	3220

$$\text{Mean} \approx \frac{\text{total weight}}{\text{number of children}}$$

total weight ← 4th column total

number of children ← 2nd column total

$$= \frac{3220}{60} = 53.7 \text{ kg (3 s.f.)}$$

d) Find the difference between the highest and lowest class boundaries.

Estimated range $= 80 - 30 = 50$ kg

This is the largest possible range. The actual range is likely to be smaller, but you can't tell with grouped data.

This time there are two columns to add

With frequency tables there was just one column to add, with grouped frequency tables there are two. It's still easy enough though, as long as you remember what the columns are and how to find them.

Interpreting Data

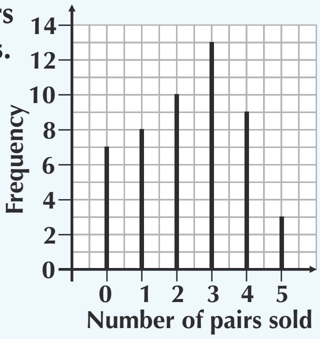
This page is about getting information from data and recognising when it might be misleading.

You can **Find Averages** from **Diagrams**

GRADE 4

EXAMPLE:

This vertical line graph shows information on the number of pairs of penguin slippers a shop sells each day for 50 consecutive days. Calculate the mean number of pairs sold each day.



Fill in a frequency table and add a third column to find the total number of pairs sold — see p.176.

Number of pairs	Frequency	No. of pairs × Frequency
0	7	0
1	8	8
2	10	20
3	13	39
4	9	36
5	3	15
Total	50	118

$$\text{Mean} = \frac{\text{total number of pairs sold}}{\text{total number of days}} = \frac{118}{50} = 2.36$$

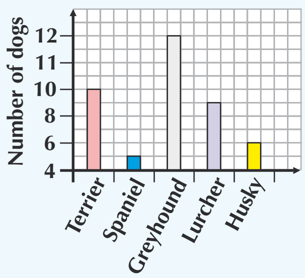
Watch Out for **Misleading Diagrams**

GRADE 3

At first glance, a diagram might look perfectly fine. But at second glance, well, not so fine...

EXAMPLE:

This bar chart shows the numbers of dogs of different breeds at a rescue centre.



a) **Write down three things that are wrong with the bar chart.**

1) The 'number of dogs' axis doesn't start at zero.
2) The 'number of dogs' axis has inconsistent numbering.
3) The 'breed of dog' axis has no label.

b) **The 'Husky' bar is twice as high as the 'Spaniel' bar. Explain why these bar heights could be misleading in the context of this data.**

The bar heights suggest that there are twice as many Huskies as Spaniels. But reading the scale, there are 6 Huskies and 5 Spaniels.

Be Careful with Measures of **Average** and **Range**

GRADE 4

Outliers are data values that don't fit the general pattern — they're a long way from the rest of the data. Outliers can have a big effect on the mean or range of a data set, so you get a misleading value.

EXAMPLE:

The data below shows the number of songs Fred downloads each week for ten weeks.

0, 1, 3, 3, 5, 6, 7, 8, 8, 20

See p.167 for averages and range.

a) **Fred works out that the range of his data is 20. Comment on this value as a measure of the spread.**

A range of 20 isn't a true reflection of the spread of the whole data set, because most of the data is much closer together. The highest value of 20 has a big effect on increasing the range.

b) **Explain why the mode isn't a helpful measure of average for this data.**

The data has two modes, 3 and 8, so this doesn't give you a good idea of the average value.

Comparing Data Sets

You can compare data sets using averages and range, or by drawing suitable diagrams.

Compare Data Sets Using Averages and Range (Grade 4)

Say which data set has the higher/lower value and what that means in the context of the data.

EXAMPLE:

Some children take part in a 'guess the weight of the baby hippo' competition. Here is some information about the weights they guess.

Compare the distributions of the weights guessed by the boys and the girls.

Boys:	Girls:
Mean = 40 kg	Mean = 34 kg
Median = 43 kg	Median = 33 kg
Range = 42 kg	Range = 30 kg

1) Compare averages:
The boys' mean and median values are higher than the girls', so the boys generally guessed heavier weights.

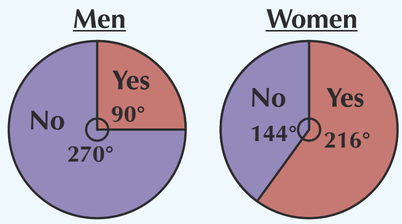
2) Compare ranges:
The boys' guesses have a bigger range, so the weights guessed by the boys show more variation.

Compare Data Sets Using Diagrams (Grade 4)

The type of diagram you should use depends on what you want to show.

EXAMPLE:

Harry carried out a survey into whether or not people like olives. He draws these pie charts to show his results.



a) **Can you tell from the pie charts whether more women said 'yes' than men? Explain your answer.**

No, you can't tell whether more women said 'yes'. You can see that a higher proportion of women said 'yes' but you don't know how many men and women the pie charts represent.

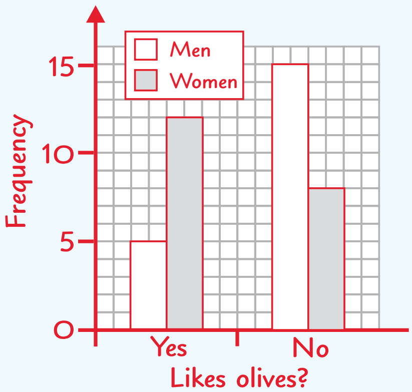
b) **Harry surveyed 20 men and 20 women. Draw a suitable diagram to compare the numbers of men and women giving each answer.**

A dual bar chart is suitable — it shows the numbers of men and women side by side.

Use the pie charts to work out the frequency of each answer. E.g. find the fraction of the total, then multiply by 20.

Men: 'Yes' = $\frac{90}{360} \times 20 = 5$, 'No' = $\frac{270}{360} \times 20 = 15$

Women: 'Yes' = $\frac{216}{360} \times 20 = 12$, 'No' = $\frac{144}{360} \times 20 = 8$



You could be asked to compare any type of chart or diagram

Make sure you're comfortable with all the different types of charts and diagrams. In the exam you might have to compare two of the same type of diagram or two different types of diagram. Sometimes you'll need to work out the values of the averages and range (see p.167) before comparing data sets.

Warm-up and Worked Exam Questions

Think of the exam as a big race. In order to do your best, you should really warm up first. So give these questions a go and get your brain all ready for the big day.

Warm-up Questions

1) 50 people were asked how many times a week they play sport. The table to the right shows the results.
   a) Find the median. b) Find the mode.

No. of times sport played	Frequency
0	8
1	15
2	17
3	6
4	4
5 or more	0

2) Here are the heights of some adults to the nearest 0.1 cm. Design and fill in a grouped frequency table to record the data.
   150.4 163.5 156.7 164.1
   182.8 175.4 171.2 169.0
   173.3 185.6 167.0 162.6

3) The grouped frequency table below represents data from 79 random people.

Height (cm)	$145 \le x < 155$	$155 \le x < 165$	$165 \le x < 175$	$175 \le x < 185$
Frequency	18	22	24	15

   a) Estimate the range of heights. b) Which group contains the median?
   c) State the modal group.

4) The following temperatures were recorded in a city at eight times during a year:
   12°C, 30°C, 30°C, 8°C, 17°C, 11°C, 4°C, 15°C
   Explain why the mode is not a good average to use for this data.

Worked Exam Question

This worked exam question is just like one that could come up in the exam.
But the one in the exam won't have the answers filled in so make the most of it now.

1 The table shows the times it took 32 pupils at a school to run a 200 m sprint. GRADE 5

a) Calculate an estimate for the mean time taken.

Time (t seconds)	Frequency	Mid-interval value	Frequency × Mid-interval value
$22 < t \le 26$	4	24	4 × 24 = 96
$26 < t \le 30$	8	28	8 × 28 = 224
$30 < t \le 34$	13	32	13 × 32 = 416
$34 < t \le 38$	6	36	6 × 36 = 216
$38 < t \le 42$	1	40	1 × 40 = 40
Total:	32	Total:	992

Estimate of mean = 992 ÷ 32 = 31

31 seconds
[4 marks]

All pupils with a time of 34 seconds or less qualified for the next round.

b) Anya says that fewer than 20% of the pupils failed to qualify for the next round. Comment on Anya's statement and show working to support your answer.

6 + 1 = 7 people failed to qualify, 7 out of 32 = 7 ÷ 32 × 100 = 21.875%

Over 20% of pupils failed to qualify, so Anya's statement is incorrect.

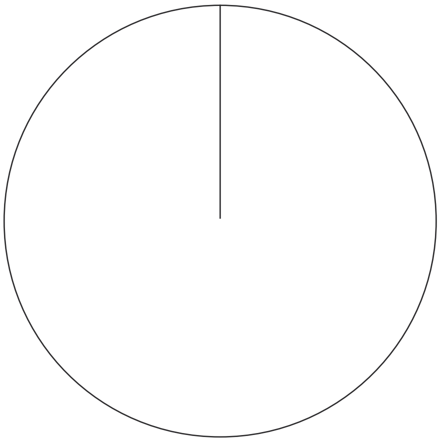
[2 marks]

Exam Questions

2 A survey was carried out in a local cinema to find out which flavour of popcorn people bought. The results are in the table below.

a) Draw and label a pie chart to represent the information.

Type of popcorn	Number sold
Plain	12
Salted	18
Sugared	9
Toffee	21

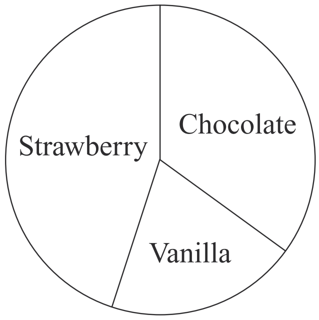


[4 marks]

Another survey was carried out to find out which flavour of ice cream people bought. The results are shown in the pie chart on the right.

Chris compares the two pie charts and says, "The results show that more people chose strawberry ice cream than toffee popcorn."

b) Explain whether or not Chris is right.



....

....

....

[1 mark]

3 This data shows the amount of rainfall in mm that fell on an island during a 12-day period in June.

a) Work out the range of the rainfall and comment on this value as a measure of the spread of the data.

0	8
1	7 9
2	3 6 9
3	0 1 4 7 8
6	3

Key
0 | 8 means
8 mm of rain

Don't be put off by the way the data is displayed — look at the key to work out how to read off the values.

....

....

[3 marks]

In November the median amount of rainfall was 22 mm and the range was 20 mm.

b) Compare the rainfall in June with the rainfall in November.

....

....

....

[3 marks]

Revision Questions for Section Seven

Here's the inevitable list of straight-down-the-middle questions to test how much you know.

- Have a go at each question... but only tick it off when you can get it right without cheating.
- And when you think you could handle pretty much any statistics question, tick off the whole topic.

Probability (p153-156)

1) I pick a random number between 1 and 50.
   Find the probability that my number is a multiple of 6.
2) The probability of a spinner landing on red is 0.3. What is the probability it doesn't land on red?
3) I flip a fair coin 3 times. a) Using H for heads and T for tails list all the possible outcomes.
   b) What is the probability of getting exactly one head?
4) What are the formulas for: a) relative frequency? b) expected frequency?
5) 160 people took a 2-part test. 105 people passed the first part and of these, 60 people passed the second part. 25 people didn't pass either test.
   a) Show this information on a frequency tree. b) Find the relative frequency of each outcome.
   c) If 300 more people do the test, estimate how many of them would pass both parts.

Harder Probability and Venn Diagrams (p160-162)

6) The table shows the probabilities of a biased dice landing on each number. What is the probability of it landing on 1 or 4?

Number	1	2	3	4	5	6
Probability	0.2	0.15	0.1	0.3	0.15	0.1

7) I have a standard pack of 52 playing cards. Use a tree diagram to find the probability of me picking two cards at random and getting no hearts if the first card is replaced.
8) 100 people were asked whether they like tea or coffee. Half the people said they like coffee, 34 people said they like tea, 20 people said they like both.
   a) Show this information on a Venn diagram.
   b) If one of the 100 people is randomly chosen, find the probability of them liking tea or coffee.

Collecting Data and Finding Averages (p165-167)

9) What is a sample and why does it need to be representative?
10) Is 'eye colour' descriptive, discrete or continuous data?
11) Complete this frequency table for the data below.

Pet	Tally	Frequency

Cat, Cat, Dog, Dog, Dog, Rabbit, Fish, Cat, Rabbit, Rabbit, Dog, Dog, Cat, Cat, Dog, Rabbit, Cat, Fish, Cat, Cat

12) Find the mode, median, mean and range of this data: 2, 8, 11, 15, 22, 24, 27, 30, 31, 31, 41

Graphs and Charts (p170-173)

13) How do you find frequencies from a pictogram?
14) The table opposite shows how some students rated a film. Draw a suitable diagram to show the number of students giving each rating.

Film rating	Terrible	Bad	OK	Good	Amazing
Students	40	30	40	45	25

15) a) Draw a line graph to show the time series data in this table.
    b) Describe the repeating pattern in the data.

Quarter	1	2	3	4	1	2	3	4
Sales (1000's)	1	1.5	1.7	3	0.7	0.9	1.2	2.2

16) Use data from Q14 to draw a suitable diagram showing the proportion of students giving each rating.
17) Sketch graphs to show: a) weak positive correlation, b) strong negative correlation, c) no correlation

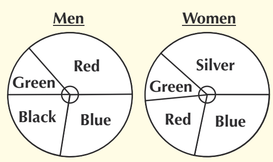
Frequency Tables and Averages (p176-177)

18) For this grouped frequency table showing the lengths of some pet alligators:
    a) find the modal class, b) find the class containing the median,
    c) estimate the mean.

Length (y, in m)	Frequency
$1.4 \le y < 1.5$	4
$1.5 \le y < 1.6$	8
$1.6 \le y < 1.7$	5
$1.7 \le y < 1.8$	2

Interpreting and Comparing Data Sets (p178-179)

19) Explain the effect that outliers can have on the mean and range of data.
20) These pie charts show the results of a survey on the colour of people's cars. Compare the popularity of each colour of car amongst men and women.



Practice Paper 1: Non-calculator
As final preparation for the exams, we've included three full practice papers to really put your Maths skills to the test. Paper 1 is a non-calculator paper — Paper 2 and Paper 3 (on pages 196 and 209) require a calculator. There's a whole page on formulas on p.244. Good luck…

Candidate Surname	Candidate Forename(s)

Centre Number	Candidate Number	Candidate Signature

GCSE

Mathematics Foundation Tier

Paper 1 (Non-Calculator)

Practice Paper

Time allowed: 1 hour 30 minutes

You must have:
Pen, pencil, eraser, ruler, protractor, pair of compasses.
You may use tracing paper.

You are **not allowed** to use a calculator.

Instructions to candidates

- Use **black** ink to write your answers.
- Write your name and other details in the spaces provided above.
- Answer **all** questions in the spaces provided.
- In calculations, show clearly how you worked out your answers.
- Do all rough work on the paper.

Information for candidates

- The marks available are given in brackets at the end of each question.
- You may get marks for method, even if your answer is incorrect.
- There are 26 questions in this paper. There are no blank pages.
- There are 80 marks available for this paper.

Answer ALL the questions.

Write your answers in the spaces provided.

You must show all of your working.

1 Write 0.113 as a fraction.
Circle your answer.

$\frac{113}{100}$ $\frac{113}{10\,000}$ $\frac{113}{1000}$ $\frac{13}{100}$

[Total 1 mark]

---

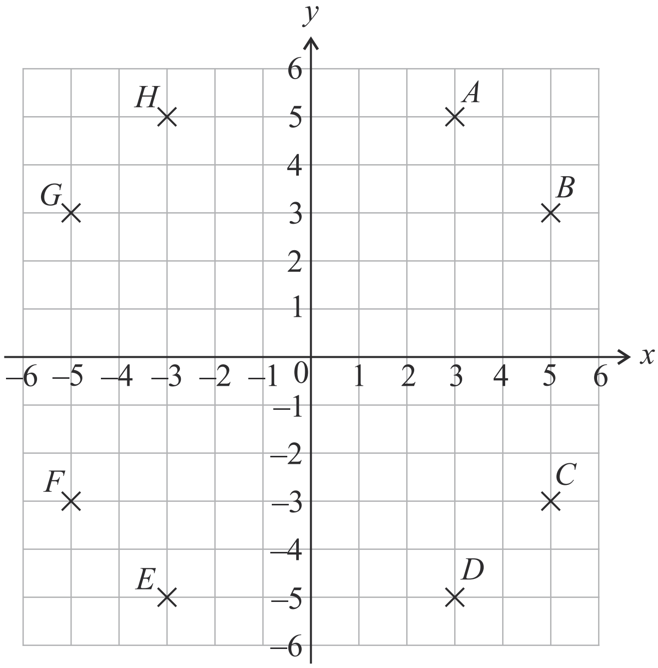
2 Write the ratio 40 : 25 in its simplest form.

....

[Total 1 mark]

---

3 Eight points are shown plotted on the grid.



(a) Circle the point that has coordinates (–5, –3).

C *E* *F* *G*

[1]

(b) Circle the equation of the straight line that passes through points *A* and *D*.

$x = 3$ $x + y = 3$ $y = 3x$ $y = 3$

[1]

[Total 2 marks]

---

4 Convert 3.97 km into m.

.... m

[Total 1 mark]

5 Karl has five number cards.

–6	6	–8	–12	2

(a) Write Karl's number cards in order, starting with the lowest.

lowest .... , ... , ... , ... , ... highest

[1]

(b) Use two of Karl's number cards to make this calculation correct.

.... – .... = 10

[1]

[Total 2 marks]

6 Beth has some 5p and 10p coins.

Coin	Number
5p	28
10p	41

She changes her coins for 50p coins at the bank.

How many 50p coins does she receive?

....

[Total 3 marks]

7 Calculate

$$\frac{1.2 - 0.2 \times 4}{0.05}$$

....

[Total 2 marks]

8

If you add a multiple of 3 to a multiple of 6, you always get a multiple of 9.

Give an example to show that this statement is not true.

....

....

[Total 1 mark]

---

9 (a) Simplify $11a + 5b - 2a + 2b$
Circle your answer.

$13a + 3b$ $\quad$ $9a + 7b$ $\quad$ $13a + 7b$ $\quad$ $16ab$

[1]

(b) Simplify $2a \times 3a$
Circle your answer.

$5a$ $\quad$ $6a$ $\quad$ $5a^2$ $\quad$ $6a^2$

[1]

[Total 2 marks]

---

10 Chloe invests £300 in a bank account.
The account pays 2% simple interest each year.

Work out how much money she has in her account after 4 years.

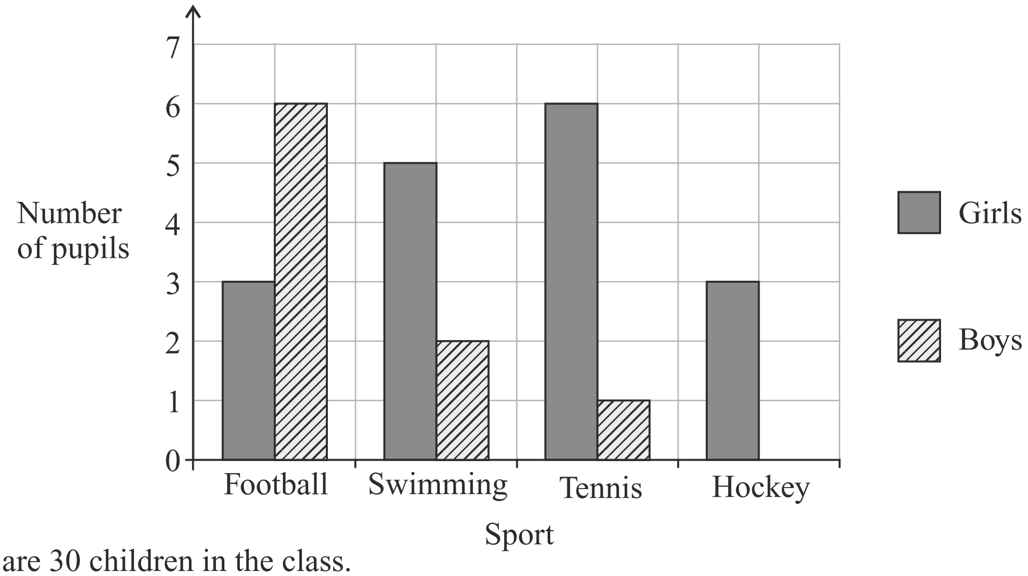
£ ....

[Total 3 marks]

---

11 The dual bar chart below shows the favourite sports of the pupils in a class.

One bar is missing.



There are 30 children in the class.

(a) Draw the missing bar to show the number of boys whose favourite sport is Hockey.

[2]

(b) One child is chosen at random from the class.

Find the probability that their favourite sport is swimming.

....

[2]

(c) What is the ratio of the number of boys who chose swimming to the number of girls who chose tennis? Give your answer in its simplest form.

....

[2]

[Total 6 marks]

---

12 Decide whether the sequence is an arithmetic or geometric progression, and write down in words the rule to get from one term to the next.

2, 8, 32, 128...

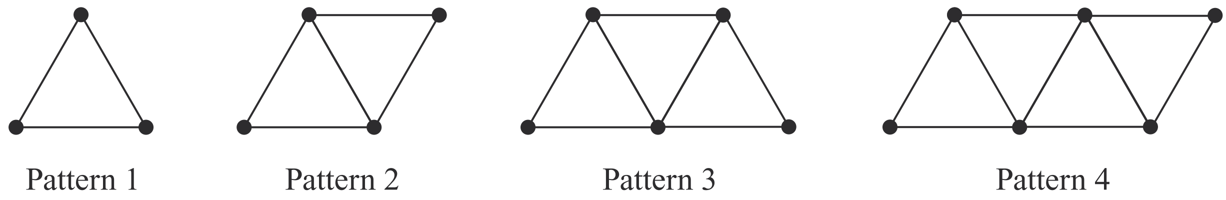
Arithmetic ☐ Geometric ☐

Rule: ....

....

[Total 2 marks]

13 The diagram shows the first four patterns in a sequence.



(a) Complete the table.

	Number of triangles	Number of dots	Number of lines
Pattern 1	1	3	3
Pattern 2	2	....	5
Pattern 3	....	5	....
Pattern 4	4	....	....

[1]

(b) Work out the number of lines in pattern 10.

....

[2]

(c) (i) Find a formula for the number of dots D in Pattern n.

....

[2]

(ii) Find the number of dots in pattern 200.

....

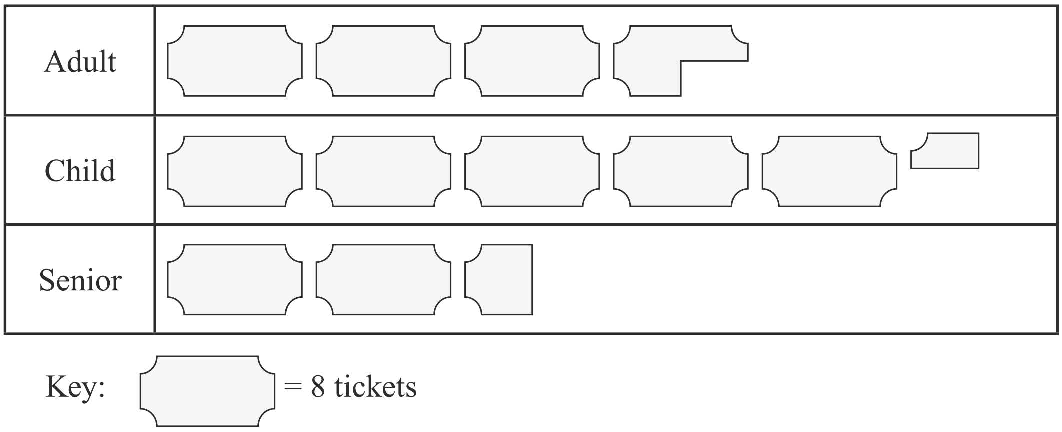
[1]

[Total 6 marks]

14 A theatre sells three types of tickets.

Ticket type	Cost
Adult	£9
Child	£5
Senior	£6.50

The pictogram shows the number of tickets of each type sold for one performance.

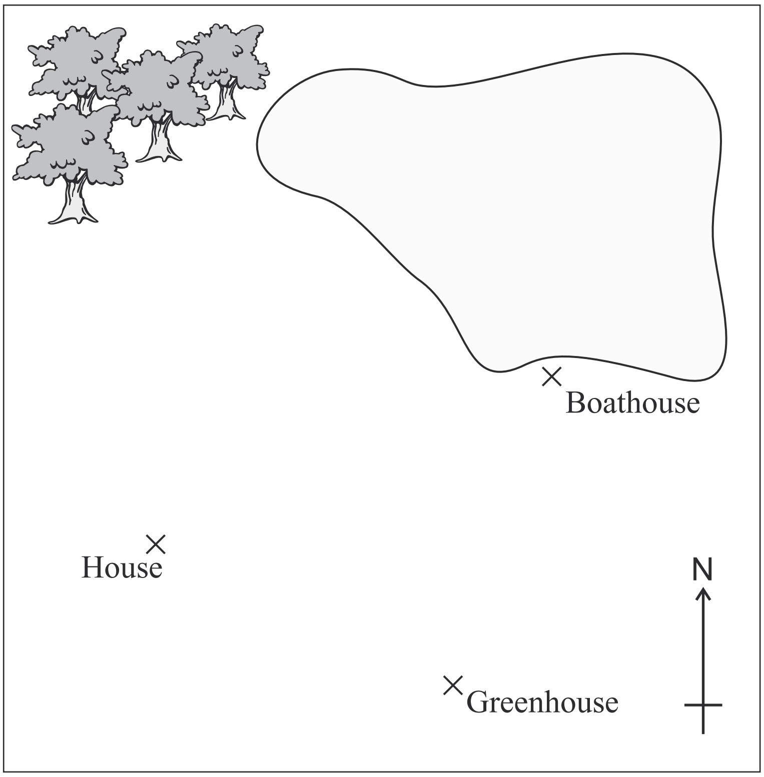


How much money did the theatre make from all the ticket sales for this performance?

£ ....

[Total 6 marks]

15 The scale drawing shows the gardens of a country house.



Scale: 1 cm = 100 metres

(a) Find the three-figure bearing of the boathouse from the house.

..................°

[1]

(b) Find the actual distance from the boathouse to the greenhouse.

.... metres

[2]

(c) A summerhouse is

450 metres from the house
700 metres from the greenhouse

Plot with a cross (×) the position of the summerhouse on the map.
Do not rub out your construction lines.

[2]

[Total 5 marks]

16 Angie makes wedding cakes with three tiers.

She needs 800 grams of sultanas to make the bottom tier of a cake.
The middle tier needs 75% of the ingredients required for the bottom tier.
The top tier needs 50% of the ingredients of the bottom tier.

Angie needs to make five wedding cakes. She has 8 kilograms of sultanas.

Does Angie have enough sultanas to make five cakes? Show your working.

[Total 5 marks]

---

17 Carrots cost 69p per kilogram.
Ahmed buys 2.785 kilograms of carrots.

(a) Estimate the cost, in pence, of his carrots.
Show the numbers you use to work out your estimate.

.... p
[2]

(b) Is your estimate in (a) bigger or smaller than the actual cost?
Tick the correct answer.

Bigger ☐ Smaller ☐

Explain your answer.

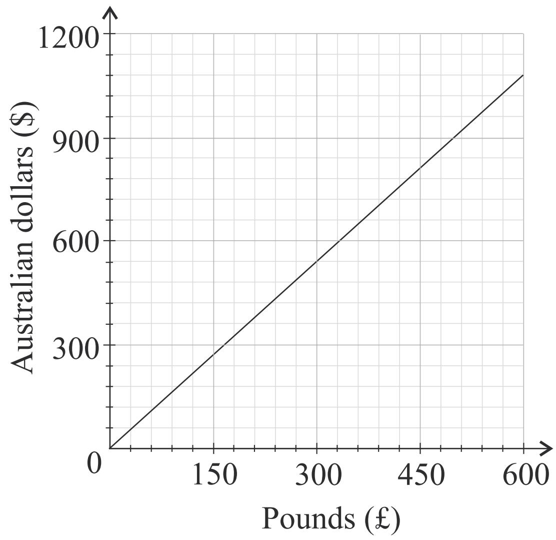
....

....
[1]

[Total 3 marks]

---

18 The graph can be used to convert between pounds (£) and Australian dollars ($).



Jack goes on holiday to Australia and China.

(a) He changes £300 into Australian dollars ($).
How many Australian dollars does he get for £300?

$ ....

[1]

(b) He spends $390 of his money while in Australia.
He converts the rest of his Australian dollars into Chinese yuan using the exchange rate

1 Australian dollar = 6 Chinese yuan

How many Chinese yuan does Jack get?

.... yuan

[2]

[Total 3 marks]

---

19 Work out the value of *k* if

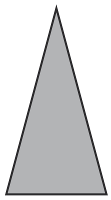
$$k \times 3^{-2} = 4$$

$k =$ ....

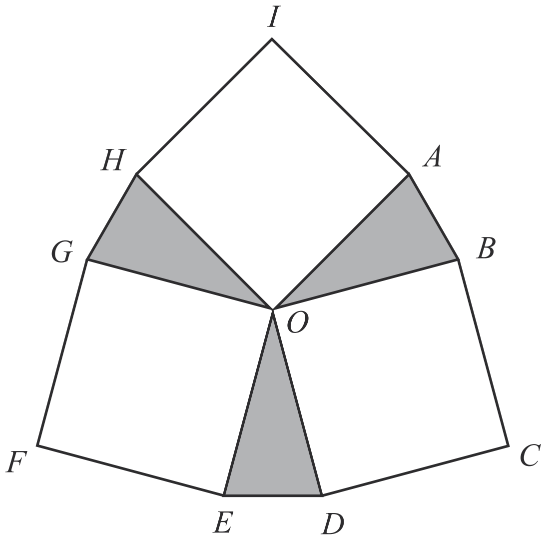
[Total 2 marks]

---

20 The diagram shows an isosceles triangle.



Three of these isosceles triangles fit together with three squares around a point O.



Show clearly that angle $OAB = 75°$.

[Total 4 marks]

---

21 Work out $1\frac{2}{3} \times 1\frac{5}{8}$. Give your answer as a mixed number.

....

[Total 3 marks]

---

22 Write 594 000 000 000 in standard form.

....

[Total 1 mark]

---

23 A school records the proportion of boys and girls in three different year groups.

In Year 9, $\frac{9}{20}$ of the pupils are girls.

In Year 10, 49% of the pupils are girls.

In Year 11, the ratio of girls : boys is 12 : 13.

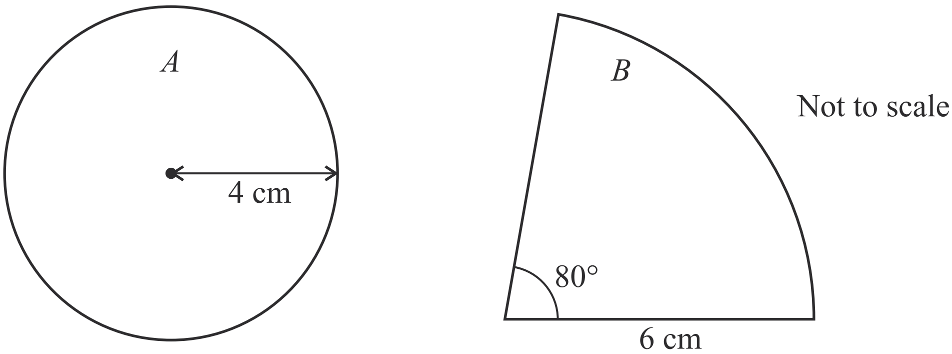
Which year has the largest proportion of girls?

....

[Total 3 marks]

---

24 The diagram shows a circle *A* and a sector *B*.

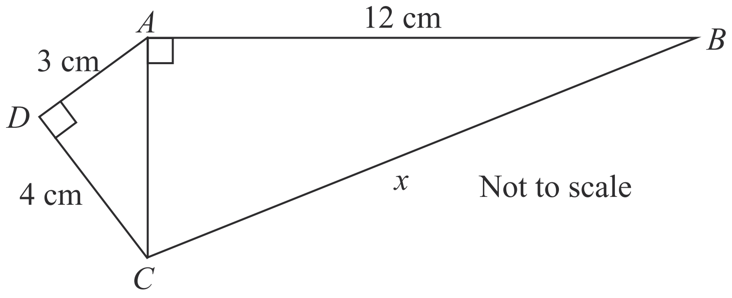


Show that the area of *A* is twice the area of *B*.

[Total 4 marks]

---

25 Quadrilateral *ABCD* is made up of two right-angled triangles, *ABC* and *ACD*.



(a) Find the length of the side labelled x.

.... cm
[4]

(b) Find the area of quadrilateral *ABCD*.

.... cm^2
[2]

[Total 6 marks]

---

26 Solve the simultaneous equations

$$3x + 2y = 17$$
$$2x + y = 10$$

x = ...

y = ...
[Total 3 marks]

[TOTAL FOR PAPER = 80 MARKS]

Practice Paper 2: Calculator
Right, here's Practice Paper 2 — you'll need a calculator for this one. Don't forget there's a page on formulas (p.244) — it tells you which formulas you'll be given in your exam and which you'll need to learn.

Candidate Surname	Candidate Forename(s)

Centre Number	Candidate Number	Candidate Signature

GCSE

Mathematics Foundation Tier

Paper 2 (Calculator)

Practice Paper

Time allowed: 1 hour 30 minutes

You must have:
Pen, pencil, eraser, ruler, protractor, pair of compasses.
You may use tracing paper.

You **may use** a calculator.

Instructions to candidates

- Use **black** ink to write your answers.
- Write your name and other details in the spaces provided above.
- Answer **all** questions in the spaces provided.
- In calculations, show clearly how you worked out your answers.
- Do all rough work on the paper.
- Unless a question tells you otherwise, take the value of π to be 3.142, or use the π button on your calculator.

Information for candidates

- The marks available are given in brackets at the end of each question.
- You may get marks for method, even if your answer is incorrect.
- There are 28 questions in this paper. There are no blank pages.
- There are 80 marks available for this paper.

Answer ALL the questions.

Write your answers in the spaces provided.

You must show all of your working.

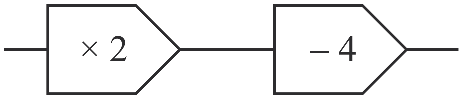
1 Write $\frac{3}{5}$ as a percentage.
Circle your answer.

6% 30% 15% 60%

[Total 1 mark]

---

2 A function is represented by this number machine.



The **output** of the machine is 20. Circle the input.

8 12 14 36

[Total 1 mark]

---

3 Complete this bill.

Barbara's Café

Menu Item	**Number Ordered**	**Cost per Item**	**Total**
Tea	2	£1.25	£2.50
Coffee	....	£1.60	£9.60
Cake	4	£...............	£5.20
Tip			£2.50
		Total cost	£...............

[Total 3 marks]

4 (a) Draw a line to match each shape to its number of **surfaces**.

cone	4
	3
sphere	
	2
cylinder	1

[2]

(b) Write down the number of **vertices** for a triangular prism.

....

[1]

[Total 3 marks]

---

5 (a) Calculate

$$\sqrt{12.2} + (1.1 + 3.6)^3$$

Write down all the digits on your calculator.

....

[1]

(b) Round your answer to (a) to two decimal places.

....

[1]

[Total 2 marks]

---

6 Kamil has these four number cards.

3	5	4	6

List the eight even numbers greater than 4000 Kamil can make by rearranging all four cards.

[Total 2 marks]

7 Circle the number below that is both a square number and a cube number.

16 27 64 25 8 100

[Total 1 mark]

---

8 (a) Expand

$$4(a + 2)$$

....
[1]

(b) Factorise

$$y^2 + 5y$$

....
[1]
[Total 2 marks]

---

9 The ages (in years) of seven children are

6 12 9 6 5 7 11

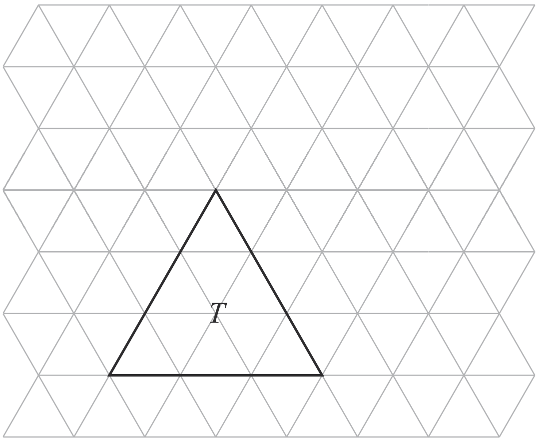
(a) Find the median age.

....
[1]

(b) Find the mean age.

....
[2]
[Total 3 marks]

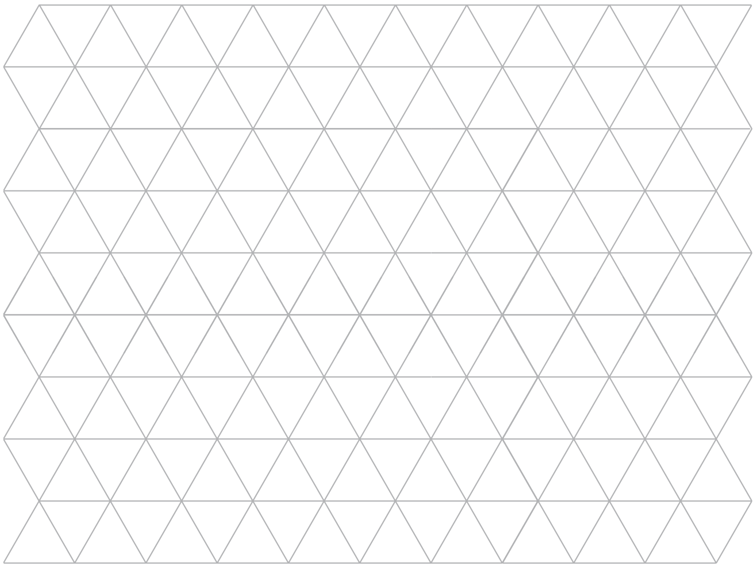
10 An equilateral triangle T is shown on the grid.



(a) Another triangle congruent to T is joined to T to form a quadrilateral.
Write down the number of lines of symmetry of the quadrilateral.

....
[1]

(b) Show on the grid how four triangles congruent to T can be joined together to form a shape with rotational symmetry of order 3.



[1]
[Total 2 marks]

---

11 Work out 185% of £3500.

£ ....
[Total 2 marks]

---

12 Sandra attends a job interview at a school.
The school refunds her travelling expenses if she uses the cheapest possible method of transport.

There are two methods of transport that Sandra can use to attend the interview.

Method 1: By car	**Method 2: By car and train**
Sandra lives 27 miles from the school.	Sandra lives 4 miles from the station. The cost of a return train ticket is £17.60.

The school refunds car travel at a rate of 40p per mile.

Which method should Sandra use to travel to her interview and home again if she wants a refund for her expenses?

Show how you work out your answer.

[Total 3 marks]

13 A car park contains 28 cars and 16 motorbikes.

$\frac{3}{4}$ of the cars and $\frac{3}{8}$ of the motorbikes are red.
A red vehicle is chosen at random.

What is the probability that it is a car? Give your answer as a fraction in its simplest form.

....

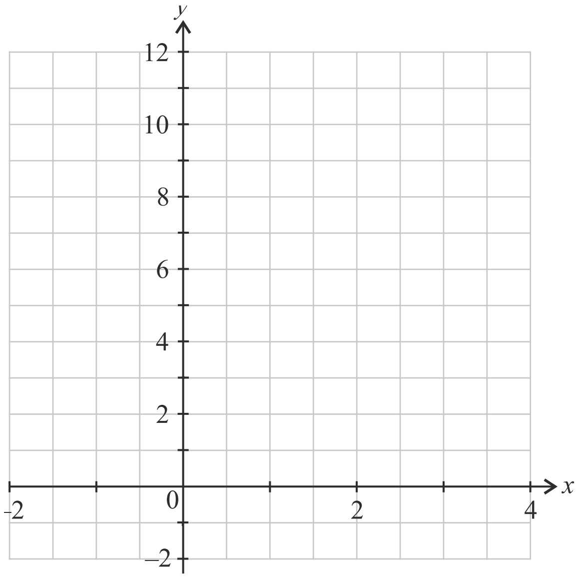
[Total 3 marks]

14 (a) Complete the table of values for $y = 7 - 2x$.

x	–2	–1	0	1	2	3	4
y		9	7			1	

[2]

(b) Draw the graph of $y = 7 - 2x$ for values of x between –2 and 4.



[2]

(c) What is the gradient of the line $y = 7 - 2x$?
Circle your answer.

–2   –1   2   7

[1]

[Total 5 marks]

15 Solve the equation $3(2x - 4) = 2x + 8$

x = ...

[Total 3 marks]

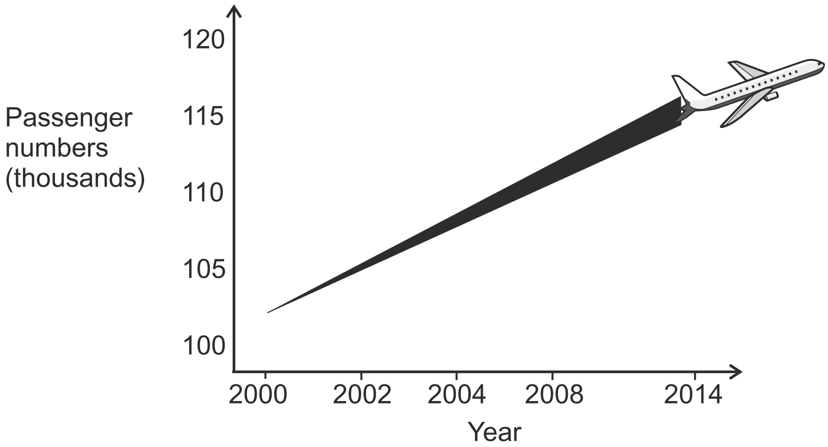
16 Kieron works out $5 \times \frac{2}{3}$ and gets the answer $\frac{10}{15}$.
Explain what mistake Kieron has made in calculating his answer.

....

....

[Total 1 mark]

17 The budget airline 'Fly By Us' produces this graph to show how their passenger numbers have increased.



Give three different criticisms of the graph.

Criticism 1 ....

....

Criticism 2 ....

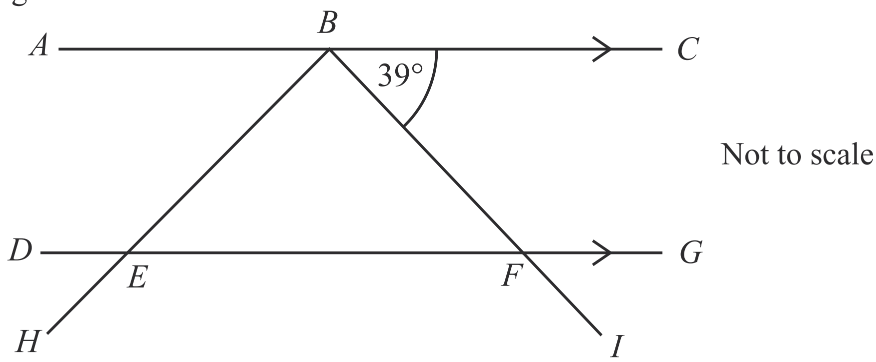
....

Criticism 3 ....

....

[Total 3 marks]

18 *AC* and *DG* are straight lines.



BH is perpendicular to *BI*.

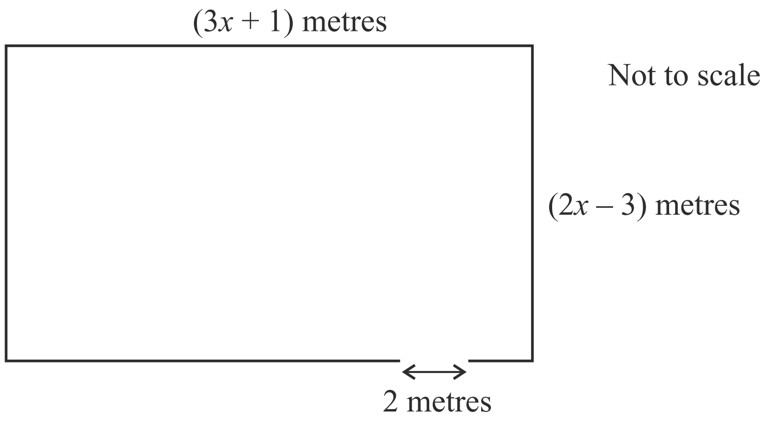
Work out the size of angle *DEH*.
Show how you work out your answer.

.... °

[Total 3 marks]

19 Jimmy has a rectangular vegetable garden measuring $(3x + 1)$ metres by $(2x - 3)$ metres.

Jimmy wants to put a fence around the outside of the garden.



He needs a 2 metre gap along one edge so that he can get in and out.

(a) Show that the length of the fence, L m, is given by the formula $L = 10x - 6$.

[2]

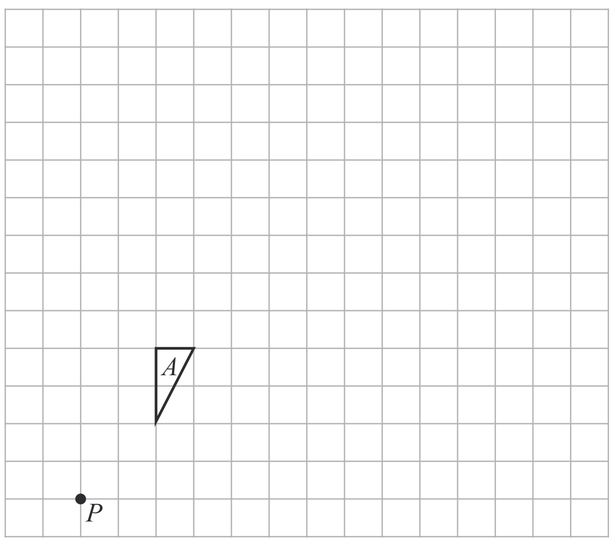
(b) Show that L is always an even number when x is a whole number.

[2]

[Total 4 marks]

20 Enlarge triangle A by scale factor 3, centre P.

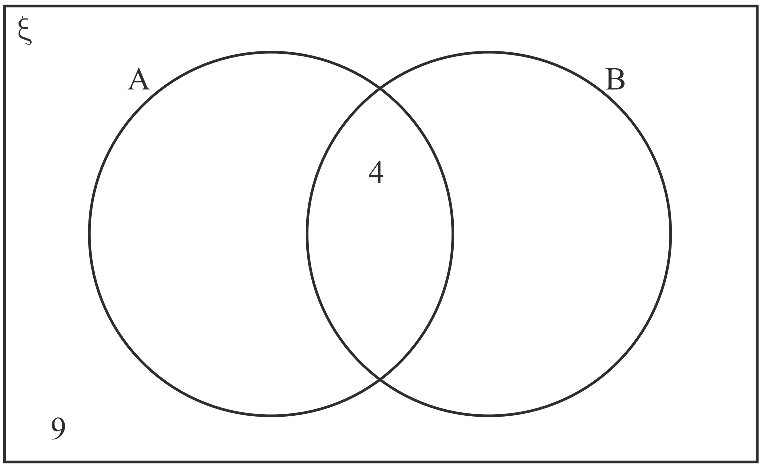
Label the image B.



[Total 2 marks]

21 $\xi = \{1, 2, 3, \ldots, 10\}$
A = $\{x : 2 < x \leq 6\}$
B = $\{x : x$ is a factor of $12\}$

Complete the Venn diagram to show the elements of each set.



[Total 3 marks]

22 Orange juice and lemonade are mixed in the ratio 3 : 5 to make orangeade.

Orange juice costs £1.60 per litre.
Lemonade costs £1.20 per litre.

What is the cost of making 18 litres of orangeade?

£ ....

[Total 4 marks]

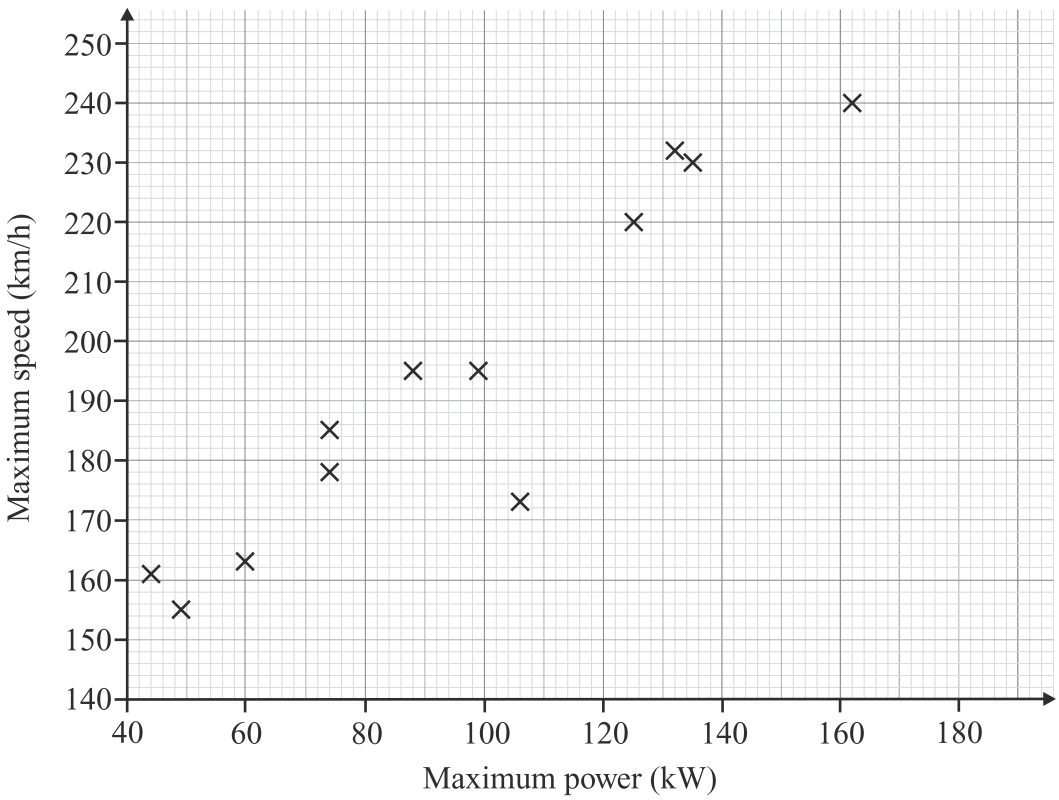
23 Make x the subject of the formula

$$y = \frac{x^2 - 2}{5}$$

....

[Total 2 marks]

24 The scatter graph shows the maximum power (in kW) and the maximum speed (in km/h) of a sample of cars.



(a) One of the cars has a maximum speed of 220 km/h.
Write down the maximum power of this car.

.... kW
[1]

(b) One of the points is an outlier as it does not fit in with the trend.
Draw a ring around this point on the graph.

[1]

(c) Ignoring the outlier, describe the correlation shown on the scatter graph.

.... correlation
[1]

(d) A different car has a maximum power of 104 kW.
By drawing a suitable line on your scatter graph, estimate the maximum speed of this car.

.... km/h
[2]

(e) Explain why it may not be reliable to use the scatter graph to estimate the maximum speed of a car with a maximum power of 190 kW.

....

....

[1]

[Total 6 marks]

25 Ollie and Amie each have an expression.

Ollie	Amie
$(x + 4)^2 - 1$	$(x + 5)(x + 3)$

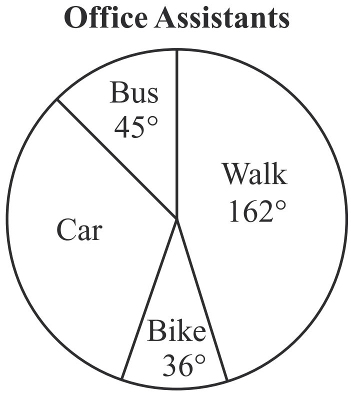
Show clearly that Ollie's expression is equivalent to Amie's expression.

[Total 3 marks]

---

26 A company consists of 80 office assistants and a number of managers.

The pie chart shows how the 80 office assistants travel to work.



(a) How many office assistants travel to work by car?

....
[2]

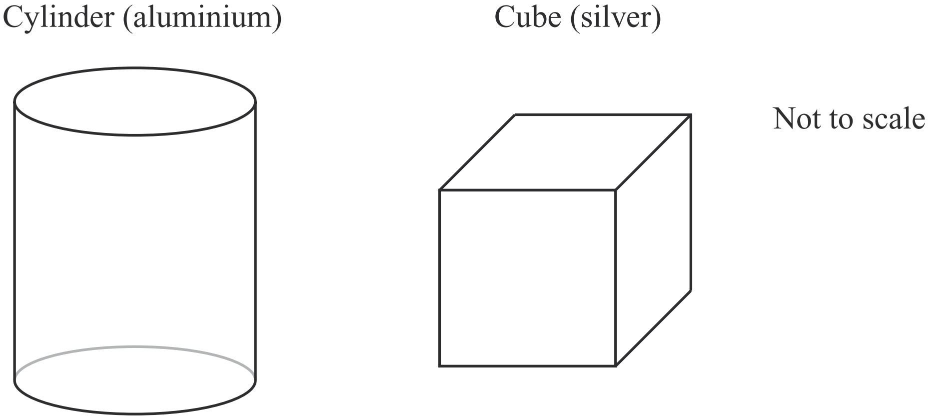
18 of the managers travel by car.
Overall, 40% of the people in the company travel by car.

(b) Work out how many people there are in the company.

....
[2]

[Total 4 marks]

27 The diagram shows a solid aluminium cylinder and a solid silver cube.



- The volume of the cylinder is 1180 cm^3.
- The cylinder and the cube have the same mass.
- The density of aluminium is 2.7 g/cm^3 and the density of silver is 10.5 g/cm^3.

(a) Calculate the mass of the cylinder.

.... g

[2]

(b) Calculate the side length of the cube. Give your answer correct to two significant figures.

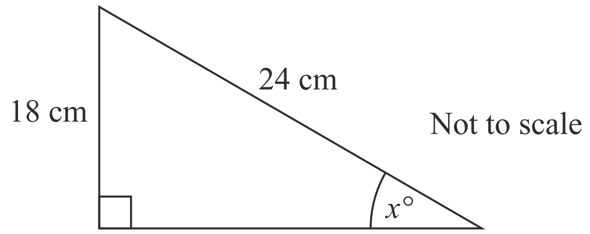
.... cm

[4]

[Total 6 marks]

---

28 The diagram shows a right-angled triangle.



Calculate the value of x. Give your answer correct to 1 decimal place.

x = ...

[Total 3 marks]

[TOTAL FOR PAPER = 80 MARKS]

Practice Paper 3: Calculator
Finally, here's Practice Paper 3 — you'll be pleased to know that you can use your calculator for this one too. Take a look at the formulas page on p.244 if you need a reminder of which formulas you need to learn and which you'll be given in the exam.

Candidate Surname	Candidate Forename(s)

Centre Number	Candidate Number	Candidate Signature

GCSE

Mathematics Foundation Tier

Paper 3 (Calculator)

Practice Paper

Time allowed: 1 hour 30 minutes

You must have:
Pen, pencil, eraser, ruler, protractor, pair of compasses.
You may use tracing paper.

You **may use** a calculator.

Instructions to candidates
- Use **black** ink to write your answers.
- Write your name and other details in the spaces provided above.
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Information for candidates
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- You may get marks for method, even if your answer is incorrect.
- There are 28 questions in this paper. There are no blank pages.
- There are 80 marks available for this paper.

Answer ALL the questions.

Write your answers in the spaces provided.

You must show all of your working.

1 Write one of the signs <, =, or > on each answer line to make a true statement.

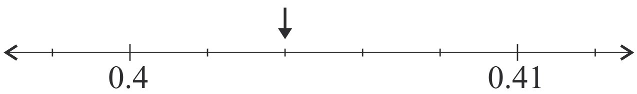
0.4 .... 0.34

$\frac{3}{4}$ 0.75

7% 0.7

[Total 2 marks]

2 The diagram shows part of a number line.



Circle the number the arrow points to.

0.42 0.44 0.402 0.404

[Total 1 mark]

3 Round 20 758 to the nearest 100.

....

[Total 1 mark]

4 What number is 12 less than –4.2?

....

[Total 1 mark]

5 Circle the number below that has exactly four factors.

2 3 5 8 9 12

[Total 1 mark]

6 24 pupils take a violin exam. Their marks are shown below.

110	103	115	134	121	98
128	112	107	112	125	132
114	102	125	93	120	120
106	111	99	98	127	115

The certificate each pupil receives depends upon their mark.

Result of exam	**Mark**
Fail	Under 100
Pass	100 – 119
Merit	120 – 129
Distinction	130 and above

(a) Complete the table to show the number of pupils achieving each result.
The first row has been filled in for you.

Result of exam	**Tally**	**Frequency**
Fail	| | | |	4
Pass		
Merit		
Distinction		
	Total:	**24**

[2]

(b) What fraction of the pupils failed the exam? Give your fraction in its simplest form.

....

[2]

(c) Draw on the grid a suitable diagram to show the number of pupils achieving each result.

[3]

[Total 7 marks]

7 A pencil case contains 10 coloured pencils.

1 pencil is yellow.
2 pencils are red.
The other pencils are either green or blue.

Carla picks one coloured pencil at random.
She has the same chance of picking a green pencil as a red pencil.

Circle the word that describes the probability of picking:

(a) a black pencil,

impossible unlikely evens likely

[1]

(b) a blue pencil.

impossible unlikely evens likely

[1]

[Total 2 marks]

---

8 Here are the names of four types of quadrilateral.

Parallelogram	Square	Trapezium	Kite

Choose from this list the quadrilateral that has:

(a) exactly one pair of parallel sides,

....

[1]

(b) no lines of symmetry, but rotational symmetry of order 2.

....

[1]

[Total 2 marks]

---

9 Circle the vector that translates a shape 5 units **left**.

$\begin{pmatrix} -5 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$ $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$

[Total 1 mark]

---

10 Rick multiplies three different numbers together and gets 90.
One of his numbers is a square number, and the other two are prime numbers.
What are the three numbers he uses?

....

[Total 3 marks]

---

11 Nigel sees this recipe for cupcakes.

Recipe for 12 cupcakes
140 grams butter
140 grams flour
132 grams sugar
2 eggs
1 tablespoon milk

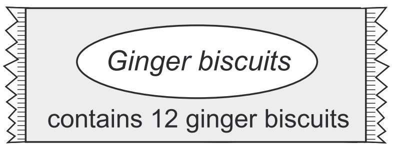
Nigel wants to make 30 cupcakes. How much sugar does he need?

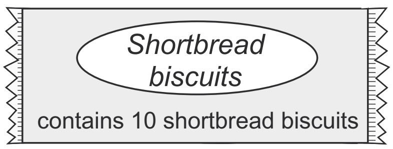
.... g

[Total 2 marks]

---

12 Ajay buys some packets of ginger biscuits.
Jane buys some packets of shortbread biscuits.





Ajay and Jane buy the same number of biscuits.

What is the smallest number of packets of shortbread biscuits Jane could have bought?

.... packets

[Total 3 marks]

---

13 Mary is preparing cream teas for 30 people.

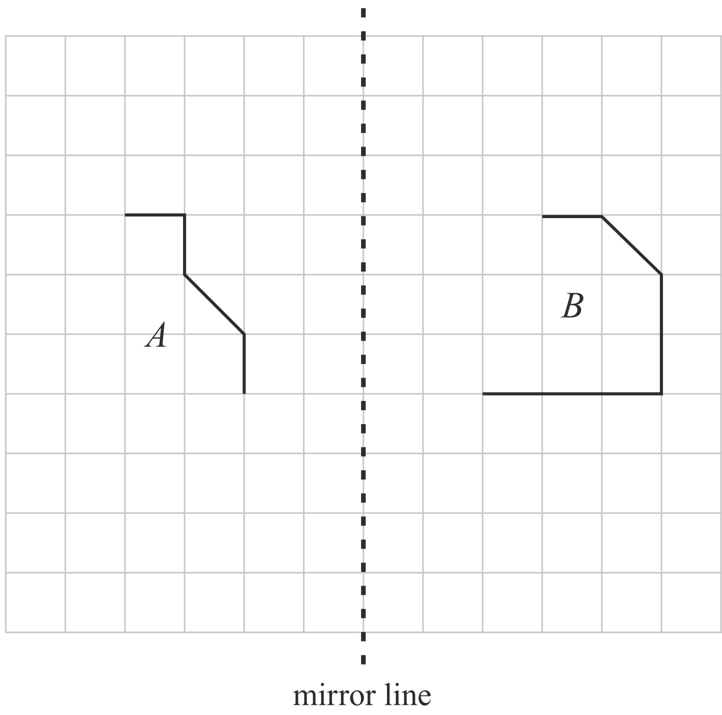
Each person needs 2 scones, 1 tub of clotted cream and 1 small pot of jam.

She has £35 to buy everything.
A pack of 10 scones costs £1.35
A pack of 6 tubs of clotted cream costs £2.95
Each small pot of jam costs 40p

Will she have enough money? Show how you work out your answer.

....
[Total 5 marks]

14 The grid shows part of two shapes, *A* and *B*.

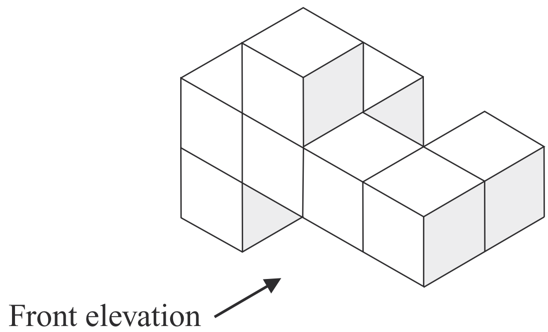


B is the reflection of *A* in the mirror line.

Complete both shapes.

[Total 2 marks]

15 The diagram shows an object made from 8 centimetre cubes.



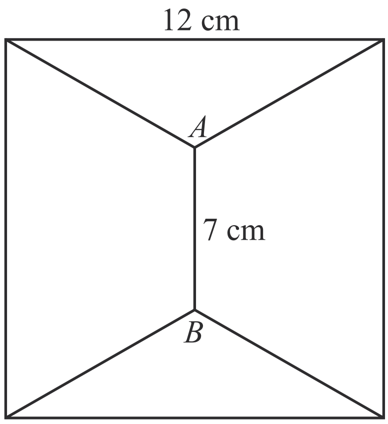
Draw on the grids below the plan view and the front elevation of the object.

Plan view

Front elevation

[Total 2 marks]

16 Two congruent trapeziums and two triangles fit inside a square of side 12 cm as shown.



Not to scale

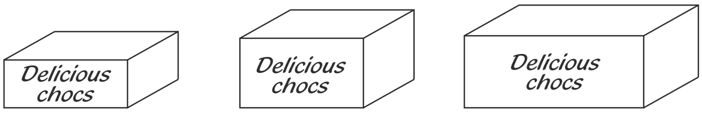
$AB = 7$ cm

Work out the area of each trapezium.

.... cm^2

[Total 2 marks]

17 A chocolate manufacturer makes boxes of chocolates in three different sizes.



Box A Box B Box C

Box A contains c chocolates.
Box B contains 4 more chocolates than Box A.
Box C contains twice as many chocolates as Box B.
Altogether there are 60 chocolates.

Work out how many chocolates there are in each box.

Box A: ....

Box B: ....

Box C: ....

[Total 5 marks]

18 Simplify

(a) $y \times y \times y$

....

[1]

(b) $n^6 \div n^2$

....

[1]

(c) $(a^4)^3$

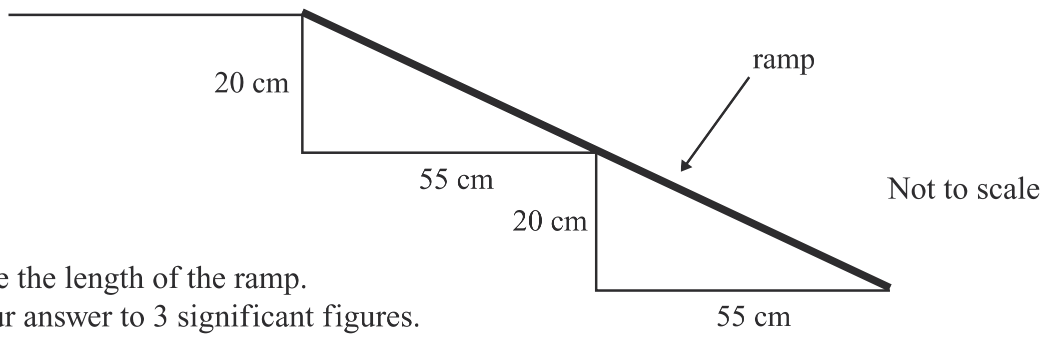
....

[1]

[Total 3 marks]

---

19 The diagram shows a ramp placed against two steps.



Calculate the length of the ramp.
Give your answer to 3 significant figures.

.... cm

[Total 3 marks]

---

20 A route between Guilford and Bath has a distance of 180 kilometres.
Dave drives from Guilford to Bath. He takes 3 hours.

Olivia drives the same route. Her average speed is 15 kilometres per hour faster than Dave's.

(a) How long does it take Olivia to drive from Guilford to Bath?
Give your answer in hours and minutes

.... hours .... minutes

[3]

(b) Why is it important to your calculation that Olivia drives the same route as Dave?

....

....

[1]

[Total 4 marks]

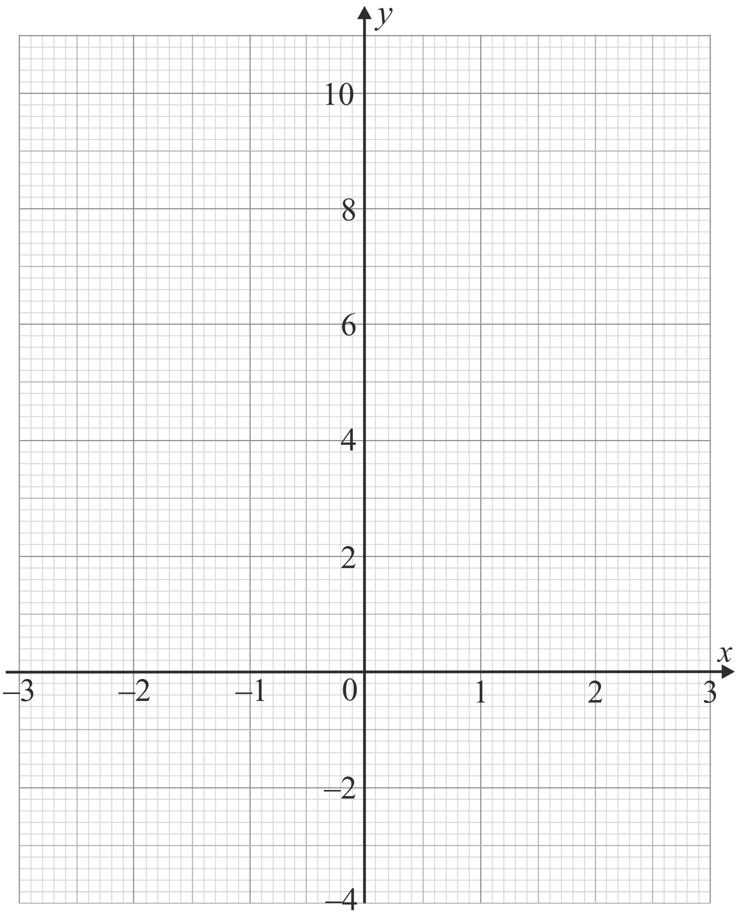
---

21 (a) Complete the table of values for $y = x^2 + x - 2$.

x	-3	-2	-1	0	1	2	3
y		0	-2	-2			10

[2]

(b) Draw on the grid the graph of $y = x^2 + x - 2$ for values of x between -3 and 3.



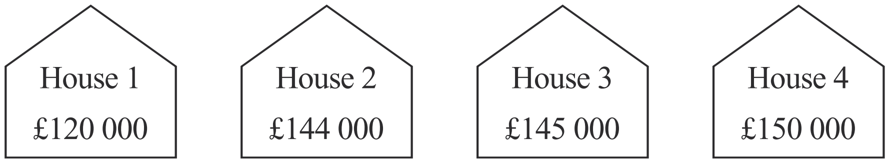
[2]

[Total 4 marks]

22 The ratio of angles in a triangle is $2:3:5$.
Show that this a right-angled triangle.

[Total 3 marks]

23 The values of four houses at the start of 2013 are shown.



(a) Which house has a value 25% higher than House 1?

House ....

[1]

(b) At the start of 2015, the value of House 2 is £161 280.
Find the percentage increase in the value of House 2.

.... %

[3]

[Total 4 marks]

---

24 Anna and Carl each think of a sequence of numbers.

Anna's sequence

4th term = 17

Term-to-term rule is
Add 3

Carl's sequence

Term-to-term rule is
Add 6

The 1st term of Anna's sequence is double the 1st term of Carl's sequence.

Work out the 5th term of Carl's sequence.

....

[Total 3 marks]

25 (a) Factorise $x^2 + 7x - 18$.

....

[2]

(b) Solve the equation $x^2 + 7x - 18 = 0$.

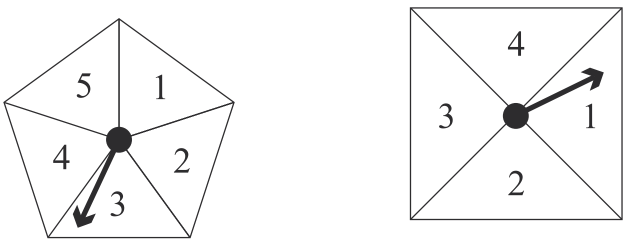
$x =$ .... or $x =$ ....

[1]

[Total 3 marks]

---

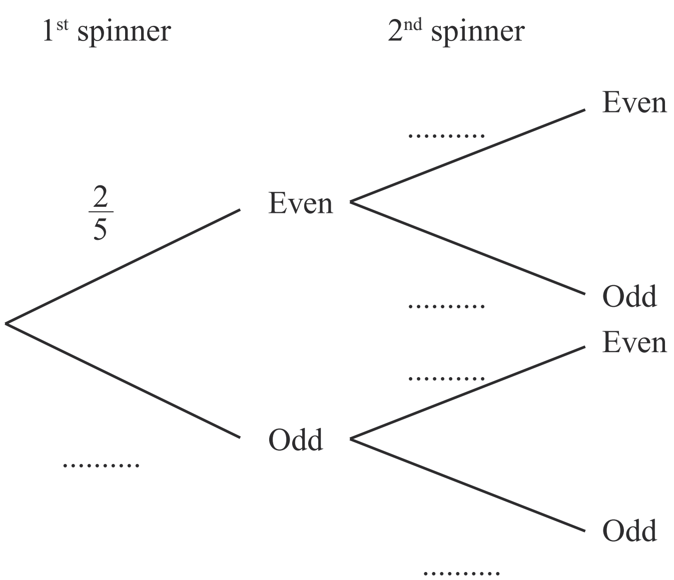
26 George has two fair spinners.



1st spinner 2nd spinner

He spins each spinner once and records whether the score is an odd or an even number.

(a) Complete the tree diagram to show the probabilities.



[2]

(b) Work out the probability that George spins two odd numbers.

....

[2]

[Total 4 marks]

27 The line L passes through the points (–2, –7) and (3, 8).
Find the equation of line L.

....

[Total 4 marks]

---

28 The grouped frequency table below shows the weights of 25 rabbits in a pet shop.

Weight (w g)	**Frequency**
$800 \leq w < 1000$	5
$1000 \leq w < 1200$	8
$1200 \leq w < 1400$	9
$1400 \leq w < 1600$	3

Estimate the mean weight.

.... g

[Total 3 marks]

[TOTAL FOR PAPER = 80 MARKS]

Section One — Number

Page 7 (Warm-up Questions)

1 55

2 19p or £0.19

3 a) 1230 b) 48

4 a) 0.245 b) 5

5 a) 336 b) 832 c) 179.2 d) 6.12

6 a) 12 b) 121 c) 56 d) 30

7 a) 12 b) –6 c) 9 d) –3

Page 8 (Exam Questions)

3 a) 81 ***[1 mark]***
b) 64 ***[1 mark]***

4 E.g. 288 ÷ –3 = –96
–96 ÷ 12 = –8
So the third number is –8
[3 marks available — 1 mark for a correct method, 1 mark for at least one correct calculation, 1 mark for the correct answer]
You could have worked out –3 × 12 (= –36) and divided by this instead, but the division would be pretty tricky.

5 £200 – £5 = £195
$\begin{array}{r} 0\,1\,3 \\ 15\overline{)1^{1}9^{4}5} \end{array}$, so each ticket costs £13
[3 marks available — 1 mark for subtracting £5 from £200, 1 mark for dividing £195 by 15, 1 mark for the correct final answer]

6 Total miles travelled = (30 × 2) + (28 × 2) + (39 × 2) + (40 × 2)
= 60 + 56 + 78 + 80
= 274 miles
Expenses for miles travelled = 274 × 30p = 8220p = £82.20
Expenses for food = 4 × £8 = £32
Total expenses = £82.20 + £32 = £114.20
[4 marks available — 1 mark for finding total miles, 1 mark for multiplying total miles by 30 or 0.3(0), 1 mark for finding food expenses, 1 mark for the correct final answer]

Page 13 (Warm-up Questions)

1 31, 37

2 27 ÷ 3 = 9. So 27 is not a prime number because it divides by 3 and 9.

3 1, 2, 4, 5, 8, 10, 20, 40

4 2 and 5

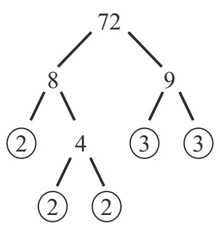
5 20 *(multiples of 4 are: 4, 8, 12, 16, 20, ..., multiples of 5 are: 5, 10, 15, 20, ...)*

6 12 *(factors of 36 are: 1, 2, 3, 4, 6, 9, 12, 18, 36, factors of 96 are: 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96)*

Page 14 (Exam Questions)

3 Jack is incorrect as there are four prime numbers (101, 103, 107 and 109) between 100 and 110.
[2 marks available — 1 mark for stating that Jack is incorrect, 1 mark for providing evidence]
Writing one prime number between 100 and 110 is enough.

4 E.g. 37 (3 + 7 = 10, which is 1 more than 9, a square number)
[2 marks available — 2 marks for a correct answer, otherwise 1 mark for a prime of two or more digits]

5


72 = 2 × 2 × 2 × 3 × 3
[2 marks available — 1 mark for a correct method, 1 mark for all prime factors correct]

6 a) LCM = $3^7 \times 7^3 \times 11^2$ ***[1 mark]***
b) HCF = $3^4 \times 11$ ***[1 mark]***

Page 20 (Warm-up Questions)

1 $\frac{3}{4}$

2 $\frac{2}{6}$ and $\frac{5}{15}$

3 a) $\frac{4}{15}$
b) $\frac{2}{5} \div \frac{2}{3} = \frac{2}{5} \times \frac{3}{2} = \frac{6}{10} = \frac{3}{5}$
c) $\frac{2}{5} + \frac{2}{3} = \frac{6}{15} + \frac{10}{15} = \frac{16}{15} = 1\frac{1}{15}$
d) $\frac{2}{3} - \frac{2}{5} = \frac{10}{15} - \frac{6}{15} = \frac{4}{15}$

4 0.7

5 $66.\dot{6}\%$ (66.666...%) or $66\frac{2}{3}\%$

6 $\frac{2}{5}$

7 $0.\dot{2}8571\dot{4}$

Page 21 (Exam Questions)

3 (12 400 ÷ 8) × 3 = 1550 × 3 = 4650
[2 marks available — 1 mark for dividing by 8 or multiplying by 3, 1 mark for the correct answer]

4 65% = 0.65, $\frac{2}{3}$ = 0.666..., $\frac{33}{50}$ = 0.66
So order is 0.065, 65%, $\frac{33}{50}$, $\frac{2}{3}$
[2 marks available — 2 marks for all four numbers in the correct order, otherwise 1 mark for writing the numbers in the same form (either decimals, percentages or fractions)]

5 a) $1\frac{1}{8} \times 2\frac{2}{5} = \frac{9}{8} \times \frac{12}{5}$ ***[1 mark]*** $= \frac{108}{40}$ ***[1 mark]***
$= 2\frac{7}{10}$ ***[1 mark]***
[3 marks available in total — as above]
b) $1\frac{3}{4} \div \frac{7}{9} = \frac{7}{4} \times \frac{9}{7}$ ***[1 mark]*** $= \frac{63}{28}$ or $\frac{9}{4}$ ***[1 mark]***
$= 2\frac{1}{4}$ ***[1 mark]***
[3 marks available in total — as above]

6 $\frac{1}{4}$ = 25%, so Jenny pays 1 – 25% – 20% – 20%
= 1 – 65% = 35% ***[1 mark]***
£17.50 = 35% ***[1 mark]***, so 1% = £17.50 ÷ 35 = £0.50.
The total bill was £0.50 × 100 ***[1 mark]*** = £50 ***[1 mark]***.
[4 marks available in total — as above]

Page 26 (Warm-up Questions)

1 a) 3.2 b) 1.8
c) 2.3 d) 0.5
e) 9.8

2 a) 3 b) 5
c) 2 d) 7
e) 3

3 a) 350 *(the decider is 2, so keep the 5, and fill the missing place with zero)*
b) 500 *(the decider is 6, so round the 4 up to 5, and fill the missing places with zeros)*
c) 12.4 *(the decider is 8, so round the 3 up to 4)*
d) 0.036 *(the decider is 6, so round the 5 up to 6)*

4 a) 2900 b) 500
c) 100

5 a) 100 *(This is approximately (30 – 10) × 5)*
b) Overestimate *(you rounded all the numbers up, so your estimate will be bigger than the actual answer)*

6 137.5 g

7 a) $375 \le x < 385$
b) $0.455 \le y < 0.465$

8 a) 37.9
b) 2.01

Page 27 (Exam Questions)

2 a) 428.6 light years ***[1 mark]***
b) 430 light years ***[1 mark]***

3 a) E.g. (£4.95 × 28) + (£11 × 19) ≈ (£5 × 30) + (£10 × 20) = £150 + £200 = £350
[2 marks available — 1 mark for rounding each value sensibly, 1 mark for a sensible estimate]
b) E.g. This is a sensible estimate as it is very close to the actual value of £347.60 ***[1 mark]***.

4 E.g. $\frac{12.2 \times 1.86}{0.19} \approx \frac{10 \times 2}{0.2} = \frac{20}{0.2} = 100$
[2 marks available — 1 mark for rounding to suitable values, 1 mark for the correct final answer using your values]

5 Minimum weight = 56.5 kg ***[1 mark]***
Maximum weight = 57.5 kg ***[1 mark]***
[2 marks available in total — as above]

6 Smallest possible value of $a = 3.8 - 0.05 = 3.75$
Largest possible value of $a = 3.8 + 0.05 = 3.85$
So error interval is $3.75 \le a < 3.85$
[2 marks available — 1 mark for 3.75 ≤ a, 1 mark for a < 3.85]

Page 32 (Warm-up Questions)

1 a) 43
b) 238.328
c) 10^4

2 a) 4^5 (= 1024) b) 7^3 (= 343)
c) q^8

3 a) 36 b) $\frac{1}{16}$

4 a) 14
b) 21
c) 3

5 1.2 cm

6 17.12422442

7 a) 432 000 000
b) 3.87×10^{-4}

8 1.9×10^{-3}

9 a) 3×10^4
b) 1.9×10^{10}

Page 33 (Exam Questions)

2 $\sqrt{6.25} = 2.5$ cm ***[1 mark]***

3 a) $A = 4.834 \times 10^9 = 4\,834\,000\,000$ ***[1 mark]***
b) C, B, A (5.21×10^3, 2.4×10^5, 4.834×10^9) ***[1 mark]***

4 $\frac{3^4 \times 3^7}{3^6} = \frac{3^{(4+7)}}{3^6} = \frac{3^{11}}{3^6} = 3^{(11-6)} = 3^5$
[2 marks available — 1 mark for a correct attempt at adding or subtracting powers, 1 mark for the correct final answer]

5 a) $6^{(5-3)} = 6^2 = 36$ ***[1 mark]***
b) $(2^4 \times 2^7) = 2^{(4+7)} = 2^{11}$
$(2^3 \times 2^2) = 2^{(3+2)} = 2^5$, so $(2^3 \times 2^2)^2 = (2^5)^2 = 2^{10}$
So $(2^4 \times 2^7) \div (2^3 \times 2^2)^2 = 2^{11} \div 2^{10} = 2^1 = 2$
[2 marks available — 1 mark if each bracket has been correctly simplified, 1 mark for the correct answer]

6 time (s) = distance (miles) ÷ speed (miles/s)
= $(9 \times 10^7) \div (2 \times 10^5)$ seconds ***[1 mark]***
= 450 seconds ***[1 mark]***
[2 marks available in total — as above]

Page 34 (Revision Questions)

1 A square number is a whole number multiplied by itself.
The first ten are: 1, 4, 9, 16, 25, 36, 49, 64, 81 and 100.

2 0.1

3 £38

4 a) £120 b) £0.50 = 50p

5 a) 1377 b) 26
c) 62.7 d) 0.35

6 a) –16 b) 7 c) 20

7 41, 43, 47, 53, 59

8 The multiples of a number are its times table.
a) 10, 20, 30, 40, 50, 60
b) 4, 8, 12, 16, 20, 24

9 a) $210 = 2 \times 3 \times 5 \times 7$
b) $1050 = 2 \times 3 \times 5 \times 5 \times 7 = 2 \times 3 \times 5^2 \times 7$

10 a) 14 b) 40

11 a) $\frac{25}{16}$ or $1\frac{9}{16}$ b) $\frac{44}{15}$ or $2\frac{14}{15}$
c) $\frac{23}{8}$ or $2\frac{7}{8}$ d) $\frac{11}{21}$

12 a) 320 b) £60

13 Amy

14 a) i) $\frac{4}{100} = \frac{1}{25}$ ii) 4%
b) i) $\frac{65}{100} = \frac{13}{20}$ ii) 0.65

15 a) Recurring decimals have a pattern of numbers which repeats forever.
b) $0.\dot{2}$

16 a) 17.7
b) 6700
c) 4 000 000

17 a) 100
b) 1400

18 a) $150 \le x < 250$
b) $24.6 \le y < 24.7$

19 7^5

20 $\frac{1}{25}$

21 a) 11 b) 4
c) 56 d) 10^5

22 a) 421.875 b) 4.8
c) 8 d) 11

23 1. The front number must always be between 1 and 10.
2. The power of 10, n, is how far the decimal point moves.
3. n is positive for big numbers, and negative for small numbers.

24 a) 3.56×10^9 b) 0.00000275

25 a) 2×10^3 b) 1.2×10^{12}

Section Two — Algebra

Page 39 (Warm-up Questions)

1 a) $10b$ b) $3x + 8y$ c) $9 + 3\sqrt{5}$

2 a) $-60rs$ b) $21m - 14$ c) $4p^2 + 8pq$

3 $5(x + 8) + 2(x - 12) = 5x + 40 + 2x - 24 = 7x + 16$

4 a) $x^2 + x - 30$ b) $2y^2 + 17y - 9$
c) $x^2 - 6x + 9$ d) $16y^2 + 40y + 25$

5 a) $6(2x + 5)$ b) $3y(2 + 5y)$
c) $(x + 5)(x - 5)$ d) $(6x + 7y)(6x - 7y)$

Page 40 (Exam Questions)

3 a) $4p$ ***[1 mark]***
b) $2m$ ***[1 mark]***
c) $4p + 3r$
[2 marks available — 1 mark for 4p and 1 mark for 3r]

4 $6x + 3 = (3 \times 2x) + (3 \times 1) = 3(2x + 1)$ ***[1 mark]***

5 a) $(x + 2)(x + 4) = x^2 + 4x + 2x + 8 = x^2 + 6x + 8$
[2 marks available — 1 mark for expanding the brackets correctly, 1 mark for simplifying]

b) $(y + 3)(y - 3) = y^2 - 3y + 3y - 9 = y^2 - 9$
[2 marks available — 1 mark for expanding the brackets correctly, 1 mark for simplifying]

c) $(2z - 1)(z - 5) = 2z^2 - 10z - z + 5 = 2z^2 - 11z + 5$
[2 marks available — 1 mark for expanding the brackets correctly, 1 mark for simplifying]

6 a) $x^2 - 49 = x^2 - 7^2 = (x + 7)(x - 7)$
[2 marks available — 2 marks for the correct final answer, otherwise 1 mark for attempting to use the difference of two squares]

b) $9x^2 - 100 = (3x)^2 - 10^2 = (3x + 10)(3x - 10)$
[2 marks available — 2 marks for the correct final answer, otherwise 1 mark for attempting to use the difference of two squares]

c) $y^2 - m^2 = (y + m)(y - m)$
[2 marks available — 2 marks for the correct final answer, otherwise 1 mark for attempting to use the difference of two squares]

Page 47 (Warm-up Questions)

1 a) $x = 6$ b) $x = 14$ c) $x = 3$ d) $x = 15$

2 a) $x = 5$ b) $y = 7$

3 $v = 29$

4 a) $y = 25$ b) $x = 11$

5 Ali = 13 tickets, Ben = 26 tickets, Joe = 34 tickets

6 3, 15 and 30

7 $x = 16$

8 $v = 3(u + 2)$ or $v = 3u + 6$

9 $d = \frac{c}{6} + 2$

Page 48 (Exam Questions)

2 a) $S = 4m^2 + 2.5n$
$S = (4 \times 2 \times 2) + (2.5 \times 10)$
$S = 16 + 25 = 41$
[2 marks available — 1 mark for correct substitution of m and n, 1 mark for correct final answer]

b) $S = 4m^2 + 2.5n$
$S = (4 \times 6.5 \times 6.5) + (2.5 \times 4)$
$S = 169 + 10 = 179$
[2 marks available — 1 mark for correct substitution of m and n, 1 mark for correct final answer]

3 a) $40 - 3x = 17x$
$40 = 20x$ ***[1 mark]***
$x = 40 \div 20 = 2$ ***[1 mark]***
[2 marks available in total — as above]

b) $2y - 5 = 3y - 12$
$-5 + 12 = 3y - 2y$ ***[1 mark]***
$y = 7$ ***[1 mark]***
[2 marks available in total — as above]

4 The sides of an equilateral triangle are all the same length, so
$4(x - 1) = 3x + 5$ ***[1 mark]***
$4x - 4 = 3x + 5$
$x = 9$ ***[1 mark]***
So each side is $(3 \times 9) + 5 = 32$ cm long ***[1 mark]***.
[3 marks available in total — as above]
To check your answer, put your value of x into the expression for the other side of the triangle — you should get the same answer.

5 $\frac{a + 2}{3} = b - 1$
$a + 2 = 3b - 3$ ***[1 mark]***
$a = 3b - 5$ ***[1 mark]***
[2 marks available in total — as above]

6 Call the number of Whitewater fans f. Redwood fans $= 3 \times f = 3f$.
Difference $= 3f - f = 2f$, so $2f = 7000$, so $f = 3500$.
Total fans $= 3f + f = 4f = 4 \times 3500 = 14\ 000$.
[3 marks available — 1 mark for the expressions for the number of fans for each team, 1 mark for forming and solving the equation to find f, 1 mark for the correct answer]

Page 52 (Warm-up Questions)

1 a) 18, 22 b) 81, 243 c) 17, 23

2 Rule = multiply the previous term by 2; next 2 terms = 24, 48
Rule = add 3, add 6, add 9...; next 2 terms = 21, 33

3 a) Arithmetic — to get from one term to the next, you add the same number to the previous term each time.

b) Geometric — to get from one term to the next, you multiply the previous term by the same number each time.

4 a) $6n + 3$ b) 51

c) No, as the solution to $6n + 3 = 62$ doesn't give an integer value of n.

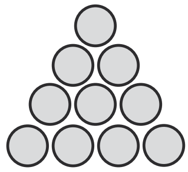
5 $n = -1, 0, 1, 2, 3, 4$

6 a) $x < 6$ b) $x \geq 3$

Page 53 (Exam Questions)

2 $-3, -2, -1, 0, 1$
[2 marks available — 2 marks for all 5 numbers correct, otherwise 1 mark for the correct answer with one number missing or one number incorrect]

3 a)



[1 mark]

b) The number of circles added increases by one each time, so the tenth triangle number is:
$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$.
[2 marks available — 1 mark for 55 and 1 mark for correct reasoning]

4 Second term = 7 – 3 = 4
Fourth term = 4 + 7 = 11
Fifth term = 7 + 11 = 18
[2 marks available — 2 marks for all three terms correct, otherwise 1 mark for at least one term correct]

5 $2 \xrightarrow{+4} 6 \xrightarrow{+6} 12 \xrightarrow{+8} 20$

The difference is increasing by 2, so the next term is:
20 + 10 = 30
[2 marks available — 1 mark for spotting the pattern, 1 mark for the correct answer]

6 Largest possible value of $p = 45$
Smallest possible value of $q = 26$ ***[1 mark for both]***
Largest possible value of $p - q = 45 - 26 = 19$ ***[1 mark]***.
[2 marks available in total — as above]

Page 57 (Warm-up Questions)

1 a) $(x + 5)(x - 3)$ b) $(x + 1)(x - 3)$
c) $(x + 4)(x + 3)$

2 a) $x = -2$ or $x = -5$ b) $x = 2$ or $x = -7$
c) $x = 2$ or $x = 3$

3 $x = 1, y = 2$

4 $x = 3, y = -1$

5 a) E.g. 25 ($= 5^2$) b) E.g. 2 and 3 ($2 \times 3 = 6$)

6 LHS: $(x + 2)^2 + (x - 2)^2 = (x^2 + 4x + 4) + (x^2 - 4x + 4)$
$= 2x^2 + 8$
$= 2(x^2 + 4) =$ RHS

Page 58 (Exam Questions)

3 a) 16 is a factor of 48 ***[1 mark]***
b) E.g. 4 + 16 = 20, which is even ***[1 mark]***
c) E.g. 38 is not a multiple of 4, 6 or 8 ***[1 mark]***

4 $x + 3y = 11$ (1) $\xrightarrow{\times 3}$ $3x + 9y = 33$ (3) ***[1 mark]***
$3x + y = 9$ (2)
(3) – (2):

$$\begin{array}{rl} & 3x + 9y = 33 \\ - & \underline{3x + y = 9} \\ & 8y = 24 \\ & y = 3 \ \textbf{\textit{[1 mark]}} \end{array} \qquad \begin{array}{l} x + 3y = 11 \\ x + (3 \times 3) = 11 \\ x = 11 - 9 \\ x = 2 \ \textbf{\textit{[1 mark]}} \end{array}$$

[3 marks available in total — as above]

5 6 and 2 multiply to give 12 and subtract to give 4,
so if $x^2 + 4x - 12 = 0$,
$(x + 6)(x - 2) = 0$
[1 mark for correct numbers in brackets, 1 mark for correct signs]
$x + 6 = 0$ or $x - 2 = 0$
$x = -6$ or $x = 2$
[1 mark for both solutions]
[3 marks available in total — as above]

6 $2(18 + 3q) + 3(3 + q) = 36 + 6q + 9 + 3q$
$= 9q + 45$
$= 9(q + 5)$

$2(18 + 3q) + 3(3 + q)$ can be written as 9 × a whole number (where the whole number is $(q + 5)$), so it is a multiple of 9.
[3 marks available — 1 mark for expanding brackets and simplifying, 1 mark for writing the expression as 9(q + 5), 1 mark for explaining why this is a multiple of 9]

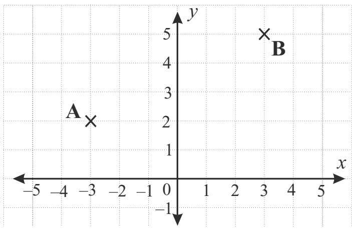
Page 59 (Revision Questions)

1 a) $3e$ b) $12f$
2 a) $7x - y$ b) $3a + 9$
3 a) m^3 b) $7pq$ c) $18xy$
4 a) $6x + 18$ b) $-9x + 12$ c) $5x - x^2$
5 $6x$
6 a) $2x^2 - x - 10$ b) $25y^2 + 20y + 4$
7 Putting in brackets (the opposite of multiplying out brackets).
8 a) $8(x + 3)$ b) $9x(2x + 3)$ c) $(6x + 9y)(6x - 9y)$
9 a) $x = 7$ b) $x = 16$ c) $x = 3$
10 a) $x = 4$ b) $x = 2$ c) $x = 3$
11 $Q = 8$
12 14
13 37 marbles
14 $6x$ cm
15 $v = \dfrac{W - 5}{4}$
16 a) 31, rule is add 7
b) 256, rule is multiply by 4
c) 19, rule is add previous two terms.
17 $6n - 2$
18 Yes, it's the 5th term.
19 a) x is greater than minus seven.
b) x is less than or equal to six.
20 $k = 1, 2, 3, 4, 5, 6, 7$
21 a) $x < 10$ b) $x \leq 7$
22 a) $(x + 2)(x + 8)$ b) $(x + 1)(x - 7)$
23 $x = -6$ or $x = 3$
24 $x = 2, y = 5$
25 E.g. 19
26 $3(y + 2) + 2(y + 6)$
$= 3y + 6 + 2y + 12$
$= 5y + 18 = 5(y + 3) + 3.$
$5(y + 3)$ is a multiple of 5, so $5(y + 3) + 3$ is not a multiple of 5.

Section Three — Graphs

Page 66 (Warm-up Questions)

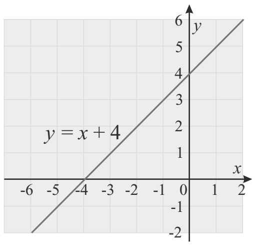
1 a)



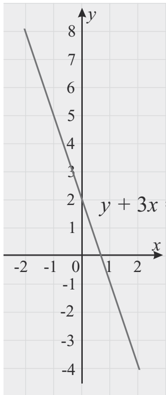
b) Midpoint = (0, 3.5)

2 a) Yes b) No c) Yes d) Yes
Try to rearrange each equation into the form y = mx + c — if you can do this then it's a straight line, if you can't it's not.

3



4



5 –5
6 –2
7 a) $y = \frac{1}{2}x + 5$ b) $y = \frac{1}{2}x + 10$

Page 67 (Exam Questions)

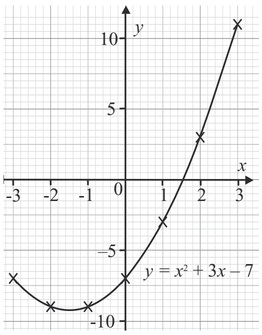
2 a) $\left(\frac{1+3}{2}, \frac{3+(-1)}{2}\right) = (2, 1)$
[2 marks available — 1 mark for correct method and 1 mark for correct final answer]
A correct method here is to find the averages of the x- and y-coordinates. Or, you could identify the midpoint of AB on the graph to get your answer — but the first way is much safer.
b) Comparing coordinates of point **C** and midpoint of **CD**:
x-distance = 2 – 0 = 2
y-distance = 1 – –1 = 2
So to get from the midpoint to point **D**, move up 2 and right 2.
So point **D** is (2 + 2, 1 + 2) = (4, 3)
[2 marks available — 1 mark for each correct coordinate]

3 Find the gradient: $\dfrac{\text{change in } y}{\text{change in } x} = \dfrac{4 - 1}{1 - 0} = 3$
Line crosses y-axis at 1, so equation of line is $y = 3x + 1$.
[3 marks available — 3 marks for a fully correct answer, otherwise 2 marks for a correct gradient, or 1 mark for a correct method to find the gradient]

4 The lines are parallel, so their gradients are equal: m = 4 ***[1 mark]***
When $x = -1$, $y = 0$, so put this into $y = 4x + c$
$0 = (4 \times -1) + c$, so c = 4 ***[1 mark]***
So equation of line is $y = 4x + 4$ ***[1 mark]***
[3 marks available in total — as above]

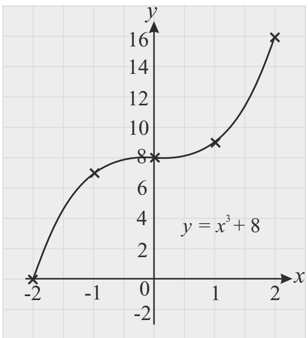
Page 74 (Warm-up Questions)

1 a)



b) (–1.5, –9.25)

2



3 $x = 6$, $y = 8$

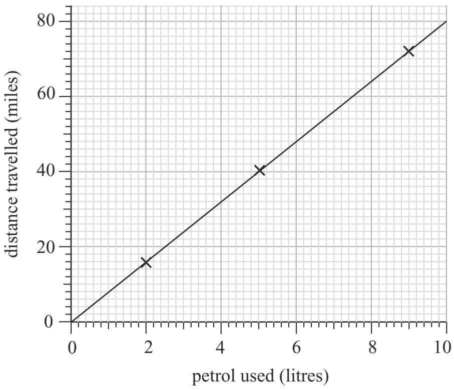
4 a) 15 minutes b) 12 km/h

5 a) 36 litres (allow 35.5-36.5 litres)
b) 4.4 gallons (allow 4.3-4.5 gallons)

6 Graph B

Pages 75-76 (Exam Questions)

2 a)



[2 marks available — 1 mark for all points plotted correctly, 1 mark for straight line joining points]

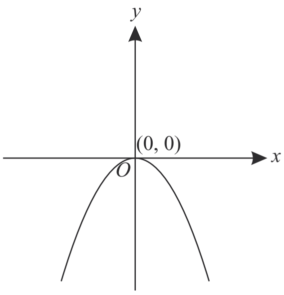
b) Gradient = $\frac{\text{change in } y}{\text{change in } x} = \frac{80-0}{10-0} = 8$

[2 marks available — 1 mark for correct method to find gradient, 1 mark for correct answer]

c) Distance travelled in miles per litre of petrol used ***[1 mark]***

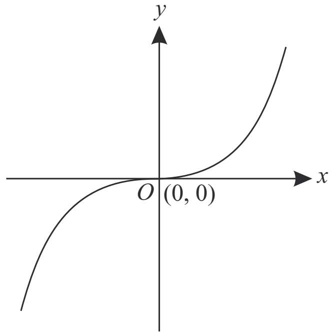
3 a) 0 ***[1 mark]***
b) –2, 0 ***[1 mark]***
c) (–1, –1) ***[1 mark]***

4 a)



[2 marks available — 1 mark for correct shape, 1 mark for labelling (0, 0)]

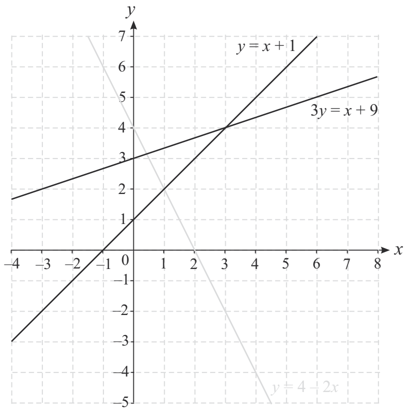
b)



[2 marks available — 1 mark for correct shape, 1 mark for labelling (0, 0)]

5 a) $x = 1$ ***[1 mark]***

b)



$x = 3$, $y = 4$

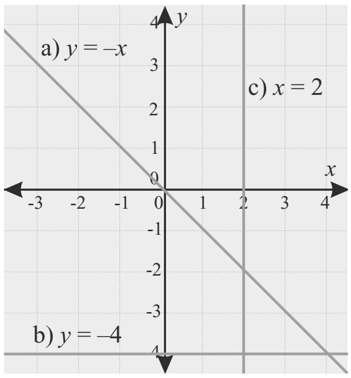
[3 marks available — 2 marks for correctly drawing the line 3y = x + 9, 1 mark for the correct answer]

Page 77 (Revision Questions)

1 A(5, –3), B(4, 0), C(0, 3), D(–4, 5), E(–2, –3)

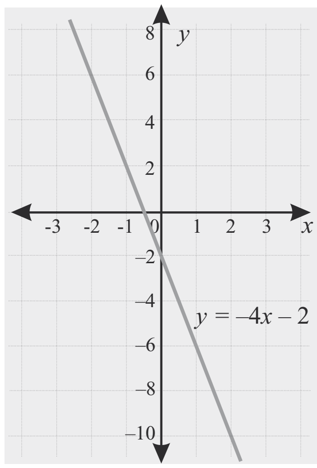
2 Midpoint = (2, 1.5)

3



4 E.g.

x	–2	0	2
y	6	–2	–10



5 a) 2 b) $y = x + 15$

6 $y = 2x - 6$

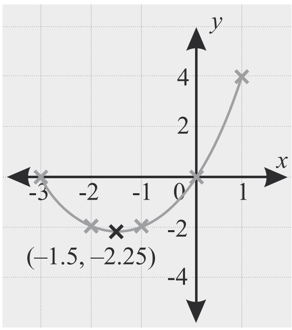
7 $y = x - 9$

8 They are both "bucket shaped" graphs. $y = x^2 - 8$ is like a "u" whereas $y = -x^2 + 2$ is like an "n" (or an upturned bucket).

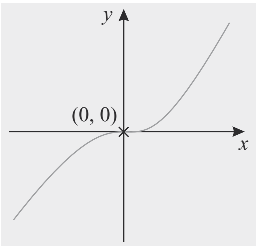
9 a)

x	–3	–2	–1	0	1
y	0	–2	–2	0	4

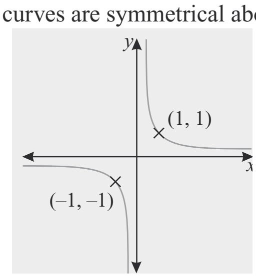
b)



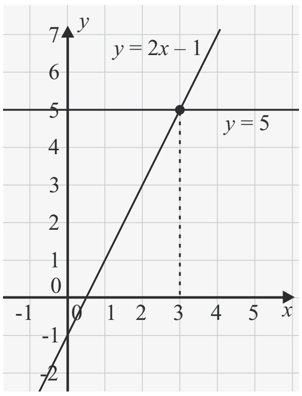
10 a) A graph with a "wiggle" in the middle. E.g.



b) A graph made up of two curves in opposite corners. The curves are symmetrical about the lines $y = x$ and $y = -x$. E.g.



11



$x = 3$

12 The object has stopped.

13 a) Ben drove fastest on his way home. b) 15 minutes

14 You would have to find the gradient, as the gradient = the rate of change.

15 a) 20 minutes b) £20

c) 26 minutes d) 67p — allow between 65p and 69p

Section Four — Ratio, Proportion, and Rates of Change

Page 81 (Warm-up Questions)

1 a) 1 : 2 b) 4 : 9 c) 2 : 9 d) 16 : 7 e) 5 : 4

2 1 : 4.4

3 300 g ÷ 2 = 150 g; 150 g × 3 = 450 g of flour

4 a) 12 : 6 = 2 : 1 b) $\frac{12}{18} = \frac{2}{3}$

5 20

6 £1000 : £1400

7 45, 60, 75

(3 + 4 + 5 = 12 parts, so 180 ÷ 12 = 15 per part.)

Page 82 (Exam Questions)

2 a) Shortest side of shape A = 3 units
Shortest side of shape B = 6 units
Ratio of shortest sides = 3 : 6 = 1 : 2
[2 marks available — 1 mark for finding the shortest sides of the triangles, 1 mark for the correct answer]

b) Area of shape A = $\frac{1}{2} \times 3 \times 4$ = 6 square units ***[1 mark]***
Area of shape B = $\frac{1}{2} \times 6 \times 8$ = 24 square units ***[1 mark]***
Ratio of areas = 6 : 24 = 1 : 4 ***[1 mark]***
[3 marks available in total — as above]

3 Donations account for 14 parts = £21 000
So 1 part = £21 000 ÷ 14 = £1500 ***[1 mark]***
Bills are 5 parts so cost £1500 × 5 = £7500 ***[1 mark]***
£21 000 – £7500 = £13 500 ***[1 mark]***
[3 marks available in total — as above]
Careful here — you are given a part : whole ratio in the question.

4 Mr Appleseed's Supercompost is made up of 4 + 3 + 1 = 8 parts, so contains: $\frac{4}{8}$ soil, $\frac{3}{8}$ compost and $\frac{1}{8}$ grit.
16 kg of Mr Appleseed's Supercompost contains:
$\frac{4}{8} \times 16$ = 8 kg of soil
$\frac{3}{8} \times 16$ = 6 kg of compost
$\frac{1}{8} \times 16$ = 2 kg of grit
Soil costs £8 ÷ 40 = £0.20 per kg.
Compost costs £15 ÷ 25 = £0.60 per kg.
Grit costs £12 ÷ 15 = £0.80 per kg.
16 kg of Mr Appleseed's Supercompost costs:
(8 × 0.2) + (6 × 0.6) + (2 × 0.8) = £6.80
[5 marks available — 1 mark for finding the fractions of each material in the mix, 1 mark for the correct mass of one material, 1 mark for the correct masses for the other two materials, 1 mark for working out the price per kg for each material, 1 mark for the correct answer]

Page 86 (Warm-up Questions)

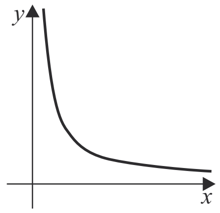
1 £1.28

2 a) 80p b) 32

3 525 g
In the 250 g jar you get 250 g ÷ 125p = 2 g per p,
in the 350 g jar you get 350 g ÷ 210p = 1.666... g per p,
in the 525 g jar you get 525 g ÷ 250p = 2.1 g per p.

4 1.5 hours

5 Direct proportion graphs are straight lines and they go through the origin.

6 

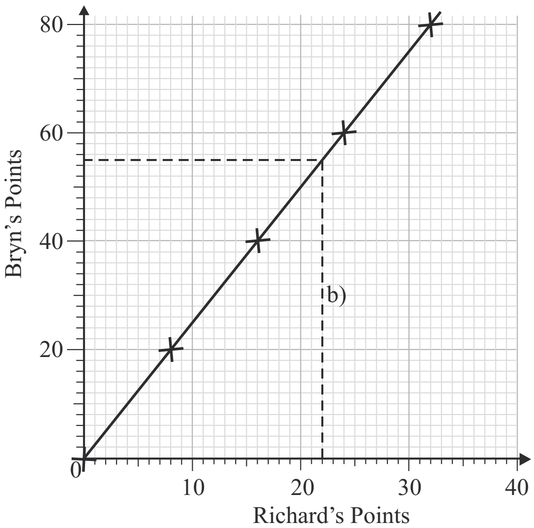
Page 87 (Exam Questions)

3 250 ml bottle: 250 ÷ 200 = 1.25 ml per penny
330 ml bottle: 330 ÷ 275 = 1.2 ml per penny
525 ml bottle: 525 ÷ 375 = 1.4 ml per penny
So the 525 ml bottle is the best value for money.
[3 marks available — 3 marks for finding the correct amounts per penny for all three bottles and the correct answer, otherwise 2 marks for two correct amounts per penny or 1 mark for one correct amount per penny]
You could also compare the cost per ml of each bottle.

4 1 bottle of water costs £52.50 ÷ 42 = £1.25 ***[1 mark]***
There are £35 ÷ £1.25 = 28 girls in the club ***[1 mark]***
[2 marks available in total — as above]

5 a) 250 people can be catered for 6 days
1 person can be catered for 6 × 250 = 1500 days
300 people can be catered for 1500 ÷ 300 = 5 days
[2 marks available — 1 mark for a correct method, 1 mark for the correct answer]

b) For a 1-day cruise it could cater for 6 × 250 = 1500 people
For a 2-day cruise it could cater for 1500 ÷ 2 = 750 people
So it can cater for 750 – 250 = 500 more people
[3 marks available — 1 mark for a correct method to find the number of people catered for on a 2-day cruise, 1 mark for the correct number of people catered for on a 2-day cruise, 1 mark for the correct final answer]

6 a) 

[2 marks available — 1 mark for two points marked correctly, 1 mark for the correct straight line]
Use the ratio to work out the coordinates of a few points to plot. E.g. If Richard scored 8 points, Bryn scored 8 × $\frac{5}{2}$ = 20 points.

b) 55 points (see graph) ***[1 mark]***

Page 93 (Warm-up Questions)

1	£17	2	74%
3	9	4	£138
5	£205	6	25%
7	£6.10	8	£3376.53

Pages 94 (Exam Questions)

2 20% increase = 1 + 0.2 = 1.2
20% increase of £33.25 = 1.2 × £33.25 = £39.90
[2 marks available — 1 mark for a correct method, 1 mark for the correct answer]

3 He normally gets 240 ÷ 40 = 6 packs ***[1 mark]***
40% cheaper = 1 – 0.4 = 0.6
So the stickers are 40p × 0.6 = 24p per pack this week ***[1 mark]***
He can buy 240 ÷ 24 = 10 packs this week ***[1 mark]***
So he can get 10 – 6 = 4 more packs ***[1 mark]***
[4 marks available in total — as above]

4 A ratio of 3 : 7 means 3 out of 10 = 30% of the animals are cats
40% of 30% = 0.4 × 30% = 12% are black cats ***[1 mark]***
100% – 30% = 70% are dogs ***[1 mark]***
50% of 70% = 0.5 × 70% = 35% are black dogs ***[1 mark]***
So, 35% + 12% = 47% are black animals ***[1 mark]***
[4 marks available in total — as above]

5 Multiplier = 1 + 0.06 = 1.06
After 1 year she will owe: £750 × 1.06 = £795
After 2 years she will owe: £795 × 1.06 = £842.70
After 3 years she will owe: £842.70 × 1.06 = £893.262
= £893.26 (to the nearest penny)
[3 marks available — 1 mark for working out the multiplier, 1 mark for a correct method, 1 mark for the correct answer]

Page 100 (Warm-up Questions)

1 a) 65 mm b) 0.25 kg
2 160 kg
3 3 feet 10 inches
4 a) 320 km b) 4 feet
To do these conversions, find the conversion factor, then multiply and divide by it. Then choose the most sensible answer.
5 a) 230 000 cm^2 b) 3.45 m^2
6 99 minutes
7 4.05 pm
8 2800 N
9 8 m/s
10 3.125 g/cm^3

Pages 101 (Exam Questions)

2 4.30 pm till 5.00 pm is 30 minutes.
5.00 pm till 7.00 pm is 2 hours.
7.00 pm till 7.15 pm is 15 minutes.
So they spend:
2 hours + 30 minutes + 15 minutes = 2 hours 45 minutes
2 hours 45 minutes = 2.75 hours
2.75 × 12 = 33 hours
33 hours + 7 hours 10 minutes = 40 hours 10 minutes
[4 marks available — 1 mark for a correct method to find the time from 4.30 pm till 7.15 pm, 1 mark for finding the correct time from 4.30 pm till 7.15 pm, 1 mark for the correct total time for the first 12 days, 1 mark for the correct answer]

3 64 pints = 64 ÷ 8 = 8 gallons ***[1 mark]***
8 gallons = 4 × 2 gallons ≈ 4 × 9 litres ***[1 mark]***
= 36 litres ***[1 mark]***
[3 marks available in total — as above]

4 One book weighs 0.55 lb so 8 books will weigh
8 × 0.55 lb = 4.4 lb ***[1 mark]***
1 kg ≈ 2.2 lb
4.4 ÷ 2.2 = 2
So the eight books weigh 2 kg. ***[1 mark]***
1 kg = 1000 g
2 × 1000 = 2000
So the books weigh 2000 g. ***[1 mark]***
For 100 g postage is £0.50 so for 2000 g postage is
£0.50 × 20 = £10. ***[1 mark]***
[4 marks available in total — as above]

5 Area of face A = 2 m × 4 m = 8 m² ***[1 mark]***
Pressure = Force ÷ Area = 40 N ÷ 8 m² ***[1 mark]***
= 5 N/m² ***[1 mark]***
[3 marks available in total — as above]

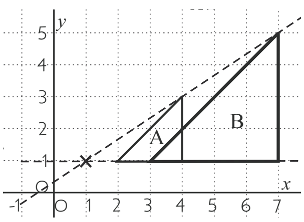
Page 102 (Revision Questions)

1 a) 9 : 11 b) 3.5 : 1
2 80 blue scarves
3 $\frac{7}{2}$ or 3.5
4 a) $\frac{5}{25}$ or $\frac{1}{5}$ b) 384
5 51 ml olive oil, 1020 g tomatoes, 25.5 g garlic powder, 204 g onions
6 960 flowers
7 See p.84
8 The 500 ml tin
9 18
10 a) 19 b) 114 c) 21.05% (2 d.p.) d) 475%
11 percentage change = (change ÷ original) × 100
12 35% decrease
13 17.6 m
14 2%
15 a) £117.13 (to the nearest penny) b) 6 years
16 a) 5600 cm³ b) 240 cm c) 336 hours
d) 12 000 000 cm³ e) 12.8 cm² f) 2750 mm³
17 9.48 pm
18 67.2 km/h
19 12 500 cm³
20 11 m²

Section Five — Shapes and Area

Page 110 (Warm-up Questions)

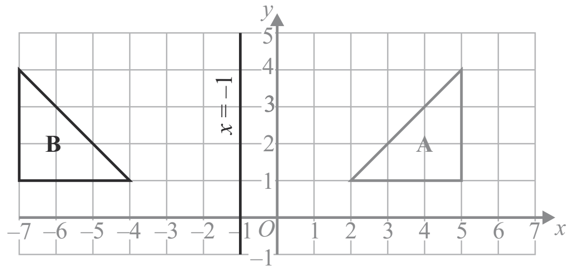
1 C 1 line of symmetry, rotational symmetry order 1
W 1 line of symmetry, rotational symmetry order 1
+ 2 lines of symmetry, rotational symmetry order 2
D 1 line of symmetry, rotational symmetry order 1
Q 0 lines of symmetry, rotational symmetry order 1
2 An equilateral triangle has 3 equal sides, 3 equal angles of 60°, 3 lines of symmetry and rotational symmetry of order 3.
3 A kite has 1 line of symmetry.
4 a) **B** and **E** are similar.
b) **A** and **D** are congruent.
5 A → B — rotation of 90° clockwise about the origin.
B → C — reflection in the line $y = x$.
C → A — reflection in the y-axis.
A → D — translation by the vector $\binom{-9}{-7}$.
6



An enlargement of scale factor 2, centre (1, 1).

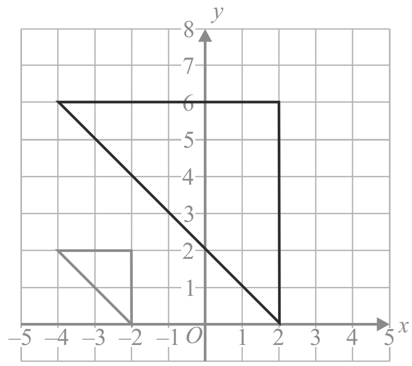
Page 111 (Exam Questions)

2 $x = 3$ ***[1 mark]***
3



[2 marks available — 2 marks for correct reflection, otherwise 1 mark for triangle reflected but in wrong position]

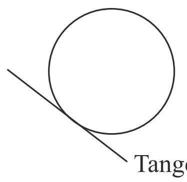
4 a) Scale factor from $EFGH$ to $ABCD$ = 9 ÷ 6 = 1.5 ***[1 mark]***
EF = 6 ÷ 1.5 = 4 cm ***[1 mark]***
[2 marks available in total — as above]
b) BC = 4 × 1.5 = 6 cm ***[1 mark]***
5



[3 marks available — 3 marks for correct enlargement, otherwise 2 marks for a correct triangle but in the wrong position or for an enlargement from the correct centre but of the wrong scale factor, or 1 mark for 2 lines enlarged by the correct scale factor anywhere on the grid]

Page 114 (Warm-up Questions)

1 42 cm
2 a) area = length × width, A = l × w
b) circumference = π × diameter, C = π × D (or C = 2πr)
c) area = base × vertical height, A = b × h
3 10.5 m² *(Area = ½ × base × vertical height = 0.5 × 3 × 7)*
4 201.06 cm² to 2 d.p. (or 201.09 cm² to 2 d.p. using π = 3.142) *(Area = $\pi r^2 = \pi \times 8^2$)*
5 a) A straight line that just touches the outside of a circle.
b)



Tangent

Page 115 (Exam Questions)

3 a) Area of trapezium = ½(8 + 11) × 6
= ½ × 19 × 6 = 57 cm²
Area of triangle = area of trapezium ÷ 3 = 57 ÷ 3 = 19 cm²
Total area of the shape = area of trapezium + area of triangle
= 57 + 19 = 76 cm²
[3 marks available — 1 mark for the area of the trapezium, 1 mark for the area of the triangle, 1 mark for correct final answer]
b) Area of triangle = ½ × base × height
19 = ½ × 8 × height ***[1 mark]***
height = 19 ÷ 4 = 4.75 cm ***[1 mark]***
[2 marks available in total — as above]

4 Area of rectangle = 6 × 8 = 48 cm^2 ***[1 mark]***
Base of triangle = 8 cm – 5 cm = 3 cm
Height of triangle = 6 cm – 2 cm = 4 cm
[1 mark for base and height]
Area of triangle = $\frac{1}{2}$ × 3 × 4 = 6 cm^2 ***[1 mark]***
Area of shaded area = 48 – 6 = 42 cm^2 ***[1 mark]***
[4 marks available in total — as above]

5 Circumference of full circle = 2 × π × 6 = 12π cm
Length of arc = $\frac{30}{360}$ × circumference of circle
= $\frac{30}{360}$ × 12π = π cm
Perimeter of sector = π + 6 + 6 = 15.1415... = 15.1 cm (3 s.f.)
Area of full circle = π × 6^2 = 36π cm^2
Area of sector = $\frac{30}{360}$ × area of circle
= $\frac{30}{360}$ × 36π = 3π cm^2 = 9.4247... = 9.42 cm^2 (3 s.f.)
[5 marks available — 1 mark for a correct method for calculating the length of the arc, 1 mark for correct arc length, 1 mark for correct perimeter of sector, 1 mark for a correct method for finding the area of the sector, 1 mark for correct area of sector]

Page 122 (Warm-up Questions)

1 A cuboid has 6 faces, 8 vertices and 12 edges.

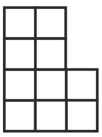
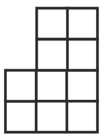
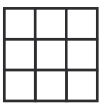
2 7 cm

3 8 cm^3 *(8 × 1 cm^3, or v = l × w × h = 2 cm × 2 cm × 2 cm)*

4 672 cm^3 *(area of triangle × length = ½ × 12 × 8 × 14)*

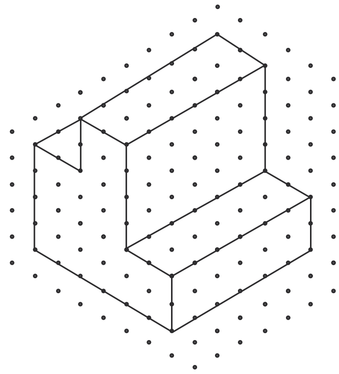
5 860π m^3

6 Volume of sphere = $\frac{4}{3}\pi r^3$
= $\frac{4}{3} \times \pi \times 3^3$
= 36π cm^3
Volume of cylinder = $\pi r^2 h$
= $\pi \times 6^2 \times 10$
= 360π cm^3
$\frac{\text{Volume of cylinder}}{\text{Volume of sphere}} = \frac{360\pi}{36\pi}$
$= \frac{360}{36} = 10$

7 a)

b)

c)


Pages 123-124 (Exam Questions)

2 Split the shape into two cuboids, looking at the front elevation. The bottom cuboid has 4 × 2 × 4 = 32 cubes in it. The cuboid at the top has 2 × 2 × 4 = 16 cubes in it. So there are 32 + 16 = 48 cubes in the shape.
[2 marks available — 1 mark for a correct calculation, 1 mark for the correct answer]
You might have split your shape up differently — as long as your working is correct and you get the correct answer, you'll get all the marks.

3

[2 marks available — 2 marks for a correct diagram, otherwise 1 mark for the correct cross-section but wrong length]

4 Volume of sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times 15^3$ ***[1 mark]*** = 4500π cm^3
= 14 137.166... = 14 100 cm^3 (3 s.f.) ***[1 mark]***
[2 marks available in total — as above]

5 a) Volume = 90 × 40 × 30 ***[1 mark]*** = 108 000 cm^3 ***[1 mark]***
[2 marks available in total — as above]
b) Volume of cuboid = length × width × height
108 000 = 120 × width × 18
108 000 = 2160 × width
width = 108 000 ÷ 2160 = 50 cm
[2 marks available — 1 mark for correctly rearranging the formula to find the width, 1 mark for the correct answer]

6 a) Volume of water in paddling pool = π × r^2 × h
= π × 100^2 × 40 ***[1 mark]*** = 400 000π cm^3 ***[1 mark]***
[2 marks available in total — as above]
b) Time it will take to fill to 40 cm = 400 000π ÷ 300 ***[1 mark[***
= 4188.790... seconds
Convert to minutes = 4188.790... ÷ 60
= 69.813... = 70 minutes (to the nearest minute) ***[1 mark]***
[2 marks available in total — as above]

Page 125 (Revision Questions)

1 H: 2 lines of symmetry,
rotational symmetry order 2
Z: 0 lines of symmetry,
rotational symmetry order 2
T: 1 line of symmetry,
rotational symmetry order 1
N: 0 lines of symmetry,
rotational symmetry order 2
E: 1 line of symmetry,
rotational symmetry order 1
✕: 4 lines of symmetry,
rotational symmetry order 4
S: 0 lines of symmetry,
rotational symmetry order 2

2 2 angles the same, 2 sides the same, 1 line of symmetry, no rotational symmetry.

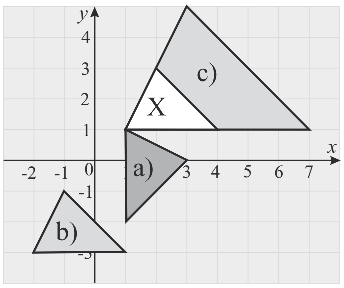
3 2 lines of symmetry, rotational symmetry order 2

4 Congruent shapes are exactly the same size and same shape. Similar shapes are the same shape but different sizes.

5 a) D and G b) C and F

6 b = 89, y = 5

7 a) Translation of $\begin{pmatrix} -2 \\ -4 \end{pmatrix}$.
b) Reflection in x = 0 (the y-axis).

8 

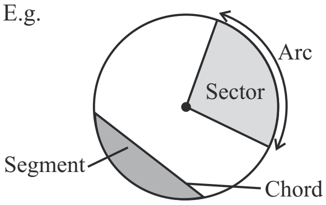
9 Area = ½(a + b) × h

10 36 cm^2

11 52 cm^2

12 Area = 153.94 cm^2 (2 d.p.)
Circumference = 43.98 cm (2 d.p.)

13 E.g.



14 Area = 7.07 cm^2 (2 d.p.)
Perimeter = 10.71 cm (2 d.p.)

15 a) faces = 5, edges = 8, vertices = 5
b) faces = 2, edges = 1, vertices = 1
c) faces = 5, edges = 9, vertices = 6

16 150 cm^2

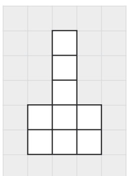
17 125.7 cm^2 (1 d.p.)

18 Volume = $\pi r^2 h$

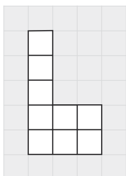
19 360 cm^3

20 a) 113.10 cm^3 (2 d.p.)
b) 4.5 s (1 d.p.)

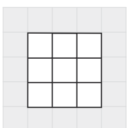
21 Front:



Side:



Plan:



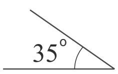
Section Six — Angles and Geometry

Page 131 (Warm-up Questions)

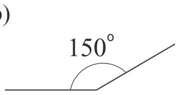
1 a) Any angle less than 90°
b) Any angle greater than 90° and less than 180°
c) Any angle greater than 180° and less than 360°
d) 90°

2 a) 37° b) 287°
Accept answers within 2°

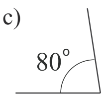
3 a)



b)



c)



4 x = 110º, y = 40º
Using angles in an isosceles triangle and angles on a straight line.

5 a = 120°, b = 60°
Using the rule for allied angles (60° + a = 180°) and using the rule for corresponding angles (b = 60°).

6 144°

Page 132 (Exam Questions)

3 a = 75° ***[1 mark]***
because vertically opposite angles are equal. ***[1 mark]***
[2 marks available in total — as above]

4 70° + 90° + 97° = 257°
Angle ADC = 360° – 257° = 103°
(angles in a quadrilateral add up to 360°)
[2 marks available — 1 mark for a correct method, 1 mark for the correct answer]

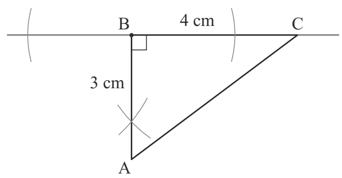
5 Exterior angle = 180° – 150° = 30° ***[1 mark]***
Number of sides = 360° ÷ 30° ***[1 mark]***
= 12 ***[1 mark]***
[3 marks available in total — as above]

6 Angle BCG = Angle CGE = x *(alternate angles)*
So 78° + x = 180° ***[1 mark]***
x = 180° – 78°
x = 102° ***[1 mark]***
[2 marks available in total — as above]
There are other ways to find x. For instance angles ACB and CGF are corresponding angles. You can then use angles on a straight line to find x.

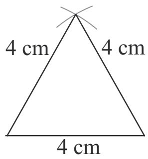
7 The polygon is split into 5 triangles.
Angles in a triangle add up to 180°
Angles in polygon = 5 × 180°
= 900°
[3 marks available — 3 marks for correct explanation, otherwise 1 mark for stating angles in triangle add up to 180° and 1 mark for an attempt at adding to find angles in polygon]

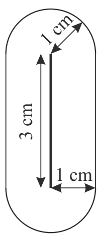
Page 140 (Warm-up Questions)

1 a)

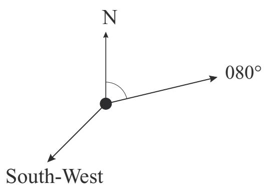


b) 5 cm

2 

3 

4 a) 1100 m *(5.5 × 200)* b) 2.5 cm *(500 ÷ 200)*

5 

Pages 141-142 (Exam Questions)

3 a) Drawing of dining table is 4 cm long.
So 4 cm is equivalent to 2 m.
$2 \div 4 = 0.5$
Therefore scale is 1 cm to 0.5 m ***[1 mark]***

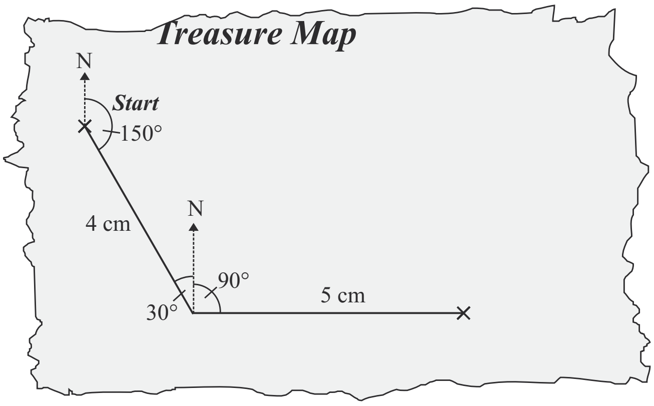
b) On drawing, dining table is 3 cm from shelves.
So real distance = $3 \times 0.5 = 1.5$ m ***[1 mark]***

c) The chair and the space around it would measure 4 cm × 5 cm on the diagram and there are no spaces that big, so no, it would not be possible.
[2 marks available — 1 mark for correct answer, 1 mark for reasoning referencing diagram or size of gaps available]

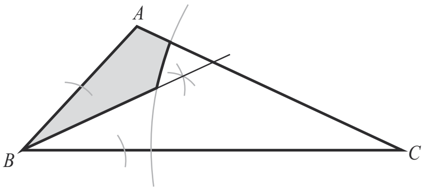
4 260° (allow 258°-262°) ***[1 mark]***
It's easier to measure the 100° angle and subtract it from 360°.

5 180° – 79° = 101° *(allied angles)* ***[1 mark]***
360° – 101° = 259°
Ruth travels on a bearing of 259°. ***[1 mark]***
[2 marks available in total — as above]

6 Using the scale 1 cm = 100 m:
400 m = 4 cm and 500 m = 5 cm



[3 marks available — 1 mark for line on accurate bearing of 150°, 1 mark for line on accurate bearing of 090°, 1 mark for accurate 4 cm and 5 cm line lengths]

7


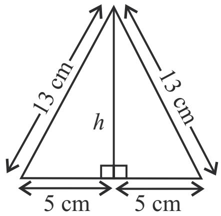
[4 marks available — 1 mark for arc with radius of 6.5 cm with centre at C, 1 mark for construction arcs on AB and BC for angle bisector at ABC, 1 mark for correct angle bisector at ABC, and 1 mark for the correct shading]
Remember to leave in your construction lines.

Page 149 (Warm-up Questions)

1 10.3 m
2 3.8 m
3 $y = 4.24$ cm
4 3 m
5 $\begin{pmatrix} 1 \\ 5 \end{pmatrix}$
6 $\mathbf{c} + \mathbf{d}$

Page 150 (Exam Questions)

3 The triangle can be split into two right-angled triangles.



Let h be the height of the triangle:
$13^2 = 5^2 + h^2$ ***[1 mark]***
$h^2 = 169 - 25 = 144$
$h = \sqrt{144}$ ***[1 mark]***
$h = 12$ cm ***[1 mark]***
[3 marks available in total — as above]

4 $\sin x = \frac{14}{18}$ ***[1 mark]***
$x = \sin^{-1}\left(\frac{14}{18}\right)$ ***[1 mark]***
$x = 51.0575... = 51.1°$ (1 d.p) ***[1 mark]***
[3 marks available in total — as above]

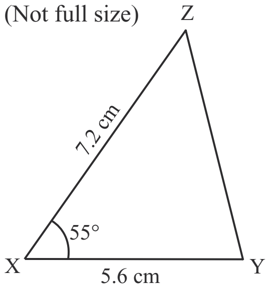
5 a) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$ ***[1 mark]***
$= 2\mathbf{c} + 2\mathbf{d}$ ***[1 mark]***
[2 marks available in total — as above]

b) $\overrightarrow{AL} = \frac{1}{2} \times \overrightarrow{AC} = \frac{1}{2} \times 2\mathbf{c} + \frac{1}{2} \times 2\mathbf{d}$ ***[1 mark]***
$= \mathbf{c} + \mathbf{d}$ ***[1 mark]***
[2 marks available in total — as above]

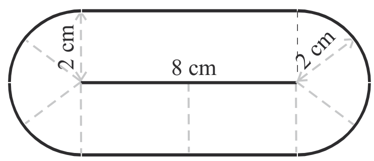
c) $\overrightarrow{BL} = \overrightarrow{BA} + \overrightarrow{AL}$ ***[1 mark]***
$= -2\mathbf{c} + \mathbf{c} + \mathbf{d}$
$= -\mathbf{c} + \mathbf{d}$ ***[1 mark]***
[2 marks available in total — as above]

Pages 151-152 (Revision Questions)

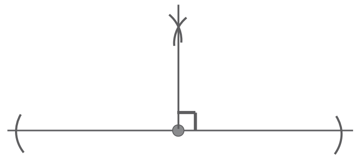
1 An obtuse angle
2 360°
3 a) 154° b) 112° c) 58°
4 140°
5 60°
6 1080°
7 162°
8 (Not full size)

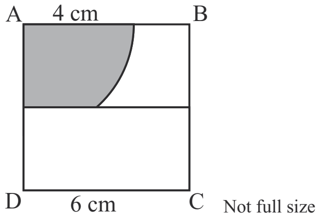


9 A circle

10


(Not full size)

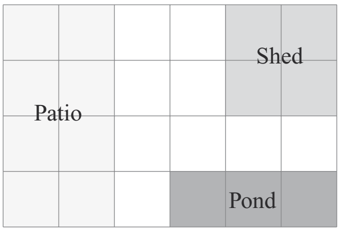
11


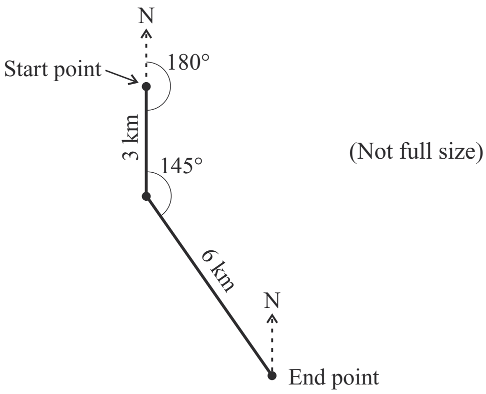
12 

13 295°

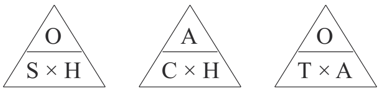
14 280°

15 26 km

16 

17 

18 14.5 cm

19 

20 $x = 33.4°$ (1 d.p.)

21 $x = 51.8°$ (1 d.p.)

22 $12\sqrt{3}$ cm

23 $\tan 45° + \sin 60° = 1 + \frac{\sqrt{3}}{2}$

$$= \frac{2}{2} + \frac{\sqrt{3}}{2}$$

$$= \frac{2 + \sqrt{3}}{2}$$

24 a) $\begin{pmatrix} -3 \\ -8 \end{pmatrix}$ b) $\begin{pmatrix} 20 \\ -10 \end{pmatrix}$

25 $2\mathbf{a} + 3\mathbf{b}$

26 $5\mathbf{a} - 7\mathbf{b}$

27 $-\mathbf{i} + \mathbf{j}$

Section Seven — Probability and Statistics

Page 157 (Warm-up Questions)

1 $\frac{3}{10}$ or 0.3

2 $\frac{5}{6}$

3 a) HH, HT, TH, TT

b) $\frac{1}{2}$ or 0.5

4

Score	Relative frequency
1	0.14
2	0.137
3	0.138
4	0.259
5	0.161
6	0.165

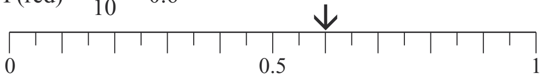
5 $\frac{15}{120} = 0.125$

6 150

Pages 158-159 (Exam Questions)

3 10 – 4 = 6 red counters.

P(red) = $\frac{6}{10} = 0.6$



[2 marks available — 2 marks for correctly drawn arrow, otherwise 1 mark for finding the correct probability of picking a red counter]

You could also work out the probability of a blue counter (0.4) and subtract it from 1 to get the probability of a red counter.

4 a) (Hockey, Netball), (Hockey, Choir), (Hockey, Orienteering), (Orchestra, Netball), (Orchestra, Choir), (Orchestra, Orienteering), (Drama, Netball), (Drama, Choir), (Drama, Orienteering).

[2 marks available — 2 marks for listing all 9 correct combinations, otherwise 1 mark if at least 5 combinations are correct]

b) There are 9 combinations and 1 of them is hockey and netball, so P(hockey and netball) = $\frac{1}{9}$ ***[1 mark]***

c) There are 9 combinations and 3 of them involve drama on Monday, so P(drama on Monday) = $\frac{3}{9} = \frac{1}{3}$ ***[1 mark]***

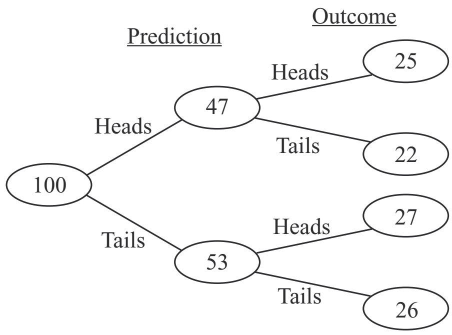
You could also count the choices for Monday — there are 3, and 1 of them is drama.

5 a) EHM, EMH, HME, HEM, MEH, MHE

[2 marks available — 2 marks for listing all 6 correct combinations, otherwise 1 mark if at least 3 combinations are correct]

b) There are 6 possible combinations and in 3 of them she does Maths before English (HME, MEH, MHE).

So P(Maths before English) = $\frac{3}{6} = \frac{1}{2}$ ***[1 mark]***

6 a) 

[2 marks available — 1 mark for the correct numbers for the predictions, 1 mark for the correct numbers for the outcomes]

b) She predicted the flip correctly 25 + 26 = 51 times out of 100 ***[1 mark]***, so relative frequency = $\frac{51}{100} = 0.51$ ***[1 mark]***

[2 marks available in total — as above]

7 a) Relative frequency of hitting the target with a left-handed throw = $\frac{12}{20} = \frac{3}{5}$ or 0.6.

[2 marks available — 1 mark for a correct method, 1 mark for the correct answer]

b) E.g. The estimated probability is more reliable for his right hand because he threw the ball more times with that hand. ***[1 mark]***

Page 163 (Warm-up Questions)

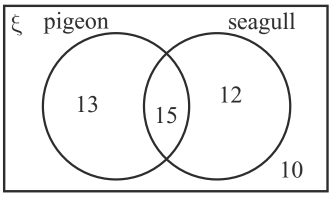
1 $\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$

2 $0.8 \times 0.9 = 0.72$

3 a) $0.3 + 0.35 = 0.65$ b) $0.3 \times 0.1 = 0.03$

4 $\frac{6}{10} \times \frac{6}{10} = \frac{36}{100} = \frac{9}{25}$

5 a)



b) P(saw seagull) = $\frac{27}{50}$

Page 164 (Exam Questions)

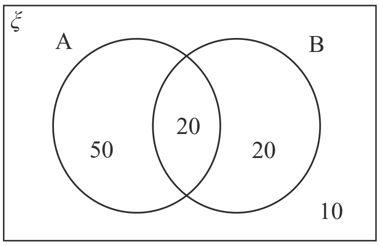
2 a) P(4 or 5) = P(4) + P(5)
= 0.25 + 0.1 ***[1 mark]***
= 0.35 ***[1 mark]***
[2 marks available in total — as above]

b) P(1 and 3) = P(1) × P(3)
= 0.3 × 0.2 ***[1 mark]***
= 0.06 ***[1 mark]***
[2 marks available in total — as above]

3 a) P(no prize) = 1 – 0.3 = 0.7 ***[1 mark]***

b) P(no prize on either game) = P(no prize) × P(no prize)
= 0.7 × 0.7 ***[1 mark]***
= 0.49 ***[1 mark]***
[2 marks available in total — as above]

4 a) Number of students who only like apples = 70 – 20 = 50
Number of students who only like bananas = 40 – 20 = 20
Number of students who don't like either
= 100 – 50 – 20 – 20 = 10



[3 marks available — 3 marks for a fully correct Venn diagram, lose 1 mark for each incorrect value]

b) 50 + 20 + 20 = 90 students out of 100 like apples or bananas,
so $P(A \cup B) = \frac{90}{100} = \frac{9}{10}$ or 0.9.
[2 marks available — 1 mark for 90 students liking either fruit, 1 mark for the correct answer.]

Page 168 (Warm-up Questions)

1 Two from, e.g: sample too small, one city centre not representative of the whole of Britain, only done in one particular place.

2 Discrete data
E.g.

Cinema visits	Tally	Frequency
0-9		
10-19		
20-29		
30-39		
40-49		
50 or over		

3 a) This question is ambiguous. "A lot of television" can mean different things to different people.

b) This is a leading question, inviting the person to agree.

c) The answers to this question do not cover all possible options.

4 First, order the numbers:
–14, –12, –5, –5, 0, 1, 3, 6, 7, 8, 10, 14, 18, 23, 25
Mean = 5.27 (2 d.p.), Median = 6, Mode = –5, Range = 39

5 2, 4 and 6

Page 169 (Exam Questions)

2 a) E.g.

Number of chocolate bars	Tally	Frequency
0-2		
3-5		
6-8		
9-11		
12 or more		

[2 marks available — 1 mark for a suitable tally table, 1 mark for non-overlapping classes that cover all possible values]

b) E.g. Faye's results are likely to be unrepresentative because she hasn't selected her sample at random from all the teenagers in the UK. Also, her sample is too small to represent the whole population. So Faye can't use her results to draw conclusions about teenagers in the UK.
[2 marks available — 1 mark for a correct comment based on sample size, 1 mark for stating that Faye can't draw conclusions about teenagers in the UK with reasoning]

3 a) Yes, the mean number is higher than 17 because the 11th data value is higher than the mean of the original 10 values.
[1 mark]

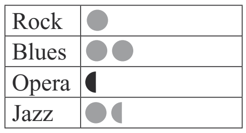
b) You can't tell if the median number is higher than 15, because you don't know the other data values.
[1 mark]

4 a) Mode = 1 ***[1 mark]***

b) Median = (25 + 1) ÷ 2 = 13th value ***[1 mark]***.
13th value is shown by the 2nd bar, so median = 1 ***[1 mark]***.
[2 marks available in total — as above]

Page 174 (Warm-up Questions)

1 a) 15 b)



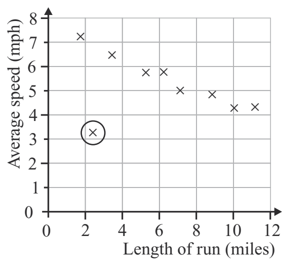
2

	Walk	Car	Bus	Total
Male	15	21	13	49
Female	18	11	22	51
Total	33	32	35	100

3 Rom Com = 23 × 6° = 138°
Western = 25 × 6° = 150°
Action = 12 × 6° = 72°

4 a) There is a strong negative correlation. The longer the run, the slower Sam's speed.

b)

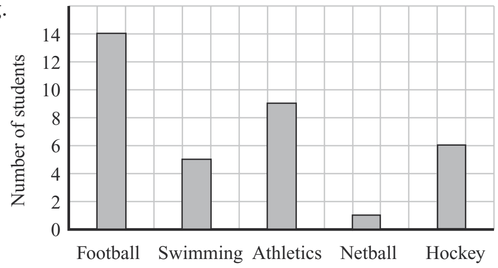


c) Approximately 5 mph (±0.5 mph)

d) The estimate should be reliable because **[either]** 8 miles is within the range of the known data **[or]** the graph shows strong correlation.

Page 175 (Exam Questions)

2 E.g.



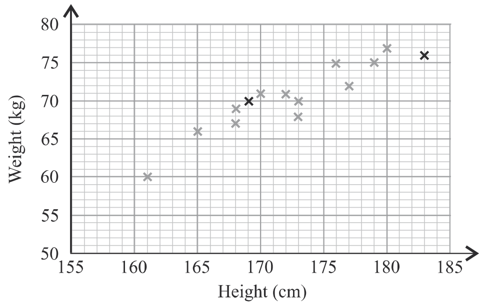
[4 marks available — 1 mark for a suitable scale starting from zero on the vertical axis, 1 mark for correctly labelling the vertical axis, 1 mark for bars of equal width and all bar heights correct, 1 mark for all bars correctly labelled]

3 a) $15:20 = 3:4$ ***[1 mark]***

b) $\frac{20}{50} \times 100$ ***[1 mark]*** = 40% ***[1 mark]***

[2 marks available in total — as above]

4 a)



[1 mark for both points plotted correctly]

b) $\frac{5}{14}$ ***[1 mark]***

c) E.g. In general, as the height increases, the weight also increases.
[1 mark for any answer indicating a positive correlation]

Page 180 (Warm-up Questions)

1 a) Median = 2 b) Mode = 2

2 E.g.

Height (h cm)	Tally	Frequency
$150 < h \leq 160$	||	2
$160 < h \leq 170$	卌	5
$170 < h \leq 180$	|||	3
$180 < h \leq 190$	||	2

3 a) Estimated range = 185 – 145 = 40

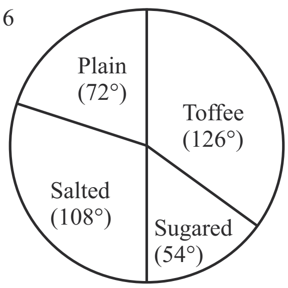
b) Median is in the group containing the 40th value, so the median group is $155 \leq x < 165$.

c) Modal Group = $165 \leq x < 175$

4 The mode is 30°C but most of the temperatures are less than 20°C, so it's not representative of all the data.

Page 181 (Exam Questions)

2 a) Total number of people = 12 + 18 + 9 + 21 = 60
Multiplier = 360 ÷ 60 = 6
Plain: 12 × 6 = 72°
Salted: 18 × 6 = 108°
Sugared: 9 × 6 = 54°
Toffee: 21 × 6 = 126°



[4 marks available — 1 mark for one sector correctly drawn, 1 mark for a second sector correctly drawn, 1 mark for a complete pie chart with all angles correct, 1 mark for correct labels]

b) E.g. Chris is not right because there is no information about the number of people in the ice-cream survey. ***[1 mark]***

3 a) Max value = 63 mm, min value = 8 mm ***[1 mark for both]***, so range = 63 – 8 = 55 mm ***[1 mark]***
E.g. a range of 55 mm isn't a good reflection of the spread of the data because most of the data is much closer together.
[1 mark for a correct comment]
[3 marks available in total — as above]
Or you could say that the single value of 63 mm has a big effect on increasing the value of the range so that it doesn't represent the spread of the rest of the data.

b) Median rainfall in June = (12 + 1) ÷ 2 = 6.5th value = (29 + 30) ÷ 2 = 29.5 mm
E.g. The rainfall was generally higher in June, as the median was higher. The rainfall in June was much more varied than in November as the range was much bigger.
[3 marks available — 1 mark for calculating the median rainfall in June, 1 mark for a correct statement comparing the medians and 1 mark for a correct statement comparing the ranges]

Page 182 (Revision Questions)

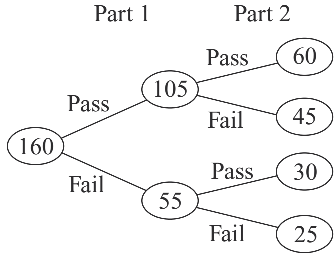
1 $\frac{8}{50} = \frac{4}{25}$

2 0.7

3 a) HHH, HHT, HTH, THH, TTH, HTT, THT, TTT b) $\frac{3}{8}$

4 a) See page 155 b) See page 156

5 a)

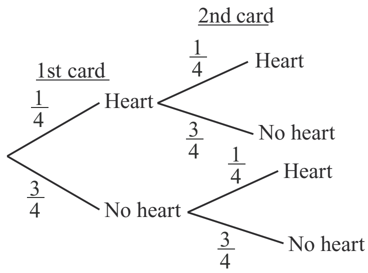


b) Relative frequency of:
pass, pass = $\frac{60}{160} = \frac{3}{8}$ or 0.375
pass, fail = $\frac{45}{160} = \frac{9}{32}$ or 0.28125
fail, pass = $\frac{30}{160} = \frac{3}{16}$ or 0.1875
fail, fail = $\frac{25}{160} = \frac{5}{32}$ or 0.15625

c) 300 × 0.375 = 113 people (to the nearest whole number)

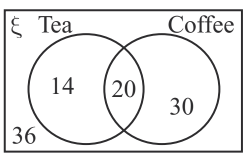
6 0.5

7



P(no hearts) = $\frac{9}{16}$

8 a)



b) $\frac{64}{100} = \frac{16}{25}$

9 A sample is part of a population. Samples need to be representative so that conclusions drawn from sample data can be applied to the whole population.

10 Descriptive data

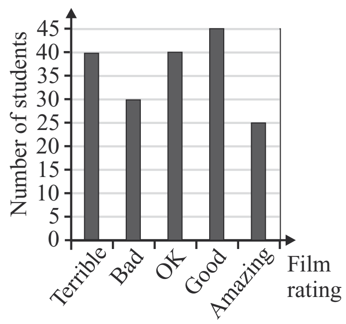
11

Pet	Tally	Frequency
Cat	𝍸 |||	8
Dog	𝍸		6
Rabbit	||||	4
Fish	||	2

12 Mode = 31, Median = 24, Mean = 22, Range = 39

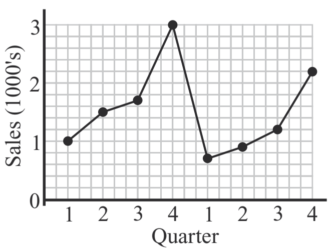
13 Count the number of symbols, then use the key to work out what frequency they represent.

14 E.g.



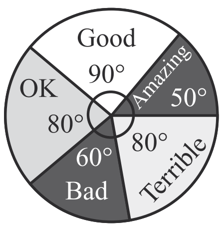
Alternatively, you could have drawn a pictogram.

15 a)

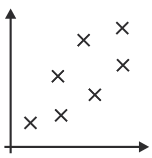


b) There is a seasonal pattern that repeats itself every 4 points. Sales are lowest in the first quarter and highest in the fourth quarter.

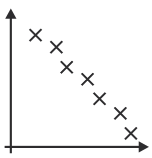
16 Draw a pie chart to show the proportions.



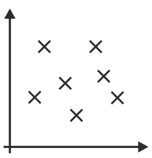
17 a)



b)



c)



18 a) Modal class is: $1.5 \leq y < 1.6$.

b) Class containing median is: $1.5 \leq y < 1.6$

c) Estimated mean = 1.58 m (to 2 d.p.)

19 Outliers can have a big effect on increasing or decreasing the value of the mean or range, so that it doesn't represent the rest of the data set very well.

20 Black cars were only owned by men and silver cars were only owned by women. So black cars were more popular amongst men and silver cars were more popular amongst women.
There are similar proportions of men and women owning blue and green cars. So blue and green cars are equally popular amongst men and women.
The proportion of men owning red cars was nearly double the proportion of women owning red cards. So red cars were almost twice as popular amongst men as women.

Practice Paper 1

1 $\frac{113}{1000}$ ***[1 mark]***

2 40 : 25 = 8 : 5 ***[1 mark]***

3 a) F ***[1 mark]***

b) $x = 3$ ***[1 mark]***

4 3.97 × 1000 = 3970 m ***[1 mark]***

5 a) –12, –8, –6, 2, 6 ***[1 mark]***

b) 2 – –8 = 10 ***[1 mark]***

6 5p × 28 = 140p
10p × 41 = 410p
140p + 410p = 550p
550p ÷ 50p = 11, so she gets 11 50p coins.
[3 marks available — 1 mark for the total of the 5p or 10p coins, 1 mark for the overall total, 1 mark for the final answer]

7 $1.2 - 0.2 \times 4 = 1.2 - 0.8 = 0.4$ ***[1 mark]***

So $\frac{1.2 - 0.2 \times 4}{0.05} = \frac{0.4}{0.05} = \frac{40}{5} = 8$ ***[1 mark]***

[2 marks available in total — as above]

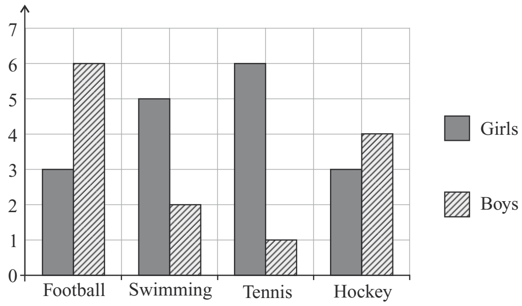
8 E.g. 6 (multiple of 3) + 6 (multiple of 6)
= 12 (not a multiple of 9) ***[1 mark]***

9 a) $9a + 7b$ ***[1 mark]***

b) $6a^2$ ***[1 mark]***

10 1% of £300 = £3, so 2% = £3 × 2 = £6 ***[1 mark]***
Interest for 4 years = £6 × 4 = £24 ***[1 mark]***
£300 + £24 = £324 ***[1 mark]***
[3 marks available in total — as above]

11 a) 3 + 6 + 5 + 2 + 6 + 1 + 3 = 26 pupils
The frequency of the final bar should be 30 – 26 = 4



[2 marks available — 2 marks for correctly drawing missing bar, otherwise 1 mark for attempting to add the seven frequencies shown on the chart]

b) Number of children whose favourite sport is swimming is 5 + 2 = 7. The probability is $\frac{7}{30}$

[2 marks available — 1 mark for the numerator (7) and 1 mark for the denominator (30)]

c) 2 boys chose swimming and 6 girls chose tennis, so the ratio is 2 : 6 = 1 : 3 in its simplest form.

[2 marks available — 2 marks for the correct answer, otherwise 1 mark for 2 : 6]

12 Geometric ***[1 mark]***
Rule: multiply the previous term by 4 ***[1 mark]***

13 a)

Pattern	Number of triangles	Number of dots	Number of lines
Pattern 1	1	3	3
Pattern 2	2	4	5
Pattern 3	3	5	7
Pattern 4	4	6	9

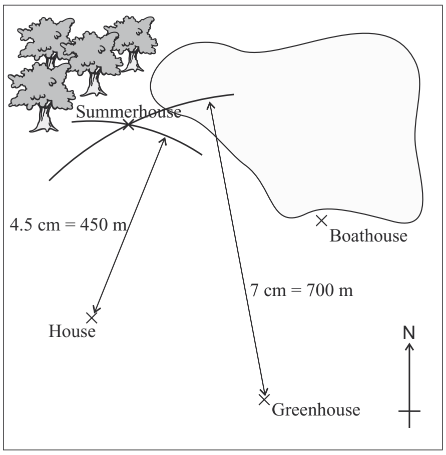
[1 mark]

b) The number of lines column increases by 2 each time. The column would continue 11, 13, 15, 17, 19, 21. Pattern 10 has 21 lines.
[2 marks available — 1 mark for finding the sequence, 1 mark for the correct number of lines in Pattern 10]
You could also find an expression for the nth term (2n + 1) and work out the value when put n = 10, (2 × 10) + 1 = 21.

c) (i) The number of dots is two more than the pattern number, so $D = n + 2$
[2 marks available — 2 marks for D = n + 2, otherwise 1 mark for n + 2, D – n = 2, or '2 more than the pattern number']
(ii) Number of dots in pattern 200 = 200 + 2 = 202 ***[1 mark]***

14 The number of tickets sold was:
Adults: (8 × 3) + 6 = 30
Child: (8 × 5) + 2 = 42
Senior: (8 × 2) + 4 = 20
Adult tickets: 30 × £9 = £270
Child tickets: 42 × £5 = £210
Senior tickets: 20 × £6.50 = £130
Total sales = 270 + 210 + 130 = £610
[6 marks available — 1 mark for each correct number of tickets sold (adult, child, senior), 1 mark for multiplying number of tickets sold by cost, 1 mark for attempting to add up the total sales, 1 mark for correct answer]

15 a) 067° (Accept answer between 065° and 069°) ***[1 mark]***
b) Distance on map = 4.3 cm
Actual distance = 4.3 × 100 = 430 metres
[2 marks available — 2 marks for an answer in the range 420 m-440 m, otherwise 1 mark for a measurement in the range 4.2 cm-4.4 cm]
c) Draw an arc of a circle with radius 4.5 cm from the house and 7 cm from the greenhouse.



[2 marks available — 2 marks for summerhouse plotted in the correct position with construction arcs shown, otherwise 1 mark for one correct arc or for the summerhouse in the correct position but construction arcs not shown]

16 The top tier needs 50% of 800 g = 400 g of sultanas ***[1 mark]***
The middle tier needs 75% of 800 g = 600 g of sultanas ***[1 mark]***
One wedding cake needs 800 + 400 + 600 = 1800 g of sultanas ***[1 mark]***
Five wedding cakes need 1800 g × 5 = 9000 g = 9 kg of sultanas ***[1 mark]***, so Angie does not have enough sultanas. ***[1 mark]***
[5 marks available in total — as above]

17 a) 69p rounds to 70p and 2.785 kg rounds to 3 kg to 1 s.f.
So estimate is 70p × 3 ***[1 mark]***
= 210p ***[1 mark]***
[2 marks available in total — as above]
If you've done 70p × 2.8 = 196p you'll still get the marks.
b) E.g. The estimate is bigger than the actual cost, as both numbers were rounded up. ***[1 mark]***

18 a) Read off the graph: £300 = \$540 ***[1 mark]***
b) After spending \$390, he is left with \$150 ***[1 mark]***
Convert to yuan: 150 × 6 = 900 yuan ***[1 mark]***
[2 marks available in total — as above]

19 $3^{-2} = \frac{1}{9}$ ***[1 mark]***
$k \times \frac{1}{9} = 4$, so $k = 4 \times 9 = 36$ ***[1 mark]***
[2 marks available in total — as above]

20 Angles in a square are 90°.
There are three squares around O, so 90° × 3 = 270°. ***[1 mark]***
Angles round a point add to 360°,
so angle AOB = (360 – 270) ÷ 3 = 30° ***[1 mark]***
The two base angles in an isosceles triangle are equal,
so angle OAB = (180 – 30) ÷ 2 ***[1 mark]***
= 75° ***[1 mark]***
[4 marks available in total — as above]

21 $1\frac{2}{3} \times 1\frac{5}{8} = \frac{5}{3} \times \frac{13}{8} = \frac{65}{24} = 2\frac{17}{24}$
[3 marks available – 1 mark for converting both numbers to improper fractions, 1 mark for multiplying, 1 mark for the correct answer]

22 594 000 000 000 = 5.94×10^{11} ***[1 mark]***

23 Year 9 — $\frac{9}{20} = \frac{45}{100} = 45\%$
Year 10 — 49%
Year 11 — $\frac{12}{12+13} = \frac{12}{25} = \frac{48}{100} = 48\%$
So Year 10 has the largest proportion of girls.
[3 marks available — 2 marks for converting two proportions to the same form as the third, and 1 mark for the correct answer.]
You could convert any two proportions to the form of the third, e.g. convert Year 10 and Year 11 into fractions.

24 Shape A: Area = $\pi \times 4^2 = 16\pi$ cm²
Shape B: Area = $\frac{80}{360} \times \pi \times 6^2 = 8\pi$ cm²
$16\pi = 2 \times 8\pi$ so the area of A is twice the area of B.
[4 marks available — 1 mark for the area of shape A, 1 mark for the correct method to find the area of shape B, 1 mark for the area of shape B, 1 mark for showing that the area of shape A is twice the area of shape B]

25 a) $AC^2 = AD^2 + DC^2 = 3^2 + 4^2 = 9 + 16 = 25$
So $AC = \sqrt{25} = 5$ cm
$x^2 = AC^2 + AB^2 = 5^2 + 12^2 = 25 + 144 = 169$
So $x = \sqrt{169} = 13$ cm
[4 marks available — 1 mark for using Pythagoras' theorem to find the length of AC, 1 mark for the correct length of AC, 1 mark for using Pythagoras' theorem to find the length of x, 1 mark for the correct length of x]
b) Area of triangle ACD = 0.5 × 4 × 3 = 6 cm²
Area of triangle ABC = 0.5 × 5 × 12 = 30 cm²
Area of quadrilateral $ABCD$ = 6 + 30 = 36 cm²
[2 marks available — 1 mark for finding the areas of triangles ACD and ABC, 1 mark for adding to find the area of the quadrilateral]

26 $2x + y = 10 \xrightarrow{\times 2} 4x + 2y = 20$ ***[1 mark]***

$$\begin{array}{rl} & 4x + 2y = 20 \\ - & \underline{3x + 2y = 17} \\ & x = 3 \end{array}$$
[1 mark]

$$\begin{aligned} 4(3) + 2y &= 20 \\ 2y &= 20 - 12 \\ 2y &= 8 \\ y &= 4 \end{aligned}$$
[1 mark]

[3 marks available in total — as above]
You could have found the value of y first then used it to find x.

Practice Paper 2

1 $\frac{3}{5} = 0.6 = 60\%$ ***[1 mark]***

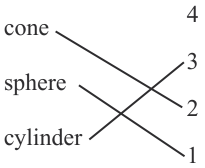
2 $20 + 4 = 24$
$24 \div 2 = 12$ ***[1 mark]***

3

Menu Item	Number Ordered	Cost per Item	Total
Tea	2	£1.25	£2.50
Coffee	6 ***[1 mark]***	£1.60	£9.60
Cake	4	£1.30 ***[1 mark]***	£5.20
Tip			£2.50
		Total cost	£19.80 ***[1 mark]***

[3 marks available in total — as above]

4 a)



[2 marks available — 2 marks for all lines correct, otherwise 1 mark for 2 lines correct]

b) 6 ***[1 mark]***

5 a) 107.3158498 ***[1 mark]***
Your calculator may display more or fewer digits than this.

b) 107.32 ***[1 mark]***
If you got a) wrong but rounded it correctly, you'll still get the mark for part b).

6 The possible numbers are:

4356 4536
5346 5364
5436 5634
6354 6534

[2 marks available — 2 marks if all 8 possible numbers are given with no errors or repetitions, otherwise 1 mark if at least 5 of the possible numbers are listed]

7 64 (4^3 and 8^2) ***[1 mark]***

8 a) $4(a + 2) = 4a + 8$ ***[1 mark]***

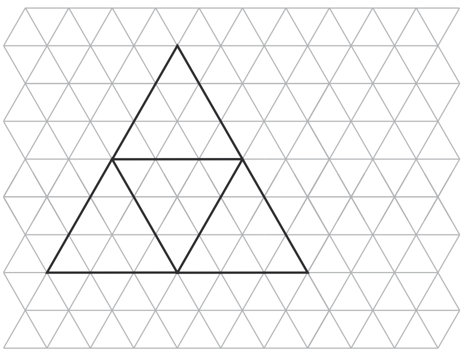
b) $y^2 + 5y = y(y + 5)$ ***[1 mark]***

9 a) Rewrite data in order: 5, 6, 6, 7, 9, 11, 12
Median is the middle (4th) value = 7 years ***[1 mark]***

b) Mean = $\frac{6+12+9+6+5+7+11}{7}$ ***[1 mark]***
$= \frac{56}{7} = 8$ years ***[1 mark]***
[2 marks available in total — as above]

10 a) Two equilateral triangles join together to form a rhombus.
So 2 lines of symmetry. ***[1 mark]***

b) E.g.



[1 mark]

11 185% of £3500 = 1.85 × 3500 ***[1 mark]***
= £6475 ***[1 mark]***
[2 marks available in total — as above]
You could also build up to 185%, by finding 50%, 5% etc. and adding these to the original value.

12 Method 1: 2 × 27 = 54 miles
Expenses = 54 × 0.40 = £21.60 ***[1 mark]***
Method 2: Expenses = (8 × 0.40) + 17.60 = £20.80 ***[1 mark]***
She should travel by the cheaper method, which is Method 2 (by car and train). ***[1 mark]***
[3 marks available in total — as above]

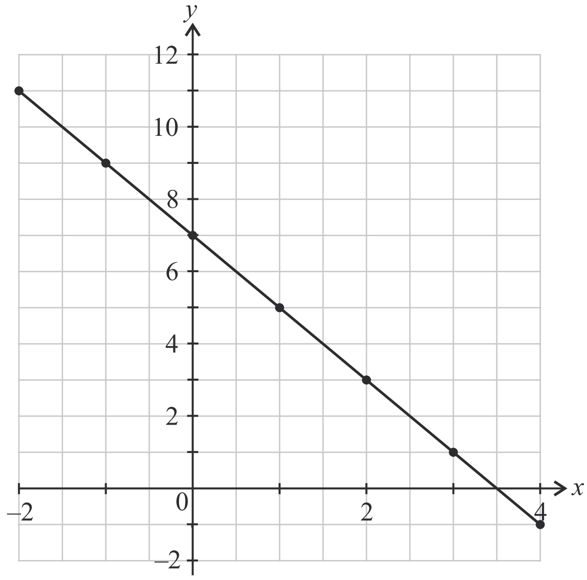
13 There are $\frac{3}{4} \times 28 = 21$ red cars and
$\frac{3}{8} \times 16 = 6$ red motorbikes ***[1 mark for both]***.
So there are 21 + 6 = 27 red vehicles in total ***[1 mark]***.
The probability of picking a car from the red vehicles is $\frac{21}{27} = \frac{7}{9}$ ***[1 mark]***.
[3 marks available in total — as above]

14 a)

x	–2	–1	0	1	2	3	4
y	11	9	7	5	3	1	–1

[2 marks available — 2 marks for all entries correct, otherwise 1 mark for at least 2 correct entries]

b)



[2 marks available — 1 mark for plotting at least 3 points correctly, 1 mark for a correct straight line]

c) –2 ***[1 mark]***

15 $3(2x - 4) = 2x + 8$
$6x - 12 = 2x + 8$ ***[1 mark]***
$4x = 20$ ***[1 mark]***
$x = 5$ ***[1 mark]***
[3 marks available in total — as above]

16 He has multiplied the denominator and numerator by 5, but he should have just multiplied the numerator by 5. ***[1 mark]***

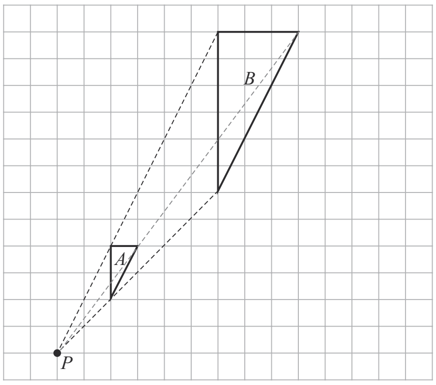
17 E.g. The vertical scale does not begin at 0 so the graph could be misinterpreted.
The horizontal scale does not increase by even amounts.
The graph is difficult to read due to the thickness of the line.
[3 marks available — 1 mark for each correct comment]

18 Angle $EBF = 90°$ ***[1 mark]***
Angles on a straight line add up to 180°,
so angle $ABE = 180 - 90 - 39 = 51°$ ***[1 mark]***
Angles ABE and DEH are corresponding angles,
so angle $DEH = 51°$ ***[1 mark]***
[3 marks available in total – as above]

19 a) $L = 2(3x + 1) + 2(2x - 3) - 2$ ***[1 mark]***
$= 6x + 2 + 4x - 6 - 2 = 10x - 6$ ***[1 mark]***
[2 marks available in total — as above]

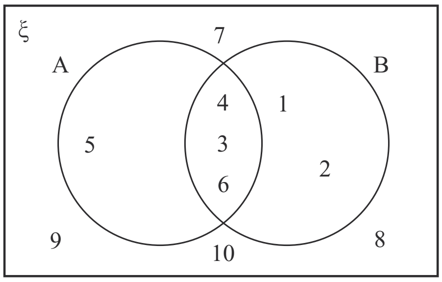
b) $L = 10x - 6 = 2(5x - 3)$, so it is always even as it can be written as 2 × a whole number, where the whole number is $(5x - 3)$
[2 marks available — 1 mark for writing the expression as 2(5x – 3) and 1 mark for explaining why this is always an even number.].

20



[2 marks available — 2 marks for image totally correct, otherwise 1 mark for 2 vertices in the correct position or for an image of the correct size but positioned incorrectly on the grid]

21 Elements of A are 3, 4, 5, 6
Elements of B are 1, 2, 3, 4, 6

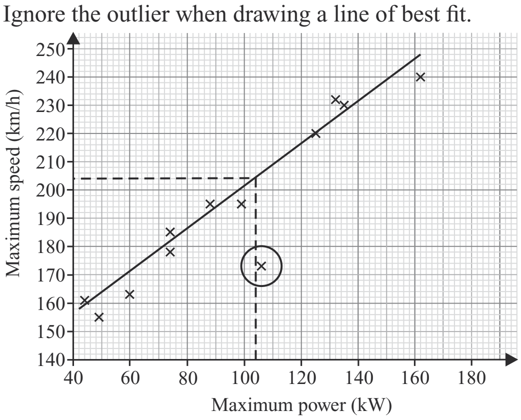


[3 marks available — 3 marks for a completely correct diagram, otherwise 1 mark for identifying the elements of sets A and B and 1 mark for correct elements in the intersection.]

22 8 litres of orangeade costs (3 × £1.60) + (5 × £1.20) = £10.80
So 1 litre of orangeade costs £10.80 ÷ 8 = £1.35
18 litres of orangeade cost £1.35 × 18 = £24.30
[4 marks available — 1 mark for using the ratio to find the cost of 8 litres of orangeade, 1 mark for dividing by 8, 1 mark for multiplying by 18 and 1 mark for the correct final answer.]
There are several different methods you could use here. Any correct method with full working shown and a correct final answer would get 4 marks.

23 $y = \dfrac{x^2 - 2}{5}$
$5y = x^2 - 2$ ***[1 mark]***
$5y + 2 = x^2$
$x = \pm\sqrt{5y + 2}$ ***[1 mark]***
[2 marks available in total— as above]

24 a) 125 kW ***[1 mark]***
b) See diagram in part d).
[1 mark for circling the point shown]
c) Strong positive correlation ***[1 mark]***
d) Ignore the outlier when drawing a line of best fit.



Maximum speed = 204 km/h (allow ± 2).
[2 marks available — 1 mark for drawing a line of best fit (ignoring the outlier), 1 mark for accurately reading from your graph the speed corresponding to a power of 104 kW]

e) 190 kW lies outside of the range of data plotted on the scatter graph. ***[1 mark]***

25 Ollie's expression:
$(x + 4)^2 - 1 = (x + 4)(x + 4) - 1 = x^2 + 4x + 4x + 16 - 1$ ***[1 mark]***
$= x^2 + 8x + 15$ ***[1 mark]***
Amie's expression:
$(x + 3)(x + 5) = x^2 + 3x + 5x + 15 = x^2 + 8x + 15$, ***[1 mark]***
so the two expressions are equivalent.
[3 marks available in total — as above]

26 a) Angle for car = 360 – 162 – 36 – 45 = 117° ***[1 mark]***
No. of office assistants that travel by car is = $\dfrac{117}{360} \times 80$
= 26 ***[1 mark]***
[2 marks available in total — as above]
b) Total number of staff that travel by car
= 26 + 18 = 44 ***[1 mark]***
44 represents 40% of the staff in the company, so the total number of staff (100%) is 44 ÷ 40 × 100 = 110 ***[1 mark]***
[2 marks available in total — as above]

27 a) Mass of cylinder = volume × density
= 1180 × 2.7 = 3186 g
[2 marks available — 1 mark for using the density formula correctly and 1 mark for the correct final answer.]
b) Mass of cube = mass of cylinder = 3186 g
Volume of cube = mass ÷ density
= 3186 ÷ 10.5 = 303.428... cm^3
Side length = $\sqrt[3]{303.428...}$
= 6.719... cm = 6.7 cm (to 2 s.f.)
[4 marks available — 1 mark for using the density formula correctly, 1 mark for finding the volume of the cube, 1 mark for attempting to find the cube root of the volume and 1 mark for the correct final answer.]

28 $\sin x = \dfrac{18}{24}$ ***[1 mark]***
$x = \sin^{-1}\left(\dfrac{18}{24}\right)$ ***[1 mark]*** = 48.590... = 48.6 (to 1 d.p.) ***[1 mark]***
[3 marks available in total — as above]

Practice Paper 3

1 $0.4 > 0.34$
$\dfrac{3}{4} = 0.75$
$7\% < 0.7$
[2 marks available — 2 marks for all correct signs, otherwise 1 mark for 2 signs correct]

2 0.404 ***[1 mark]***

3 20 800 ***[1 mark]***

4 –16.2 ***[1 mark]***

5 8 ***[1 mark]***

6 a)

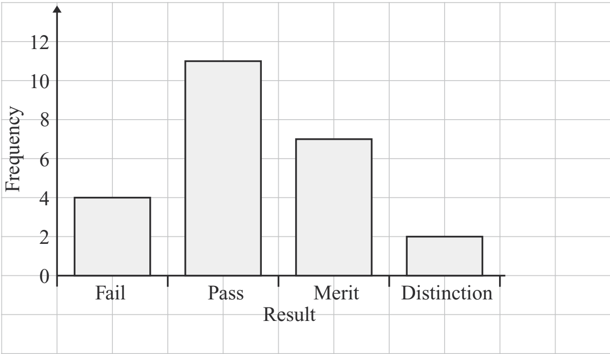
Result of exam	Tally	Frequency
Fail	||||	4
Pass	卌 卌 |	11
Merit	卌 ||	7
Distinction	||	2
	TOTAL:	**24**

[2 marks available — 1 mark for tally marks fully correct, 1 mark for correct frequencies]

b) $\dfrac{4}{24} = \dfrac{1}{6}$

[2 marks available — 2 marks for correct answer, otherwise 1 mark for $\frac{4}{24}$ or $\frac{2}{12}$]

c) E.g.



[3 marks available — 1 mark for correct vertical axis scale and labelling, 1 mark for 4 labelled bars of equal widths, 1 mark if the bars all have heights that agree with the frequencies found in (a)]

7 a) Impossible ***[1 mark]***
b) There are 5 blue pencils, so evens. ***[1 mark]***

8 a) Trapezium ***[1 mark]***
b) Parallelogram ***[1 mark]***

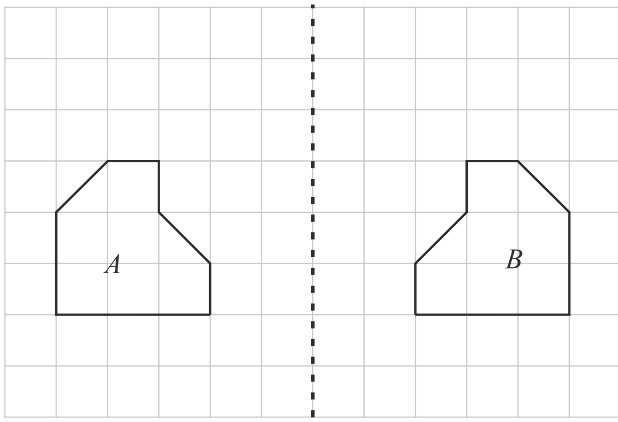
9 $\begin{pmatrix}-5\\0\end{pmatrix}$ ***[1 mark]***

10 Square numbers: 1, 4, 9, 16, 25, 36...
Prime numbers: 2, 3, 5, 7, 11, 13, 17...
$2 \times 5 \times 9 = 90$, so Rick's numbers are 2, 5, and 9.
[3 marks available — 3 marks for correct answer, otherwise 1 mark for listing at least 2 square numbers or 2 prime numbers, 1 mark for 3 numbers which multiply to give 90, but which aren't 1 square number and 2 prime numbers.]
You could also answer this by using a factor tree to find that $90 = 2 \times 3 \times 3 \times 5 = 2 \times 9 \times 5$

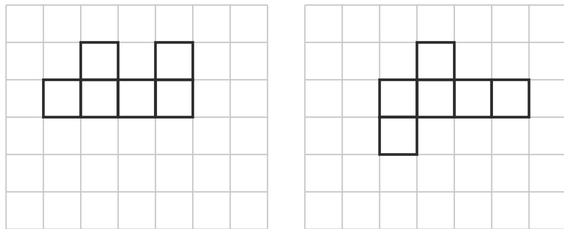
11 1 cupcake needs $132 \div 12 = 11$ g sugar.
So 30 cupcakes need $30 \times 11 = 330$ g sugar.
[2 marks available — 1 mark for a correct method, 1 mark for the correct answer]

12 Multiples of 12: 12, 24, 36, 48, 60 …
Multiples of 10: 10, 20, 30, 40, 50, 60, …
The smallest number they could each have bought is 60.
This is 6 packets of shortbread biscuits.
[3 marks available — 3 marks for the correct answer, otherwise 2 marks for finding that smallest number of biscuits they can buy is 60, or 1 mark for listing at least three multiples of 12 or 10]

13 Mary needs $2 \times 30 = 60$ scones, 30 tubs of clotted cream and 30 small pots of jam. ***[1 mark]***
$60 \div 10 = 6$ packs of scones, costing $6 \times £1.35 = £8.10$ ***[1 mark]***
$30 \div 6 = 5$ tubs of clotted cream,
costing $5 \times £2.95 = £14.75$ ***[1 mark]***
30 pots of jam costing $30 \times £0.40 = £12$ ***[1 mark]***
The total cost is $£8.10 + £14.75 + £12 = £34.85$,
so yes, she has enough money. ***[1 mark]***
[5 marks available in total — as above]

14


[2 marks available — 1 mark for each correct shape]

15


[2 marks available — 1 mark for each correct drawing]

16 Each trapezium has these dimensions:
Parallel sides: 12 cm and 7 cm
Height: 6 cm
So area $= \frac{1}{2}(12 + 7) \times 6$ ***[1 mark]***
$= 9.5 \times 6 = 57$ cm^2 ***[1 mark]***
[2 marks available in total — as above]

17 Box B contains $c + 4$ chocolates
Box C contains $2(c + 4) = 2c + 8$ chocolates
[1 mark for both expressions correct]
Total number of chocolates in Boxes A, B and C
$= c + c + 4 + 2c + 8 = 4c + 12$ ***[1 mark]***
So $4c + 12 = 60$ ***[1 mark]***
$4c = 48$
$c = 12$, so the number of chocolates in Box A is 12 ***[1 mark]***, the number in Box B is $12 + 4 = 16$ and the number in Box C is $2 \times 16 = 32$. ***[1 mark for boxes B and C correct]***
[5 marks available in total — as above]

18 a) $y \times y \times y = y^3$ ***[1 mark]***
b) Subtract the powers: $n^6 \div n^2 = n^4$ ***[1 mark]***
c) Multiply the powers: $(a^4)^3 = a^{12}$ ***[1 mark]***

19 Use Pythagoras' theorem on the triangle formed by one step and the ramp. Let x be the length of the ramp.
$20^2 + 55^2 = x^2$ ***[1 mark]***
$x^2 = 3425$
So, $x = \sqrt{3425}$ cm ***[1 mark]***
Total length of the ramp $= 2 \times \sqrt{3425}$ cm
$= 117.046...$ cm
$= 117$ cm (3 s.f.) ***[1 mark]***
[3 marks available in total — as above]

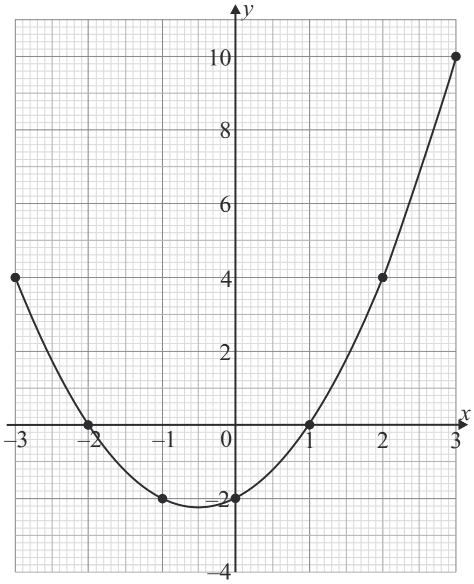
20 a) Dave's speed is $180 \div 3 = 60$ km/h ***[1 mark]***
Olivia's speed is $60 + 15 = 75$ km/h
So it takes her $180 \div 75 = 2.4$ hours ***[1 mark]***
$= 2$ hours 24 minutes ***[1 mark]***
[3 marks available in total — as above]
b) E.g. If Olivia drove a different distance at the same average speed, her journey time would be different ***[1 mark]***.

21 a)

x	-3	-2	-1	0	1	2	3
y	4	0	-2	-2	0	4	10

[2 marks available — 2 marks for all three values correct, otherwise 1 mark if one value is correct]

b)



[2 marks available — 1 mark for plotting the points from your table and 1 mark for joining with a smooth curve to form a U-shaped curve]

22 2 + 3 + 5 = 10 'parts' in the ratio. Angles in a triangle add to 180°, so 10 parts = 180° and 1 part = 18°.
So, the three angles in the triangle are (2 × 18°) = 36°, (3 × 18°) = 54° and (5 × 18°) = 90°.
One angle is 90°, so the triangle is a right-angled triangle.
[3 marks available — 1 mark for finding the size of one part of the ratio, 1 mark for finding the size of at least one angle in the triangle and 1 mark for showing that one angle is 90°.]

23 a) 25% higher than £120 000 = 1.25 × £120 000
= £150 000, so House 4 ***[1 mark]***

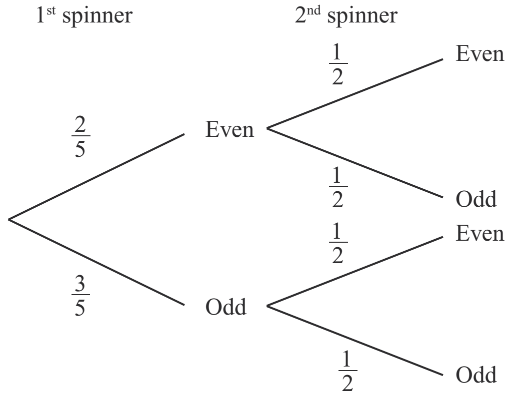
b) £161 280 – £144 000 = £17 280 ***[1 mark]***
% change = $\frac{17\,280}{144\,000}$ × 100 ***[1 mark]*** = 12% ***[1 mark]***
[3 marks available in total — as above]

24 Work back in Anna's sequence to find her 1st term:
17 – 3 – 3 – 3 = 8 ***[1 mark]***
So the 1st term of Carl's sequence is 8 ÷ 2 = 4 ***[1 mark]***
So the first five terms of Carl's sequence are 4, 10, 16, 22, 28.
His 5th term is 28. ***[1 mark]***
[3 marks available in total — as above]

25 a) $x^2 + 7x - 18 = (x + 9)(x - 2)$
[2 marks available — 2 marks for the correct factorisation, otherwise 1 mark for an answer of the form (x ± a)(x ± b) where a and b are numbers that multiply to make 18]

b) $(x + 9)(x - 2) = 0$
$x = -9$ or $x = 2$
[1 mark for correct solutions using factorisation in (a)]

26 a)



[2 marks available — 1 mark for the correct probabilities for the first spinner, 1 mark for the correct probabilities for the second spinner]

b) P(two odd numbers) = $\frac{3}{5} \times \frac{1}{2}$ ***[1 mark]***
= $\frac{3}{10}$ ***[1 mark]***
[2 marks available in total — as above]

27 Change in y = 8 – (–7) = 15
Change in x = 3 – (–2) = 5
So gradient = 15 ÷ 5 = 3
So $y = mx + c$ becomes $y = 3x + c$
Put in $x = 3$ and $y = 8$ to find the value of c:
8 = 3(3) + c, which means c = 8 – 9 = –1
The equation of the line is $y = 3x - 1$.
[4 marks available — 1 mark for a correct method for finding the gradient, 1 mark for the correct gradient, 1 mark for putting one point into the equation, 1 mark for the correct answer]

28

Weight (w g)	**Frequency**	**Mid-interval value**	**Frequency × mid-interval value**
800 ≤ w < 1000	5	900	4500
1000 ≤ w < 1200	8	1100	8800
1200 ≤ w < 1400	9	1300	11 700
1400 ≤ w < 1600	3	1500	4500
Total	25	—	29 500

Estimate of mean 29 500 ÷ 25 = 1180 g.
[3 marks available — 1 mark for attempting to find the mid-interval and frequency × mid-interval values, 1 mark for dividing the sum of the frequency × mid-interval values by 25, 1 mark for the correct answer.]

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Index

Formulas in the Exams

GCSE Maths uses a lot of formulas — that's no lie. You'll be scuppered if you start trying to answer a question without the proper formula to start you off. Thankfully, CGP is here to explain all things formula-related.

You're Given these Formulas

Fortunately, those lovely examiners give you some of the formulas you need to use.

For a sphere radius *r*, or a cone with base radius *r*, slant height *l* and vertical height *h*:

Volume of sphere $= \frac{4}{3}\pi r^3$

Volume of cone $= \frac{1}{3}\pi r^2 h$

Surface area of sphere $= 4\pi r^2$

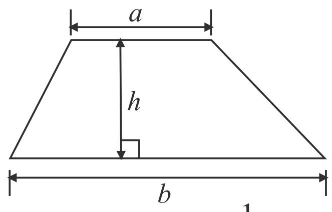
Curved surface area of cone $= \pi r l$

And, actually, that's your lot I'm afraid. As for the rest...

Learn All The Other Formulas

Sadly, there are a load of formulas which you're expected to be able to remember straight out of your head. Basically, any formulas in this book that aren't in the box above, you need to learn. There isn't space to write them all out below, but here are the highlights:

Area of parallelogram = base × vertical height



Area of trapezium $= \frac{1}{2}(a + b)h$

Exterior angle of regular polygon $= \frac{360°}{n}$

Sum of interior angles of any polygon $= (n - 2) \times 180°$

where n is the number of sides

For a circle with radius *r*:

Circumference $= 2\pi r$

Area $= \pi r^2$

Volume of prism = cross-sectional area × length

Volume of cylinder = area of circle × height $= \pi r^2 h$

Surface area of cylinder $= 2\pi rh + 2\pi r^2$

Compound Growth and Decay:

$N = N_0(\text{multiplier})^n$

Area of sector $= \frac{x°}{360°} \times$ Area of full circle

Length of Arc $= \frac{x°}{360°} \times$ Circumference of full circle

For a right-angled triangle:

Pythagoras' theorem: $a^2 + b^2 = c^2$

Trigonometry ratios:

$\sin x = \frac{O}{H}$, $\cos x = \frac{A}{H}$, $\tan x = \frac{O}{A}$

Compound Measures:

Speed $= \frac{\text{Distance}}{\text{Time}}$ Density $= \frac{\text{Mass}}{\text{Volume}}$ Pressure $= \frac{\text{Force}}{\text{Area}}$

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